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STATISTICAL TESTING OF DIRECTIONS OBSERVATIONS INDEPENDENCE

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Abstract

Independence of observations is often assumed when adjusting geodetic network. Unlike the distance observations, no dependency of environmental conditions is known for horizontal direction observations. In order to determine the dependency of horizontal direction observations, we established test geodetic network of a station and four observation points. Measurements of the highest possible accuracy were carried out using Leica TS30 total station along with precise prisms GPH1P. Two series of hundred sets of angles were measured, the first one in bad observation conditions. Using different methods, i.e. variance-covariance matrices, χ^2 test and analyses of time series, the independency of measured directions, reduced directions and horizontal angles were tested. The results show that the independence of horizontal direction observations is not obvious and certainly not in poor conditions. In this case, it would be appropriate for geodetic network adjustments to use variance-covariance matrix calculated from measurements instead of diagonal variance-covariance matrix.

Keywords: statistical independence, directions measurements, variance-covariance matrices, correlations, tests for randomness.

Introduction

When reckoning or adjusting a geodetic network, the independence of geodetic measurement (and often also equal accuracies) is assumed. Observed quantities in terrestric geodetic network are horizontal angles, zenith distances and slope distances. It is known that observed distances have to be reduced before used in adjustment, because they are highly dependent on observation conditions [**6**]. No such direct dependency is known for observations of horizontal angles or better horizontal directions. The main objective of this research is to carry out some representative geodetic observations of horizontal directions and test their statistical independence.

Test geodetic network was established on the roof of the building of the Faculty of Civil and Geodetic Engineering at the University of Ljubljana. The network consisted of three concrete pillars and two tripods. It was established with a primary goal to realistically represent networks used in practice. Measurement equipment of the highest possible accuracy was used. We measured and recorded two series of hundred sets of angles. The first series was measured in typical bad observation conditions. The measurements were carried out in the morning when the enviromental conditions were changing rapidly. Furthermore, the instrument was not acclimatized. We assume that the observation conditions of the second series were optimal. The measurements were carried out in the afternoon. The weather conditions were stable and the instrument was acclimatized. The stability of reflectors was ensured during the measurements.

The results of measurements were two series of hundred repeated observations of horizontal directions. Reduced directions were calculated as meant for sets of angle method [**1**]. Angles, as difference between successive observed directions, may also be used in adjustment [**1**], [**4**], and were calculated as well.

The data obtained during the observations were statistically tested. The following statistical methods were used: variance-covariance matrices, χ^2 test and time series analysis, where graphical presentation, autocorrelation functions and tests for randomness were included.

Methods

High accuracy measurements

For high accuracy angle observations in geodetic network, a set of angle method is usually used. Measuring in both faces eliminates most of the instrumental errors. However, environmental and random errors remain, but usually we assume they are not significant.

For our task geodetic network was established on the faculty roof. It consisted of one station point and four observation points. The position of the points is shown in Figure 1.

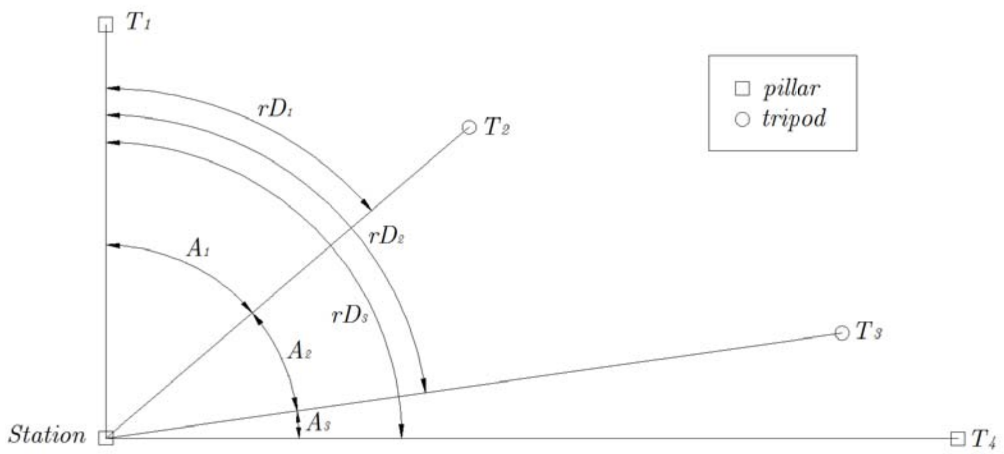

Fig. 1. Reduced directions and angles

Measurements were carried out using Leica TS30 total station and four precise prisms Leica GPH1P. Two series of hundred sets of angles each were measured using automatic target recognition (ATR) system. Specified accuracy of ATR horizontal direction observations is 1 mm or $1''$. At distances greater than 200 m, 1 mm represents less than $1''$.

Technical characteristics of Leica TS30 are shown in Table 1.

Table 1. *Tehnical characteristics of Leica TS30*

Instrument	
Operating range	-20 °C — $+50$ °C
Resolution of electronic level	$2''$
Theodolite	
Principle of reading	code principle
Standard deviation $\sigma_{ISO-THEO}$	$0.5''$
Accuracy of ATR system $\sigma_{ISO-THEO}$	$1''$ or 1 mm
Electronic distance meter	
Reference contitions: n_0, p_0, t_0	1.0002863, 1013.25 hPa, 12 °C
Standard deviation $\sigma_{ISO-EDM}$	0.6 mm ; 1 ppm

The mean values of measured directions from both faces were calculated for each set

$$D_{kij} = \frac{D_{kijI} + D_{kijII} \pm \pi}{2}, \tag{1}$$

where $k = 1, 2$ denotes series, $i = 1, 2, ..., 100$ denotes sets, $j = 1, 2, 3, 4$ denotes points $T1$-$T4$ and I, II denotes faces. Reduced directions and angles were also calculated

$$rD_{kij} = D_{kij} - D_{ki1}; \quad j = 2, 3, 4, \tag{2}$$

$$A_{ki1} = D_{ki2} - D_{ki1} \qquad A_{ki2} = D_{ki3} - D_{ki2} \qquad A_{ki3} = D_{ki4} - D_{ki3}. \tag{3}$$

Their meaning is presented in Figure 1.

Variance-covariance matrix and correlation matrix

Accuracy estimation of multiple observations is usually represented by variance-covariance matrix

$$\mathbf{\Sigma} = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \dots & \sigma_{1m} \\ \sigma_{21} & \sigma_2^2 & \sigma_{23} & \dots & \sigma_{2m} \\ \sigma_{31} & \sigma_{32} & \sigma_3^2 & \dots & \sigma_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sigma_{m1} & \sigma_{m2} & \sigma_{m3} & \dots & \sigma_m^2 \end{bmatrix},$$

where σ_{ij} are estimated covariances between i-th and j-th observation and σ_i^2 are estimated variances of the i-th observation. Nondiagonal elements of variance-covariance matrix should be close to zero for independent observations. Values in variance-covariance matrix depend on measurement units. Since we are interested only in ratios between values, we calculate correlation matrix

$$\rho_{ij} = \frac{\sigma_{ij}}{\sigma_i \sigma_j}. \tag{4}$$

When the value of correlation coefficient ρ_{ij} is close to 1 or -1, it means strong linear dependence between i-th and j-th observation. A form of correlation matrix is

$$\mathbf{R} = \begin{bmatrix} 1 & \rho_{12} & \rho_{13} & \dots & \rho_{1m} \\ \rho_{21} & 1 & \rho_{23} & \dots & \rho_{2m} \\ \rho_{31} & \rho_{32} & 1 & \dots & \rho_{3m} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \rho_{m1} & \rho_{m2} & \rho_{m3} & \dots & 1 \end{bmatrix}.$$

In the case of independent observations $\mathbf{R}$ is an identity matrix. Reduced directions and angles are functions of observed directions as shown by equations (2) and (3). Considering the variance-covariance propagation

$$\mathbf{\Sigma}_y = \mathbf{J}\mathbf{\Sigma}_x\mathbf{J}^T, \tag{5}$$

where $\mathbf{J}$ is the Jacobian, we derive to variance-covariance matrix for reduced directions and angles.

If the variance-covariance matrix of observed directions was

$$\Sigma_D = \begin{bmatrix} \sigma_1^2 & \sigma_{12} & \sigma_{13} & \sigma_{14} \\ \sigma_{12} & \sigma_2^2 & \sigma_{23} & \sigma_{24} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 & \sigma_{34} \\ \sigma_{14} & \sigma_{24} & \sigma_{34} & \sigma_4^2 \end{bmatrix}, \tag{6}$$

then for reduced directions it would be

$$\Sigma_{rD_{theor}} = \begin{bmatrix} \sigma_1^2 + \sigma_2^2 - 2\sigma_{12} & \sigma_1^2 + \sigma_{23} - \sigma_{13} - \sigma_{12} & \sigma_1^2 + \sigma_{24} - \sigma_{14} - \sigma_{12} \\ \sigma_1^2 + \sigma_{23} - \sigma_{12} - \sigma_{13} & \sigma_1^2 + \sigma_3^2 - 2\sigma_{13} & \sigma_1^2 + \sigma_{34} - \sigma_{14} - \sigma_{13} \\ \sigma_1^2 + \sigma_{24} - \sigma_{12} - \sigma_{14} & \sigma_1^2 + \sigma_{34} - \sigma_{13} - \sigma_{14} & \sigma_1^2 + \sigma_4^2 - 2\sigma_{14} \end{bmatrix} \tag{7}$$

and for angles it would be

$$\Sigma_{A_{theor}} = \begin{bmatrix} \sigma_1^2 + \sigma_2^2 - 2\sigma_{12} & \sigma_{12} + \sigma_{23} - \sigma_{13} - \sigma_2^2 & \sigma_{13} + \sigma_{24} - \sigma_{14} - \sigma_{23} \\ \sigma_{12} + \sigma_{23} - \sigma_2^2 - \sigma_{13} & \sigma_2^2 + \sigma_3^2 - 2\sigma_{23} & \sigma_{23} + \sigma_{34} - \sigma_{24} - \sigma_3^2 \\ \sigma_{13} + \sigma_{24} - \sigma_{23} - \sigma_{14} & \sigma_{23} + \sigma_{34} - \sigma_3^2 - \sigma_{24} & \sigma_3^2 + \sigma_4^2 - 2\sigma_{34} \end{bmatrix}. \tag{8}$$

Assuming the independence of observation, all nondiagonal elements equal zero, therefore equations simplify to

$$\tilde{\Sigma}_D = \begin{bmatrix} \sigma_1^2 & 0 & 0 & 0 \\ 0 & \sigma_2^2 & 0 & 0 \\ 0 & 0 & \sigma_3^2 & 0 \\ 0 & 0 & 0 & \sigma_4^2 \end{bmatrix}, \tag{9}$$

$$\tilde{\Sigma}_{rD_{theor}} = \begin{bmatrix} \sigma_1^2 + \sigma_2^2 & \sigma_1^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 + \sigma_3^2 & \sigma_1^2 \\ \sigma_1^2 & \sigma_1^2 & \sigma_1^2 + \sigma_4^2 \end{bmatrix}, \tag{10}$$

$$\tilde{\Sigma}_{A_{theor}} = \begin{bmatrix} \sigma_1^2 + \sigma_2^2 & -\sigma_2^2 & 0 \\ -\sigma_2^2 & \sigma_2^2 + \sigma_3^2 & -\sigma_3^2 \\ 0 & -\sigma_3^2 & \sigma_3^2 + \sigma_4^2 \end{bmatrix}. \tag{11}$$

Test χ^2 - Contingency table

For testing statistical independence of direction observations χ^2 test is used [**5**]. Hypotheses are:
H_0 : Variables X and Y are independent.
H_1 : Variables X and Y are dependent.
We have four observed directions and each of them is observed a hundred times ($n = 100$). Observed directions represent random variables. Since χ^2 test allows us to test independence of only two random variables at a time, we are testing all six possible combinations of two directions, denoted X and Y. Elements of each variable X and Y are classified by their size into $k_x = 5$ and $k_y = 5$ classes, respectively. Class boundaries were defined uniformly from minimum to maximum value of observations. The numbers of occurances of random vector X_t, Y_t in i, j-th class are entered in Table 2.

Table 2. *Empirical contingency table*

		X					
	class	1	2	3	4	5	$\sum Y$
	1	$\hat{n}_{11}$	$\hat{n}_{12}$	$\hat{n}_{13}$	$\hat{n}_{14}$	$\hat{n}_{15}$	$\hat{n}_{1Y}$
	2	$\hat{n}_{21}$	$\hat{n}_{22}$	$\hat{n}_{23}$	$\hat{n}_{24}$	$\hat{n}_{25}$	$\hat{n}_{2Y}$
Y	3	$\hat{n}_{31}$	$\hat{n}_{32}$	$\hat{n}_{33}$	$\hat{n}_{34}$	$\hat{n}_{35}$	$\hat{n}_{3Y}$
	4	$\hat{n}_{41}$	$\hat{n}_{42}$	$\hat{n}_{43}$	$\hat{n}_{44}$	$\hat{n}_{45}$	$\hat{n}_{4Y}$
	5	$\hat{n}_{51}$	$\hat{n}_{52}$	$\hat{n}_{53}$	$\hat{n}_{54}$	$\hat{n}_{55}$	$\hat{n}_{5Y}$
	$\sum X$	$\hat{n}_{X1}$	$\hat{n}_{X2}$	$\hat{n}_{X3}$	$\hat{n}_{X4}$	$\hat{n}_{X5}$	n

On the other hand, theoretical contingency table consists of the number of random vector X_t, Y_t which would lie in i, j-th class if variables X and Y were independent. Values are gained as a product of number of all vectors X_t, Y_t and probability that X_t, Y_t lays in i, j-th class.

$$n_{ij} = nP[X \in i \cap Y \in j] \tag{12}$$

Assuming the independence of variables, the probability $P[X \in i \cap Y \in j]$ can be written as product of $P[X \in i]$ and $P[Y \in j]$. Probability of element lying in certain class is obtained from empirical contingency table as the ratio between the number of elements lying in certain class $\hat{n}_{Xi}$ or $\hat{n}_{Yj}$ and the number of all elements n.

$$n_{ij} = n\frac{\hat{n}_{Xi}}{n}\frac{\hat{n}_{Yj}}{n} = \frac{\hat{n}_{Xi} \cdot \hat{n}_{Yj}}{n} \tag{13}$$

Test statistic is calculated as a comparison between empirical and theoretical contingency table.

$$H = \sum_{i=1}^{k_x}\sum_{j=1}^{k_y} \frac{(n_{ij} - \hat{n}_{ij})^2}{n_{ij}}. \tag{14}$$

It is distributed by the χ^2 distribution with $\nu = (k_x - 1)(k_y - 1)$ degrees of freedom. Rejection limit for $\nu = 16$ ($k_x = k_y = 5$) and the significance level of 5%, equals $\chi^2_{0.95,16} = 26.296$. When H exceeds this value, we reject null hypothesis and claim that X and Y are dependent.

All combinations of two observed horizontal directions, reduced directions and angles are going to be tested this way.

Treating observations as a time series

Measurements carried out in chronological order can be treated as time series. Graphical presentation with two procedures of smoothing will be done. Autocorrelation functions will be evaluated and plotted. Two additional tests for randomness of time series will be run at the end.

Graphical presentation

When analyzing time series, it is recommended to plot the observations. As the values of observed quantities may be of different magnitude than fluctuations of these values, the residuals from average are plotted. Random errors in observations cause fluctuations that hinder us an insight into alteration of values with time. That is the reason to smooth the series using moving average and exponential smoothing [**3**].

Moving average shows chronological order of the average of k consecutive values [**7**].

$$DP_i = \frac{1}{k}\sum_{j=0}^{k-1} x_{i+j} \tag{15}$$

Exponential smoothing uses a different approach. The first element of smoothed series equals the first observation. The next element is calculated as weighted average of the last smoothed element and the next observation [**7**].

$$EG_i = \alpha x_i + (1-\alpha)EG_{i-1} \qquad i = 2, ..., n \tag{16}$$

The value of α is usually chosen between 0.1 and 0.3, depending on the degree of smoothing we desire. Exponential smoothing is more suitable for stationary processes without noticeable trend.

Autocorrelation function

Calculation of autocorrelation function is based on the comparison of one part of time series with another part of the same series, translated for a time lag k [**2**]. The graph of autocorrelation function shows correlation between original and translated series, on translation k. Firstly, we calculate covariance.

$$\sigma(k) = \frac{1}{n-k}\sum_{i=1}^{n-k}(x_i - \bar{x}_{1,...,n-k})(x_{i+k} - \bar{x}_{k+1,...,n}); \qquad k = 0, ..., n/2. \tag{17}$$

Correlation is obtained from

$$\rho(k) = \frac{\sigma(k)}{\sigma_1\sigma_2}, \tag{18}$$

where

$$\sigma_1^2 = \frac{1}{n-k}\sum_{i=1}^{n-k}(x_i - \bar{x}_{1,...,n-k})^2; \qquad k = 0, ..., n/2, \tag{19}$$

$$\sigma_2^2 = \frac{1}{n-k}\sum_{i=1}^{n-k}(x_{i+k} - \bar{x}_{k+1,...,n})^2; \qquad k = 0, ..., n/2. \tag{20}$$

The value of autocorrelation function represents the linear time dependence of the series. In Figure 2, three typical cases are shown. Random series, linear dependent series and sine dependent series are shown along with their autocorrelation functions. Functions are plotted for all values of time lag $k = 0, ..., n$, although they should and at $k = n/2$, so the first half is emphasised. If the observations are expected to be independent, the autocorrelation function graph immediately decreases from one to about zero and then continues to fluctuate around zero.

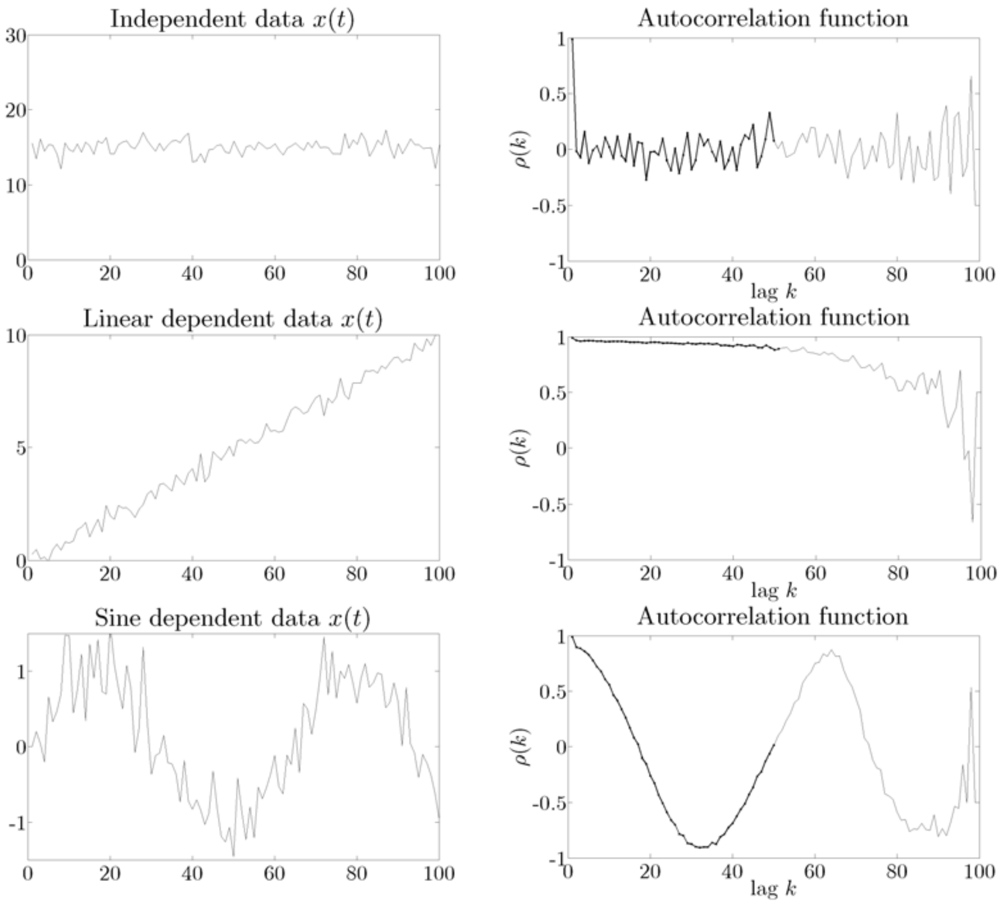

Fig. 2. Autocorrelation function for independent, linear and sine dependent data

Tests for randomness

Three basic tests for randomness were used in our research. The first two tests are mostly summarized from [**3**] and the third from [**5**].

Turning points

For each sequence of three consecutive values (x_i, x_{i+1}, x_{i+2}) from time series, we check whether the middle value represents turning point. From six possible permutations of three different values, only two cases do not form turning point ($x_i > x_{i+1} > x_{i+2}$ or $x_i < x_{i+1} < x_{i+2}$). The other four permutations represent turning point, unless two successive values are equal. The number of turning points p in series of n observation is random variable with the following expected value and variance

$$E[p] = \frac{2}{3}(n-2), \tag{21}$$

$$var[p] = \frac{8n}{45}. \tag{22}$$

For large samples, p is distributed approximately normally, therefore a test statistic can be formed as follows:

$$T = \frac{p - \frac{2}{3}(n-2)}{\sqrt{\frac{8n}{45}}}. \tag{23}$$

Hypotheses are:
H_0: Time series is random.
H_1: Time series is not random.
Rejection limits for the significance level of 5% equals -1.96 and 1.96. If test statistic T is within these borders, we can not claim that the series is not random.

Difference sign

For random series in half cases the graph between two observations should fall and in half cases it should rise. Random variable is the number of consecutive pairs from series, where the graph falls ($x_i > x_{i+1}$). This variable has the following expected value and variance

$$E[c] = \frac{n-1}{2}, \tag{24}$$

$$var[c] = \frac{n+1}{12}. \tag{25}$$

By increasing the sample, the distribution of c quickly converges to normal. For the same hypothesis as before, we form test statistic

$$U = \frac{c - \frac{n-1}{2}}{\sqrt{\frac{n+1}{12}}}. \tag{26}$$

Again critical boundaries are -1.96 and 1.96 and if the statistic U lies between them we can not say that the series is not random. Otherwise we claim that it is not.

Runs test

For Runs test we have to construct a binary sequences out of observed series. If the observed value exceeds the mean value of series, the value of binary sequence becomes 1; if not, the value of binary sequence becomes 0. For test statistic we need to count the number of zeros n_1 and ones n_2 in the sequence and the number of "runs" r, i.e. the number of "occurances of an equal value subsequence delimited by a different value" [**5**]. Hypotheses are the same as above. The expected value and variance of variable r are

$$E[r] = \frac{2n_1n_2}{n_1+n_2} + 1, \tag{27}$$

$$var[r] = \frac{2n_1n_2(2n_1n_2 - n_1 - n_2)}{(n_1+n_2)^2(n1_+n_2-1)}. \tag{28}$$

In the case of large samples, r is distributed approximately normally. We form the test statistic

$$V = \frac{r - \frac{2n_1n_2}{n_1+n_2}}{\sqrt{\frac{2n_1n_2(2n_1n_2-n_1-n_2)}{(n_1+n_2)^2(n1_+n_2-1)}}}. \tag{29}$$

Critical boundaries are -1.96 and 1.96 and if the statistic V lies between them, we can not say that the series is not random.

Results

Measurements

Measurements were carried out on 25$^{\text{th}}$ Februar 2011.The first series of a hundred sets of angles started at 8:00 and ended at 9:35 in the morning. The second series lasted from 13:40 to 15:15 in the afternoon. The first series was intentionally carried out in poor observation conditions. The instrument was not acclimatized and atmospheric conditions were not stable due to sunrise and warming of the air. Afternoon conditions were stable, the weather was clear, it was cold and windy. Each series took about an hour and half, which means approximately one set per minute. Observations were carried out automatically using ATR system. 98 sets were extracted from the first series and 99 from the second. Three sets were incomplete due to some unexplained errors. Reduced directions and angles were calculated using equations (2) and (3).

We have two series of 98 and 99 observations of four directions. Additionally, we calculated three reduced directions and three angles for each of these sets in both series.

Variance-covariance matrix and correlation matrix

Tables 3 and 4 show variance-covariance and correlation matrices for observed directions, reduced directions and angles and for both series. The units of the variance-covariance matrices are cc (= 1/10000 gon).

Comparing the obtained variance-covariance matrices with assumptions for independent observations (equations (9), (10) and (11)), we note that they are not adequate. Correlation matrix for observed directions shows a strong dependence

Table 3. *Variance-covariance and correlation matrices for the first series*

variance-covariance matrices	correlation matrices
$\mathbf{\Sigma}_{D_1} = \begin{bmatrix} 0.00569 & 0.00494 & 0.00498 & 0.00439 \\ 0.00494 & 0.00504 & 0.00459 & 0.00430 \\ 0.00498 & 0.00459 & 0.00789 & 0.00424 \\ 0.00439 & 0.00430 & 0.00424 & 0.00536 \end{bmatrix}$	$\mathbf{R}_{D_1} = \begin{bmatrix} 1.000 & 0.922 & 0.743 & 0.795 \\ 0.922 & 1.000 & 0.727 & 0.826 \\ 0.743 & 0.727 & 1.000 & 0.651 \\ 0.795 & 0.826 & 0.651 & 1.000 \end{bmatrix}$
$\mathbf{\Sigma}_{rD_1} = \begin{bmatrix} 0.00085 & 0.00035 & 0.00066 \\ 0.00035 & 0.00361 & 0.00055 \\ 0.00066 & 0.00055 & 0.00226 \end{bmatrix}$	$\mathbf{R}_{rD_1} = \begin{bmatrix} 1.000 & 0.202 & 0.472 \\ 0.202 & 1.000 & 0.193 \\ 0.472 & 0.193 & 1.000 \end{bmatrix}$
$\mathbf{\Sigma}_{A_1} = \begin{bmatrix} 0.00085 & -0.00050 & 0.00030 \\ -0.00050 & 0.00375 & -0.00336 \\ 0.00030 & -0.00336 & 0.00477 \end{bmatrix}$	$\mathbf{R}_{A_1} = \begin{bmatrix} 1.000 & -0.278 & 0.149 \\ -0.278 & 1.000 & -0.794 \\ 0.149 & -0.794 & 1.000 \end{bmatrix}$

of all four directions. According to these results we conclude that the measurements from the first series are highly dependent.

For the second series we can note that the obtained variance-covariance matrices coincide with the assumptions better. At matrix $\mathbf{\Sigma}_{A_2}$ covariances between adjacent angles are evidently higher. The correlations for observed directions in the second series are obviously smaller than in the first one. We may conclude that the correlation depends on the observation conditions.

Table 4. *Variance-covariance and correlation matrices for the second series*

variance-covariance matrices	correlation matrices
$\mathbf{\Sigma}_{D_2} = \begin{bmatrix} 0.00030 & -0.00012 & 0.00025 & 0.00012 \\ -0.00012 & 0.00047 & -0.00013 & -0.00004 \\ 0.00025 & -0.00013 & 0.00105 & 0.00021 \\ 0.00012 & -0.00004 & 0.00021 & 0.00227 \end{bmatrix}$	$\mathbf{R}_{D_2} = \begin{bmatrix} 1.000 & -0.324 & 0.436 & 0.150 \\ -0.324 & 1.000 & -0.186 & -0.035 \\ 0.435 & -0.186 & 1.000 & 0.135 \\ 0.140 & -0.035 & 0.135 & 1.000 \end{bmatrix}$
$\mathbf{\Sigma}_{rD_2} = \begin{bmatrix} 0.00102 & 0.00005 & 0.00027 \\ 0.00005 & 0.00087 & 0.00014 \\ 0.00027 & 0.00014 & 0.00232 \end{bmatrix}$	$\mathbf{R}_{rD_2} = \begin{bmatrix} 1.000 & 0.053 & 0.173 \\ 0.053 & 1.000 & 0.100 \\ 0.173 & 0.100 & 1.000 \end{bmatrix}$
$\mathbf{\Sigma}_{A_2} = \begin{bmatrix} 0.00102 & -0.00097 & 0.00022 \\ -0.00097 & 0.00179 & -0.00094 \\ 0.00022 & -0.00094 & 0.00290 \end{bmatrix}$	$\mathbf{R}_{A_2} = \begin{bmatrix} 1.000 & -0.719 & 0.126 \\ -0.719 & 1.000 & -0.412 \\ 0.126 & -0.412 & 1.000 \end{bmatrix}$

Test χ^2

All possible combinations of two observed directions, two reduced directions and two angles will be tested for both series. Null hypothesis says that the treated quantities are independent. Critical limit for the significance level of 5% equals $\chi^2_{0.95,16} = 26.296$. If test statistic is greater than limit, we claim that the tested quantities are dependent. Results are shown in Tables 5, 6 and 7.

As observed in previous section, the observations are far more dependent in the first series. The first two pairs of directions from the second series are also dependent, but their test statistic is not very high. An interesting fact is that reduced directions in second series do not show dependence even though they

Table 5. *Test χ^2 — values of testing statistic H for observed directions*

	1st series		2nd series	
	dependence		dependence	
pair	significant	not significant	significant	not significant
D_1-D_2	159.207		40.212	
D_1-D_3	79.679		35.538	
D_1-D_4	118.651			10.339
D_2-D_3	88.822			19.275
D_2-D_4	125.284			15.362
D_3-D_4	80.293			13.371

Table 6. *Test χ^2 — values of testing statistic H for reduced directions*

	1st series		2nd series	
	dependence		dependence	
pair	significant	not significant	significant	not significant
rD_1-rD_2		10.291		16.988
rD_1-rD_3	32.743			10.293
rD_2-rD_3		12.705		12.953

Table 7. *Test χ^2 — values of testing statistic H for angles*

	1st series		2nd series	
	dependence		dependence	
pair	significant	not significant	significant	not significant
A_1-A_2		15.460	89.591	
A_1-A_3	31.503			5.126
A_2-A_3	96.058		32.052	

should. The reason is small variance of the first observation which represents the covariance between all three of reduced directions (see equation (10)). For the angles, the results of the second series completely coincide with the assumptions for independendent observations.

Treating observations as a time series

Graphical presentation

Residuals of the observations, from their average, are plotted in Figure 3 for the first and in Figure 4 for the second series. Graphs of raw residuals for both series show random fluctuations of measurements. Besides raw residuals we added the results of moving average and exponential smoothing, to present the trends of observations more clearly.

In the first series it is quite obvious that all measured values are decreasing in the first third of observations. Direction D_3 is oscillating from the 20$^{\text{th}}$ observation

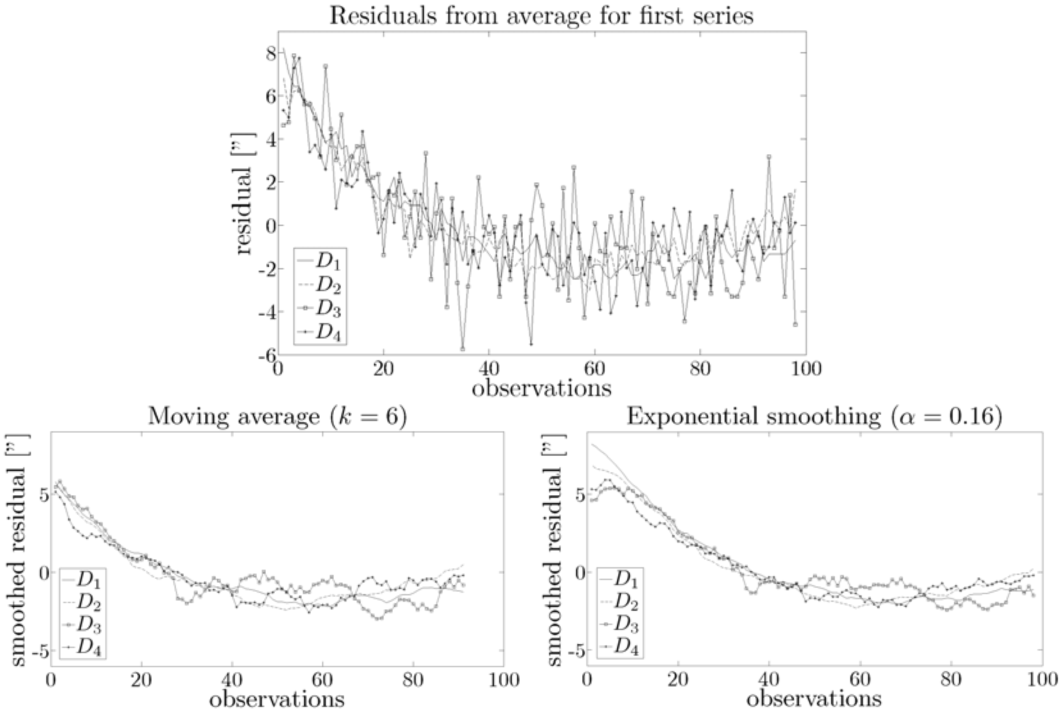

Fig. 3. Residuals from average, moving average and exponential smoothing in the first series

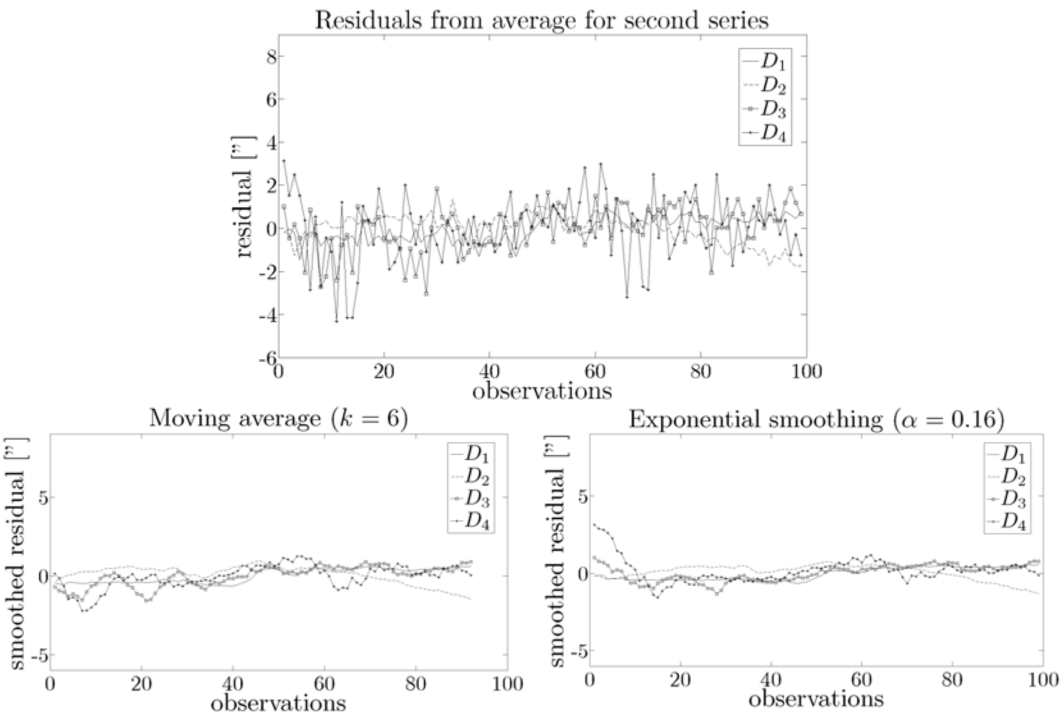

Fig. 4. Residuals from average, moving average and exponential smoothing in the second series

on, while for the other three directions we could say that they are rising again from the 60th observation on. These changes of values are not negligible.

Residuals of the second series are fluctuating due to random errors, but on the smoothed graph no significant trends can be noticed. Only the values of direction D_4 decrease drastically in the first 15 measurements. Values of D_2 are decreasing gently in the last 40 sets of angles. The oscillation of D_3 can be noticed, too.

Trends or oscillations observed in these plots do not necessary mean dependence of observations. However, they show that our measurements are not strictly random.

Autocorrelation function

Autocorrelation functions are plotted for 60 translations. In Figure 5 we can see autocorrelation functions for all four observed directions and both series. The functions of morning observations are typical for dependent variables. None of the observations steady about zero; directions D_1 and D_2 do not even decrease in the beginning. The autocorelation functions indicates that variables D_1 and D_2 are linear dependent. Autocorrelation functions of the other two directions decreases, but only to a half. In the second series, all four functions decrease in the first step and eventually they stay around zero.

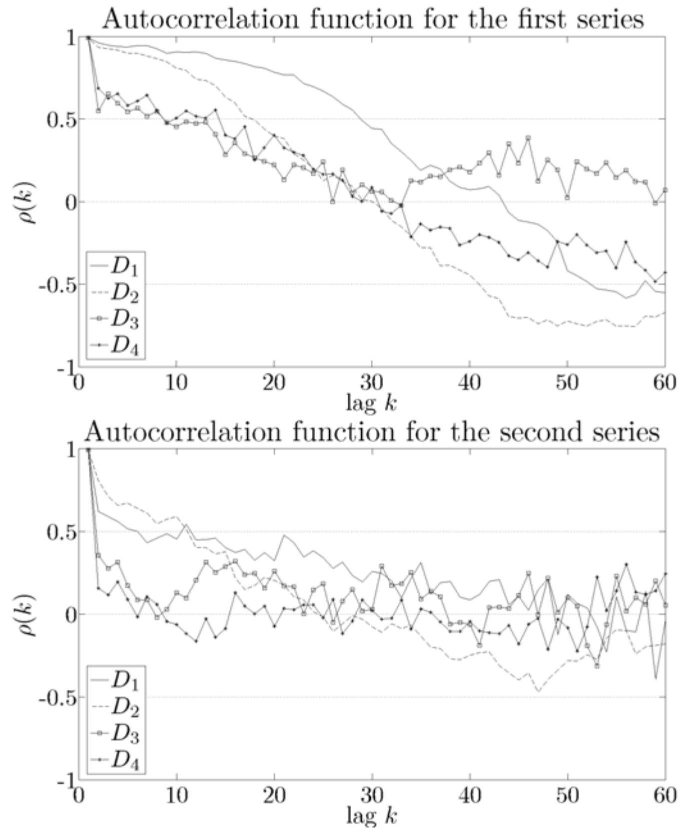

Fig. 5. Autocorrelation functions for both series

Tests for randomness

In this section the results of tests for randomness of time series are presented. The critical limits for all three tests are $(-1.96, 1.96)$ and when the test statistic lies outside this interval, we conclude that series is statistically significant not random. In Tables 8, 9 and 10, the results of tests are shown along with the values of test statistics.

Table 8. *Turning points test — results*

	1st series		2nd series	
	non-randomness		non-randomness	
T	significant	not significant	significant	not significant
D_1	−4.03		−2.19	
D_2		−1.18	−2.47	
D_3		−0.33		0.00
D_4		0.73		0.24

Table 9. *Difference sign test — results*

	1st series		2nd series	
	non-randomness		non-randomness	
U	significant	not significant	significant	not significant
D_1		−1.11		0.54
D_2	−2.00			−0.57
D_3		−0.53		−1.57
D_4		−1.22		−1.22

Table 10. *Runs test — results*

	1st series		2nd series	
	non-randomness		non-randomness	
V	significant	not significant	significant	not significant
D_1	−9.26		−6.56	
D_2	−6.46		−7.82	
D_3		−1.95	−3.71	
D_4	−2.86			−0.80

We can notice that Runs test is the strictest of all three tests. Only two observed directions not significantly non-random are presented as such also by the first two tests. None of the observations indicated significant non-randomness in all three tests.

Discussion

The test measurements carried out in the test network are comparable to the control measurements of the highest accuracy except in two aspects. Firstly, the test network is of smaller dimensions than common geodetic networks and secondly, the measurements of the first series were carried out in unacceptable observation conditions. However, we estimate that the second series of measurements are quite representative sample of high accuracy geodetic observations.

Before dealing with variance-covariance matrices of observations, we derived theoretical variance-covariance matrices of reduced directions and angles. The reduced directions and angles are not independent. The calculated values of variance-covariance matrices and corelation matrices expressed rather strong dependence of observations in the first series. The second series observations were less dependent except for the first observed direction D_1. The correlation of reduced directions was not obvious, for the variance of D_1 was small by chance. Successive pairs of angles were highly correlated as expected.

The results of χ^2 test are consistent with the described correlations.

Plots of observations show strong trend in the first third of the first series, which is attributed to acclimatization of the instrument. The magnitude of difference in values in the second series is a few times smaller than in the first. There are some fluctuations in the second series observations, but they can probably be attributed to random errors. According to autocorrelation functions, we could say that directions D_3 and D_4 are more independent than D_1 and D_2, besides the fact that first series is more correlated in general. One of the tests for randomness of time series detected non-randomness in observations.

Based on this research we conclude that the acclimatization of instrument and the choise of favorable observation conditions is essential for the independence of observed directions, which is one of the basic assumptions in geodetic network adjustments.

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