Univerza
*v Ljubljani*
Fakulteta
*za gradbeništvo*
*in geodezijo*

*Jamova 2*
*1000 Ljubljana, Slovenija*
*telefon (01) 47 68 500*
*faks (01) 42 50 681*
*fgg@fgg.uni-lj.si*

Univerzitetni program Gradbeništvo,
Konstrukcijska smer

Kandidat:
**Simon Petrovčič**

# Analiza in projektiranje tankostenskih cilindričnih silosov v skladu z Evrokod standardi

**Diplomska naloga št.: 3010**

**Mentor:**
izr. prof. dr. Boštjan Brank

**Somentor:**
prof. dr. Werner Guggenberger

Ljubljana, 23. 4. 2008

## ERRATA

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## IZJAVA O AVTORSTVU

Podpisani **SIMON PETROVČIČ** izjavljam, da sem avtor diplomske naloge z naslovom: »**ANALIZA IN PROJEKTIRANJE TANKOSTENSKIH CILINDRIČNIH SILOSOV V SKLADU Z EVROKOD STANDARDI**«.

Izjavljam, da se odpovedujem vsem materialnim pravicam iz dela za potrebe elektronskega arhiva FGG.

Ljubljana, 8.4.2008 Podpis:________________________

# IZJAVE O PREGLEDU NALOGE

Nalogo so si ogledali naslednji učitelji konstrukcijske smeri:

# BIBLIOGRAFSKO – DOKUMENTACIJSKA STRAN IN IZVLEČEK

| | |
|---|---|
| **UDK:** | **624.74.4:624.9(043.2)** |
| **Avtor:** | **Simon Petrovčič** |
| **Mentor:** | **izr. prof. dr. Boštjan Brank (UL)** |
| **Somentor:** | **prof. dr. Werner Guggenberger (TU Graz)** |
| **Naslov:** | **Analiza in projektiranje tankostenskih cilindričnih silosov v skladu z Evrokod standardi** |
| **Obseg in oprema:** | **372 str., 50 pregl. (slo.), 71 sl. (slo.), 461 en. (slo.), 28 pregl. (ang.), 52 sl. (ang.), 247 en. (ang.)** |
| **Ključne besede:** | **cilindrični silosi, obtežba, analiza, projektiranje, Evrokod** |

## Izvleček

Diplomska naloga je zamišljena kot priročnik za analizo in projektiranje tankostenskih, cilindričnih, osno simetričnih metalnih silosov v skladu z Evrokod standardi. Vsebinsko je razdeljena na tri dele. V prvem delu so obravnavane vse pomembnejše obtežbe, ki delujejo na silose. Poseben poudarek je namenjen obtežbam, ki nastanejo pri polnjenju in praznjenju ter obtežbe zaradi vetra in potresa. Za vsako od teh obtežb je na enostaven in razumljiv način (po korakih in v obliki diagramov poteka) podan postopek za izračun pritiskov po silosu. Izpeljani so izrazi za membranske sile. Pri obtežbah zaradi polnjenja in praznjenja so bistveni izrazi grafično prikazani v posplošeni (brezdimenzijski) obliki, s čimer postanejo bolj pregledni in hitro berljivi za različne geometrije silosov in za različne shranjene materiale. Raziskane so nekatere nejasnosti, ki se pojavljajo v standardih. Predstavljene so obtežne kombinacije, ki jih je potrebno upoštevati pri projektiranju. V drugem delu diplomske naloge je izračunan praktični računski primer z upoštevanjem postopkov iz prvega dela. V tretjem delu sta predstavljena dva računalniška programa, ki sta tudi bila izdelana v sklopu te naloge. Prvi je namenjen določitvi obtežbe shranjenega materiala na stene silosa, drugi pa določitvi obtežbe vetra na stene silosa. Programa sta na zgoščenki, ki je priložena nalogi. V prilogah sta tudi angleška prevoda prvega in tretjega dela.

## BIBLIOGRAPHIC – DOCUMENTALISTIC INFORMATION

**UDC:** **624.74.4:624.9(043.2)**

**Author:** **Simon Petrovčič**

**Supervisor:** **assoc. prof. dr. Boštjan Brank (UL)**

**Co-supervisor:** **prof. dr. Werner Guggenberger (TU Graz)**

**Title:** **Analysis and design of thin-walled cylindrical silo structures in accordance with EN Eurocodes**

**Notes:** **372 p., 50 tab. (slo.), 71 fig. (slo.), 461 eq. (slo.), 28 tab. (eng.), 52 fig. (eng.), 247 eq. (eng.)**

**Key words:** **cylindrical silos, loading, analysis, design, EN Eurocodes**

### Abstract

This work is meant to serve as a guidebook for analysis and design of thin-walled, cylindrical, axisymetric metal silos in accordance with EN Eurocode building codes. It consists of three parts. In the first part the important loads on silo structures are considered. An emphasis is given on loads due to the stored solid material, wind and seismic effects. Simple and clear procedures (with step-by-step instructions and flowcharts) for determining pressures acting on the silo are given. Membrane section forces are derived. For loads due to the stored solid material the key expressions are rewritten and plotted in a generalized (dimensionless) form in order to accommodate different silo geometries and different stored solid materials. Some discrepancies of the code are identified and investigated. Load combinations, which are needed for design, are also given. In the second part of the work an example of analysis and design of a silo is presented by using the procedures introduced in the first part. In the third part two computer programs, which were also developed in the scope of this work, are presented. The first program serves to determine the loads on silo walls due to effects of the stored solid material, while the second program serves to determine the wind loading on the silo structure. Both programs are located on the CD attached to this work. English translations of the first and third part are given in appendices.

## ZAHVALA

Iskreno se zahvaljujem mentorjema, izr. prof. dr. Boštjanu Branku in prof. dr. Wernerju Guggenbergerju, za vso pomoč, podporo in motivacijo, ki sta mi jo nudila pri izdelavi diplomske naloge.

Zahvaljujem se obema univerzama, Univerzi v Ljubljani in Tehniški Univerzi v Gradcu, za izkazano finančno pomoč.

Nenazadnje se zahvaljujem tudi moji mami Vidi Petrovčič in vsem bližnjim, ki so mi stali ob strani skozi vsa leta študija.

## DANKSAGUNG

Ich möchte meinen Mentoren, Herrn Ao. Prof. Dr. Boštjan Brank und Herrn Prof. Dr. Werner Guggenberger, für alle Hilfe, Unterstützung und Motivation, die mir ihrerseits bei der Ausarbeitung dieser Diplomarbeit geboten wurde, meinen aufrichtigen Dank aussprechen.

Ich bedanke mich ebenfalls bei beiden Universitäten, der Universität von Ljubljana und der Technischen Universität Graz, für die erwiesene finanzielle Hilfe.

Nicht zuletzt danke ich meiner Mutter Vida Petrovčič und allen Nahestehenden, die mir während des gesamten Studiums zur Seite gestanden haben.

# KAZALO VSEBINE

## I. DEL – OBTEŽBE NA SILOSE

**II. DEL – RAČUNSKI PRIMER**

## III. DEL - RAČUNALNIŠKA PROGRAMA

# KAZALO PRILOG

**PRILOGA A: ANGLEŠKI PREVOD I. DELA Z UVODOM**

**PRILOGA B: ANGLEŠKI PREVOD III. DELA**

**PRILOGA C: PREGLEDNICA Z LASTNOSTMI SHRANJENIH MATERIALOV**

# KAZALO PREGLEDNIC

# KAZALO SLIK

## KAZALO DIAGRAMOV

# 1 Uvod

## 1.1 Motivacija

Silosi predstavljajo skupino posebnih inženirskih objektov. Uporabljajo se v industriji in kmetijstvu, kjer služijo kratkotrajni ter srednje dolgi shrambi organskih in anorganskih sipkih materialov. Njihova kapaciteta se lahko giblje od 10 ton pa vse do 10 000 in celo več ton. Silosi imajo lahko krožni ali pravokotni prečni prerez. Njihove stene so ponavadi zgrajene iz jekla in armiranega betona, lahko pa se zanje uporabljajo tudi drugi materiali, kot na primer aluminij ali les. V tej nalogi so obravnavani samo tankostenski metalni silosi s krožnim prečnim prerezom.

Določitev obtežb na silose je relativno zapleten proces, predvsem zaradi zelo raznolike sestave materiala, ki ga hranimo v silosu. Ker so ti materiali sipki, se med procesom polnjenja in praznjenja obnašajo kot ne-Newtonske tekočine, kar privede do kompleksnih napetostnih stanj v shranjenem materialu in v stenah silosa. Zato je potrebno posebej obravnavati stanje polnjenja (ang. »filling state«) in stanje praznjenja (ang. »discharge state«). Dodatne stabilnostne in dinamične probleme povzročata obtežbi vetra in potresa. Poln silos ima ponavadi veliko maso, ki je podprta na neki višini nad tlemi z relativno podajnimi podporami, kar lahko predstavlja problem pri potresni obremenitvi. Pri praznem silosu pa lahko pride zaradi tanke zunanje stene do uklona.

Namen te diplomske naloge je pripraviti priročnik za projektiranje tankostenskih osno simetričnih silosov v skladu z Evrokod standardi. V nalogi je predstavljen pregled procesa projektiranja silosov, skupaj s praktičnim računskim primerom. Primer je analiziran in projektiran glede na zahteve ustreznih EN Evrokod standardov in glede na priporočila, podana v tej nalogi. Poseben poudarek je namenjen tudi potresni analizi silosov.

## 1.2 Struktura naloge

Naloga je vsebinsko razdeljena ne tri dele. V prvem delu so predstavljene obtežbe, ki delujejo na silose, podani so podrobni napotki za določitev najpomembnejših obtežb in izrazi pripadajoče membranske sile. Raziskane so tudi nekatere nejasnosti, ki se pojavljajo v standardih. Na koncu prvega dela so predstavljene potrebne obtežne kombinacije, ki jih je potrebno upoštevati pri projektiranju.

V drugem delu diplomske naloge je predstavljen praktični računski primer, ki upošteva napotke iz prvega dela.

V tretjem delu pa sta predstavljena računalniška programa, ki sta bila ustvarjena v sklopu te diplomske naloge. Prvi program služi določitvi obtežbe shranjenega materiala na stene silosa, drugi pa določitvi obtežbe vetra na stene silosa. Programa se nahajata na zgoščenki, ki je priložena na notranji strani zadnje platnice.

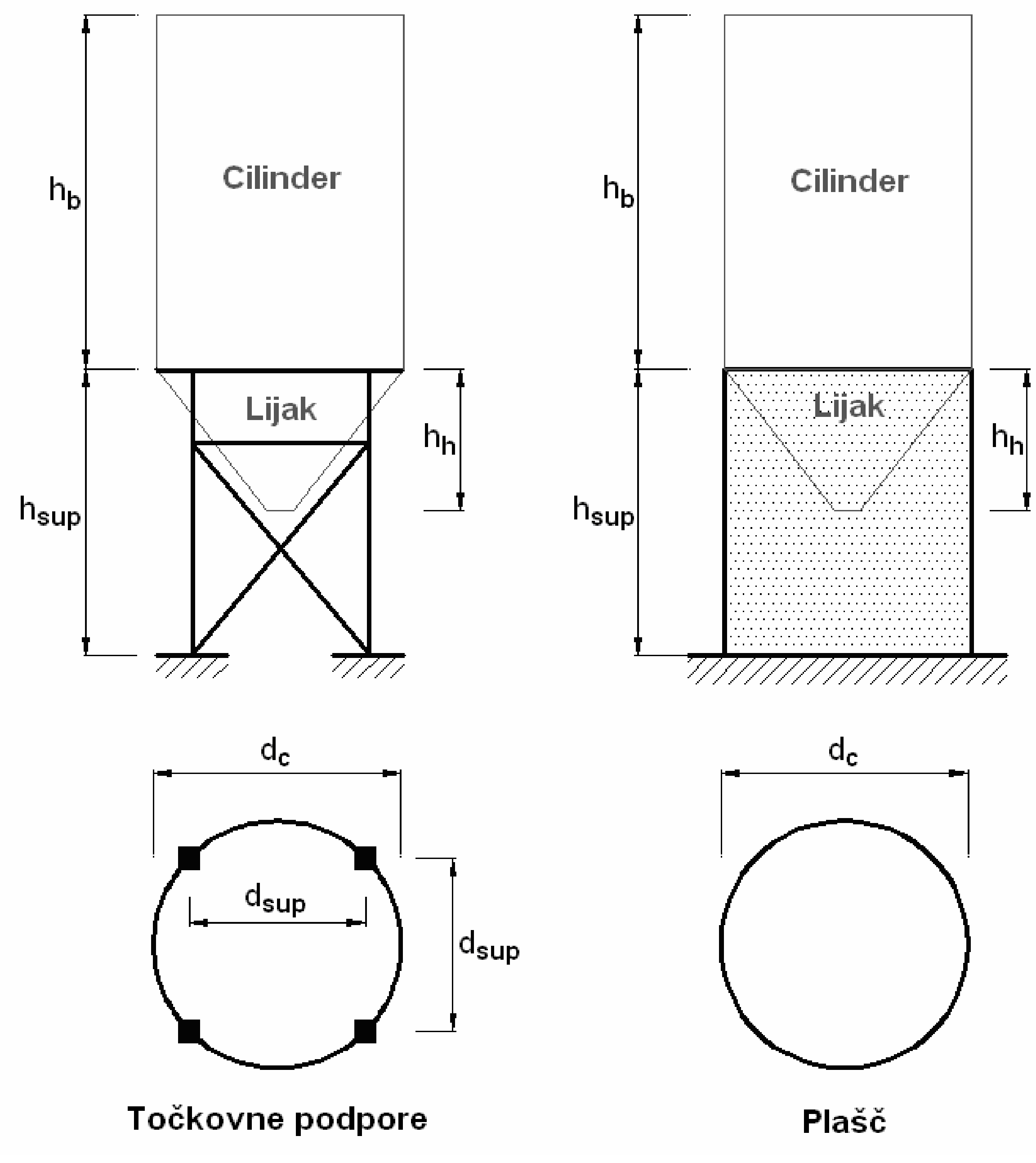
## 1.3 Tipi obravnavanih silosov

Silosi se med seboj razlikujejo po obliki, velikosti, materialu, iz katerega so narejeni, in po materialu, ki ga hranijo. Kot rečeno v uvodu, ta diplomska naloga obravnava samo tankostenske osno simetrične silose.

Veljajo torej naslednje predpostavke:

- osno simetrična geometrija silosa (krožni cilinder in stožčast lijak),
- stena je tanka in narejena iz kovine (nerjaveče jeklo, aluminij),
- dotok v silos in odtok iz silosa ležita na simetrijski osi,
- silos ima samo en prekat, v katerega se shranjuje material.

Tipičen silos, ki ga obravnavamo v tej nalogi, je tako sestavljen iz cilindričnega zgornjega dela – cilindra (ang. »cylinder« oz. »barrel«), v katerem je shranjena glavnina materiala, in stožčastega spodnjega dela – lijaka (ang. »hopper«), ki služi enakomernemu in kontroliranemu izpustu materiala. Olajšan dostop do izpusta iz silosa, ki se nahaja na dnu lijaka, je omogočen s postavitvijo silosa v dvignjen položaj. Silos, je lahko podprt točkovno (s stebri) ali pa s t.i. plaščem (ang. »skirt«), ki predstavlja podaljšanje cilindra do tal.



**Slika 1: Sestavni deli silosa in tipični podporni konstrukciji**

## 1.4 Obravnavani standardi

Pri pisanju diplomske naloge so bili obravnavani sledeči evropski EN standardi:

**Preglednica 1: Obravnavani standardi**[1]

| Standard | Opis |
|---|---|
| **EN 1990:2000** | Evrokod 0: Osnove projektiranja konstrukcij |
| **EN 1991-1-4:2005** | Evrokod 1: Vplivi na konstrukcije, Del 1.4: Splošni vplivi - Vplivi vetra |
| **EN 1991-4:2006** | Evrokod 1: Vplivi na konstrukcije, Del 4: Silosi in rezervoarji |
| **EN 1993-1-1:2005** | Evrokod 3: Projektiranje jeklenih konstrukcij, Del 1-1: Splošna pravila in pravila za stavbe |
| **prEN 1993-1-6:2004** | Evrokod 3: Projektiranje jeklenih konstrukcij, Del 1-6: Nosilnost in stabilnost lupin |
| **prEN 1993-4-1:2005** | Evrokod 3: Projektiranje jeklenih konstrukcij, Del 4-1: Silosi |
| **EN 1998-1:2004** | Evrokod 8: Projektiranje potresno odpornih konstrukcij, Del 1: Splošna pravila, potresni vplivi in vplivi na stavbe |
| **EN 1998-4:2006** | Evrokod 8 : Projektiranje potresno odpornih konstrukcij, Del 4: Silosi, rezervoarji in cevovodi |

## 1.5 Uporabljeni simboli

Simboli, ki so bili uporabljeni v nalogi in imajo fizikalen pomen, so podani v spodnjih preglednicah.

**Preglednica 2: Seznam simbolov, ki določajo geometrijo silosa**

| Simbol | Pomen |
|---|---|
| $A$ | prečni prerez silosa |
| $\beta$ | polovični kot lijaka, merjen glede na os simetrije |
| $d_c$ | premer silosa |

**se nadaljuje …**

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[1] Uporabljeni so bili izključno originalni EN standardi, ki jih izdaja CEN, in ne slovenski prevodi, ki jih izdaja SIST. Slovenski prevodi naslovov so zato zgolj informativni.

**… nadaljevanje**

| | |
|---|---|
| $d_{sup}$ | razdalja med podporami podporne konstrukcije |
| $\phi$ | kot med izbrano in fiksno smerjo znotraj prečnega prereza |
| $G_m$ | teža shranjenega materiala |
| $G_s$ | teža sten silosa |
| $h_0$ | oddaljenost od ekvivalentne ravnine, pri kateri je shranjen material še v stiku s steno silosa |
| $h_b$ | celotna višina cilindra |
| $h_c$ | nadomestna višina cilinda, merjena do ekvivalentne ravnine |
| $h_h$ | višina lijaka |
| $h_m$ | težišče silosa |
| $h_s$ | višina celotnega silosa |
| $h_{sup}$ | višina podporne konstrukcije |
| $h_{tp}$ | višina vrhnjega kupa materiala |
| $\lambda$ | vitkost cilindra |
| $r$ | notranji radij cilindra |
| $s$ | koordinata, ki poteka v smeri stene lijaka |
| $t$ | debelina stene |
| $U$ | obseg notranjega dela cilindra |
| $V$ | volumen silosa |
| $V_m$ | Volumen shranjenega materiala |
| $\omega$ | brezdimenzijski parameter dolžine |
| $x$ | koordinata, ki teče v navpični smeri, od dna lijaka navzgor |
| $\psi_{\lambda\phi}$ | faktor vitkosti |
| $z$ | koordinata, ki teče v navpični smeri, od ekvivalentne ravnine navzdol |
| $z'$ | koordinata, ki teče v navpični smeri, od vrha cilindra navzdol |
| $z_s$ | globina pod točko, kjer je material še v stiku s steno |

**Preglednica 3: Simboli, ki označujejo pritiske in membranske sile**

| Simbol | Pomen |
|---|---|
| $\Delta_{ph,s}$ | dodatni normalni pritisk, ki nastane zaradi delovanja potresne obremenitve na poln silos |
| $n_{\phi,ees}$ | obročna sila zaradi delovanja potresne obremenitve na prazen silos |
| $n_{\phi,efs}$ | obročna sila zaradi delovanja potresne obremenitve na poln silos |

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**… nadaljevanje**

| | |
|---|---|
| $n_{\phi,s}$ | obročna sila zaradi pritiska materiala, ki je shranjen v silosu |
| $n_{\phi,sneg}$ | obročna sila zaradi obtežbe snega |
| $n_{\phi,we}$ | obročna sila zaradi delovanja vetra na prazen silos |
| $n_{\phi,wf}$ | obročna sila zaradi delovanja vetra na poln silos |
| $n_{\phi,d}$ | projektna vrednost obročne sile |
| $n_{\phi,e}$ | obročna sila zaradi praznjenja shranjenega materiala |
| $n_{\phi,f}$ | obročna sila zaradi polnjenja shranjenega materiala |
| $n_{s,ees}$ | membranska sila v praznem lijaku zaradi delovanja potresne obremenitve |
| $n_{s,efs}$ | membranska sila v polnem lijaku zaradi delovanja potresne obremenitve |
| $n_{sd}$ | projektna vrednost membranske sile v lijaku |
| $n_{se}$ | membranska sila v lijaku zaradi praznjenja silosa |
| $n_{sf}$ | membranska sila v lijaku zaradi polnjenja silosa |
| $n_{z,ees}$ | vertikalna membranska sila zaradi delovanja potresne obremenitve na prazen silos |
| $n_{z,efs}$ | vertikalna membranska sila zaradi delovanja potresne obremenitve na poln silos |
| $n_{z,p}$ | vertikalna membranska sila zaradi delovanja stalnih obtežb |
| $n_{z,sneg}$ | vertikalna membranska sila zaradi delovanja snega |
| $n_{z,we}$ | vertikalna membranska sila zaradi delovanja vetra na prazen silos |
| $n_{z,wf}$ | vertikalna membranska sila zaradi delovanja vetra na poln silos |
| $n_{zd}$ | projektna vrednost vertikalne membranske sile |
| $n_{ze}$ | vertikalna membranska sila zaradi praznjenja silosa |
| $n_{zf}$ | vertikalna membranska sila zaradi polnjenja silosa |
| $p_{he}$ | horizontalni pritisk na steno cilindra zaradi praznjenja |
| $p_{hf}$ | horizontalni pritisk na steno cilindra zaradi polnjenja |
| $p_{vb}$ | komponenta, ki predstavlja enakomerni vertikalni pritisk |
| $p_{ve}$ | vertikalni pritisk v shranjenem materialu med praznjenjem |
| $p_{vf}$ | vertikalni pritisk v shranjenem materialu med polnjenjem |
| $p_{vft}$ | vertikalni pritisk v shranjenem materialu med polnjenjem na stiku cilinder - lijak |
| $p_{vho}$ | vertikalni pritisk na dnu vrhnjega kupa materiala |
| $p_{we}$ | vertikalno trenje po steni cilindra zaradi praznjenja |
| $p_{wf}$ | vertikalno trenje po steni cilindra zaradi polnjenja |
| $q_p$ | konični tlak vetra |

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**… nadaljevanje**

| | |
|---|---|
| $R_e$ | reakcija v podporah zaradi delovanja potresne obremenitve na poln silos |
| $R_s$ | reakcija v podporah zaradi delovanja snega |
| $R_{ss}$ | reakcija v podporah zaradi delovanja shranjenega materiala |
| $R_{sw}$ | reakcija v podporah zaradi delovanja lastne teže silosa |
| $R_w$ | reakcija v podporah zaradi delovanja vetra |
| $w_e$ | zunanji pritisk vetra |
| $w_{eq}$ | nadomestna, osno simetrična obtežba vetra |

**Preglednica 4: Simboli, ki se uporabljajo za določitev obtežb**

| Simbol | Pomen |
|---|---|
| $a_g$ | projektni pospešek tal |
| $C_b$ | faktor povečave vertikalne obtežbe |
| $c_0$ | faktor topologije terena |
| $c_e$ | faktor izpostavljenosti |
| $C_h$ | faktor povečave horizontalne obtežbe |
| $C_{op}$ | referenčni faktor za nesimetrično obtežbo |
| $c_{pe}$ | koeficient zunanjega tlaka vetra |
| $c_r$ | faktor hrapavosti |
| $C_S$ | faktor vitkosti |
| $C_w$ | faktor povečave trenja |
| $E_w$ | modul elastičnosti stene |
| $F_b$ | celotna prečna sila pri potresni obremenitvi |
| $F_e$ | karakteristična vrednost razmerja pritiskov v lijaku po praznjenju |
| $F_f$ | karakteristična vrednost razmerja pritiskov v lijaku po polnjenju |
| $\phi_i$ | strižni kot, oz. koz notranjega trenja shranjenega materiala |
| $\phi_r$ | kot deponiranja materiala |
| $\{\phi\}$ | vektor nihajnih oblik |
| $\varphi$ | zasuk masnega središča silosa zaradi delovanja potresne obremenitve |
| $\gamma$ | specifična teža shranjenega materiala |
| $g$ | težnostni pospešek Zemlje ($g = 9.81\ m/s^2$) |
| $\Gamma$ | masni participacijski faktor |
| $K$ | koeficient bočnega pritiska |

**se nadaljuje …**

**… nadaljevanje**

| | |
|---|---|
| $k_I$ | faktor vetrne turbulence |
| $k_m$ | togost rotacijske vzmeti |
| $k_r$ | faktor terena |
| $k_v$ | togost vodoravne vzmeti |
| $\mu$ | koeficient trenja med steno in materialom |
| $m$ | masa shranjenega materiala |
| $m_{silo}$ | masa silosa skupaj s shranjenim materialom |
| $m_{sup}$ | del mase podporne konstrukcije, ki je relevanten za seizmično analizo |
| $m_{sup,total}$ | celotna masa podporne konstrukcije |
| $q$ | faktor obnašanja |
| $\rho$ | gostota zraka |
| $Re$ | Reynoldsovo število |
| $S$ | faktor zemljine |
| $S_d\,(T)$ | projektna vrednost v spektru pospeškov |
| $T$ | nihajni čas |
| $T_B$, $T_C$ | limitni vrednosti konstantnega dela spektra pospeškov |
| $T_D$ | vrednost nihajnega časa, pri kateri se začne območje konstantne vrednosti spektralnega pomika |
| $\Theta_m$ | rotacijska masa podporne konstrukcije |
| $u$ | celotni pomik masnega središča silosa zaradi delovanja potresne obremenitve |
| $u_v$ | pomik masnega središča silosa zaradi deformacije vodoravne vzmeti |
| $u_z$ | pomik masnega središča silosa zaradi deformacije rotacijske vzmeti |
| $v_b$ | osnovna hitrost vetra |
| $\omega$ | kotna frekvenca |

# I. DEL – OBTEŽBE NA SILOSE

## 2 Vrste obtežb

Obtežbe na silose se določajo ob upoštevanju zunanje geometrije, topografskih značilnosti okolice, shranjenega materiala in materialnih tokov, ki nastajajo ob polnitvi in izpraznitvi materiala iz silosa.

V standardu *EN 1991-4:2006, Priloga A* so določene naslednje obtežbe, ki jih je potrebno upoštevati pri projektiranju silosov; obtežbe, ki so podrobneje obravnavane v tej nalogi, so napisane v mastnem tisku in označene s sivo barvo.

**Preglednica 5: Obtežbe na silosih**

| |
|---|
| Lastna teža silosove lupine, |
| **polnjenje in hramba sipkega materiala (obtežba zaradi polnjenja),** |
| **praznjenje sipkega materiala (obtežba zaradi praznjenja),** |
| vsiljene obtežbe (obtežbe zaradi nekonstrukcijskih elementov), |
| obtežba snega, |
| **obtežba vetra**, |
| temperaturna obtežba, |
| vsiljene deformacije (obtežbe zaradi posedanja temeljev), |
| **potresna obremenitev**, |
| obtežbe zaradi eksplozije prahu v silosu. |

V sledečih poglavjih so bolj podrobno obravnavane obtežbe zaradi polnjenja in praznjenja shranjenega sipkega materiala, vetra in potresa.

# 3 Obtežbe zaradi shranjenega materiala

V silosih shranjujemo organske in anorganske sipke materiale. Pri določanju obtežb na stene silosa zaradi shranjenega materiala je potrebno razlikovati med procesoma polnjenja in praznjenja silosa. Obtežbe na stene silosa pri praznjenju shranjenega materiala so nekoliko večje od obtežb, ki nastanejo pri polnjenju, oz. hrambi materiala. Vzrok temu so dodatni materialni tokovi znotraj materiala, ki se pojavijo, ko material izteka iz silosa.

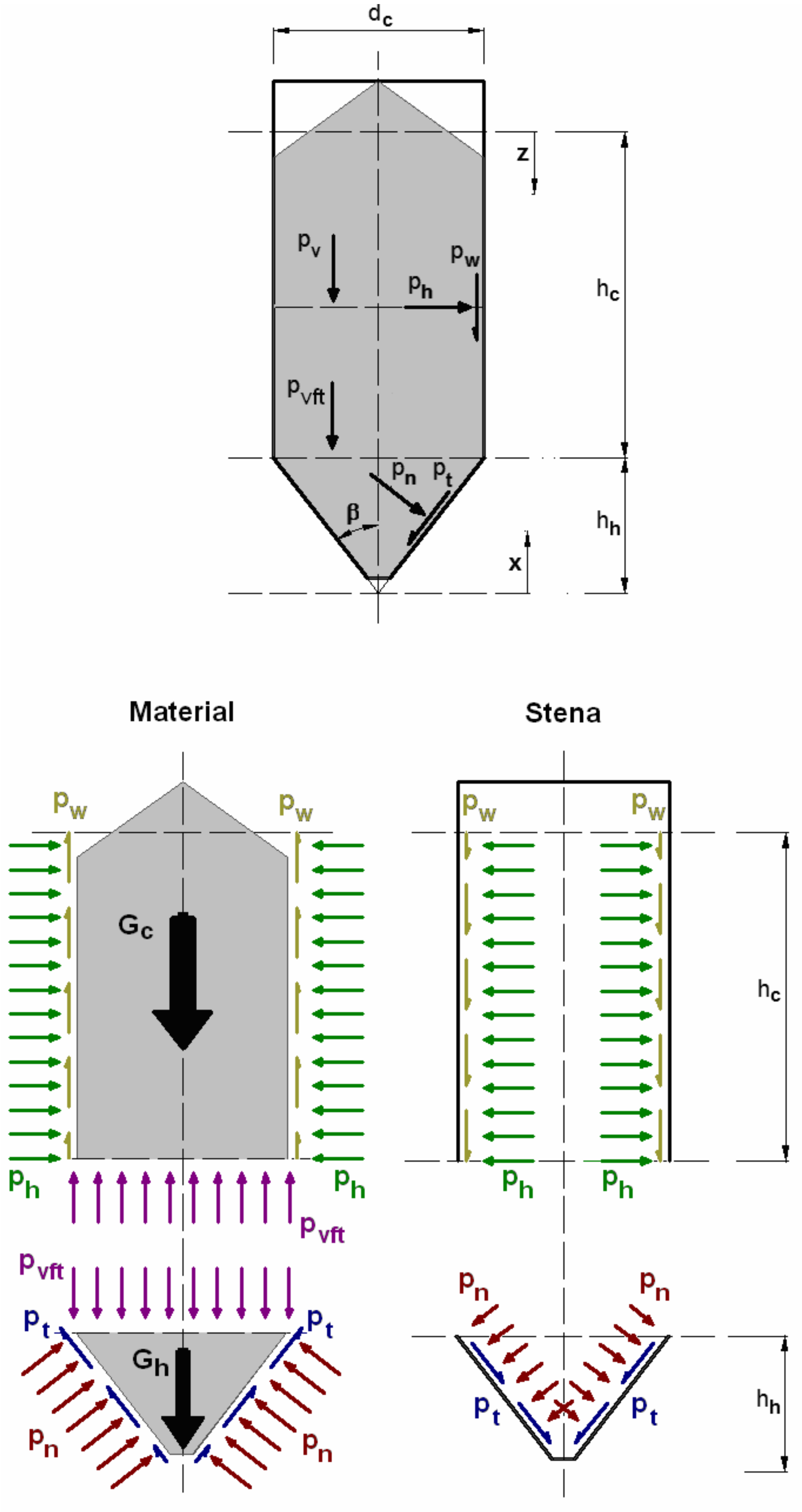
Ker imamo opravka z osno simetričnimi silosi, so ekscentričnosti, ki nastanejo med procesom polnjenja in praznjenja, zgolj naključne in izvirajo iz nepopolnosti materiala in same izdelave silosa. Zato lahko predpostavimo, da so te ekscentričnosti majhne in nimajo bistvenega vpliva na samo obremenitev sten silosa.

Obtežbe na stene silosa (z upoštevanjem majhnih ekscentričnosti) so v tej nalogi ponazorjene z osno simetrično obtežbo (ang. »axisymetric load«) in z nesimetrično obtežbo (ang. »patch load«).

## 3.1 Osno simetrična obtežba

Osno simetrična obtežba je ponazorjena z vodoravnim pritiskom $p_h$, ki deluje na notranji del navpične stene cilindra, normalnim pritiskom $p_n$, ki deluje na notranji del poševne steno v lijaku, s trenjem na navpični steni cilindra $p_w$ in poševni steni lijaka $p_t$ ter z navpičnim pritiskom $p_v$ v shranjenem materialu. Navpični pritisk v materialu na prehodu iz cilindričnega dela v lijak, je označen z $p_{vft}$.

Sili $G_c$ in $G_h$ na spodnji sliki ponazarjata težo shranjenega materiala v cilindru in lijaku.



Slika 2: Osno simetrični pritiski v shranjenem materialu in na stenah silosa

### 3.1.1 Postopek za določitev obtežbe

Postopek, ki je podan v tem poglavju, upošteva določila standarda *EN 1991-4:2006* in se uporablja za določitev osno simetrične obtežbe zaradi shranjenega materiala na stene silosa

**Preglednica 6: Pregled postopka**

| Korak | Opis koraka | Lokacija v standardu EN 1991-4 |
|---|---|---|
| 1 | Določitev lastnosti shranjenega materiala | Preglednica E.1 |
| 2 | Določitev geometrijskih parametrov silosa | Slika 1.1a |
| 3 | Določitev tipa silosa in lijaka | Poglavje 5.1(2)P in 6.1.1(2)P |
| 4 | Določitev razreda obremenitve | Preglednica 2.1 |
| 5 | Določitev ustreznih kombinacij materialnih parametrov | Preglednica 3.1 |
| 6 | Določitev obtežb na cilinder silosa | Poglavje 5 |
| 7 | Določitev obtežb na lijak silosa | Poglavje 6 |

### 3.1.1.1 Korak 1: Lastnosti shranjenega materiala

Parametri, ki določajo lastnosti shranjenega materiala, so podani v *Prilogi C*.

**Preglednica 7: Lastnosti shranjenega materiala**

| Materialni parameter | Opis |
|---|---|
| $\gamma_{min}$ | specifična teža<br>(minimum in maksimum) |
| $\gamma_{max}$ | |
| $\phi_r$ | kot deponiranja materiala |
| $\phi_{i,min} = \phi_{im} / a_\phi$ | kot notranjega trenja oz. strižni kot materiala<br>(minimum, srednja vrednost in maksimum) |
| $\phi_{im}$ | |
| $\phi_{i,max} = \phi_{im} \cdot a_\phi$ | |
| $K_{min} = K_m / a_K$ | koeficient bočnega pritiska<br>(minimum, srednja vrednost in maksimum) |
| $K_m$ | |
| $K_{max} = K_m \cdot a_K$ | |
| $\mu_{min} = \mu_m / a_\mu$ | koeficient trenja med steno in materialom<br>(minimum, srednja vrednost in maksimum) |
| $\mu_m$ | |
| $\mu_{min} = \mu_m \cdot a_\mu$ | |
| $C_{op}$ | referenčni faktor za nesimetrično obtežbo |

Simboli $\mu_m$, $K_m$, $\phi_{I,m}$ prestavljajo srednje vrednosti parametrov $\mu$, $K$, $\phi_I$. Za določitev maksimalnih pritiskov na stene cilindra in lijaka je potrebno upoštevati ustrezne kombinacije minimalnih in maksimalnih vrednosti danih materialnih parametrov. Te kombinacije so predstavljene v poglavju *3.1.1.4* (korak 5).

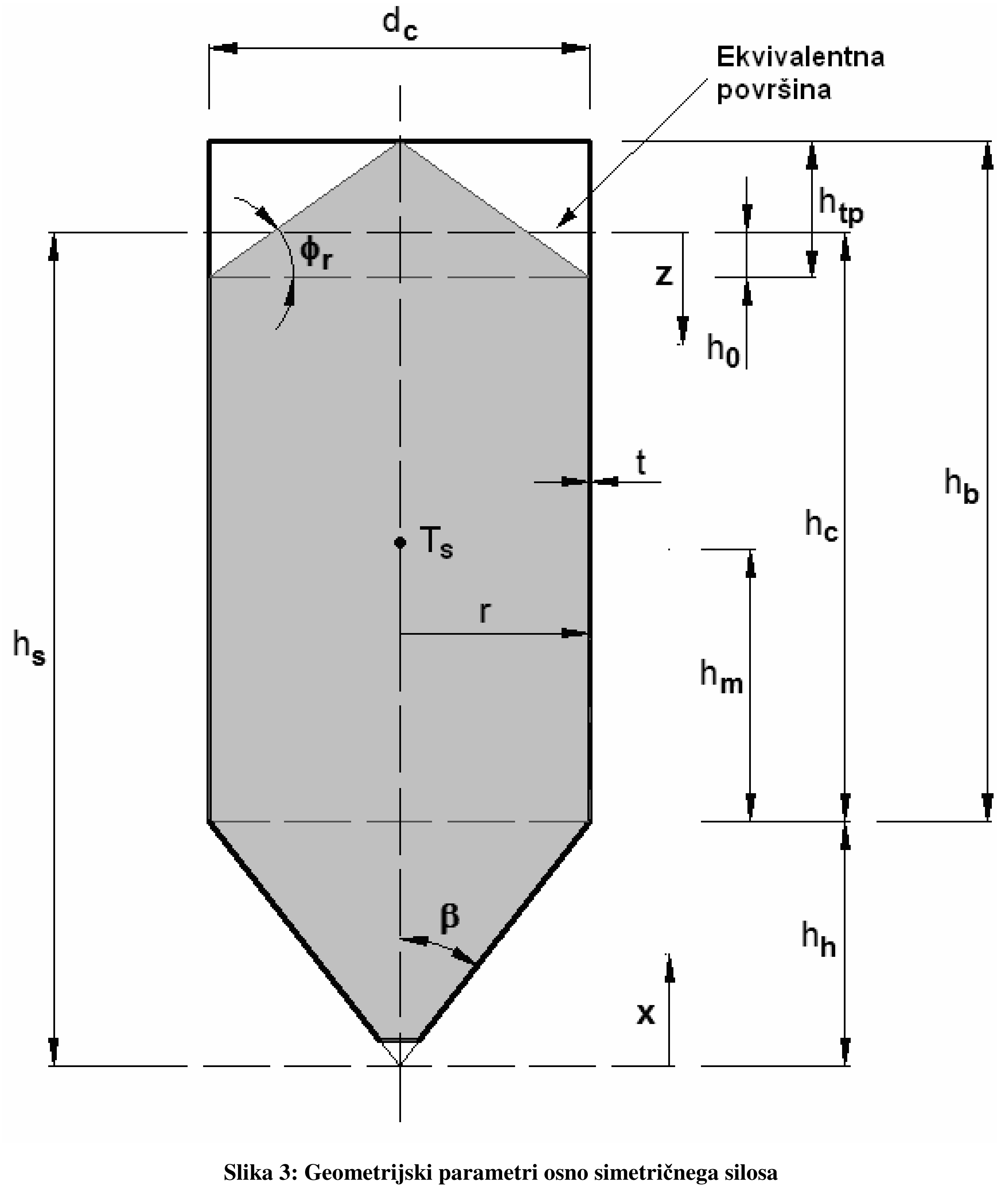
Srednjo vrednost koeficienta trenja med steno in materialom ($\mu_m$) je potrebno določiti ob upoštevanju ustrezne kategorije stene. Stene so, glede na trenje, razdeljene v štiri kategorije (od *D1* do *D4*). Različne vrednosti $\mu_m$ veljajo za različno kategorijo stene.

**Preglednica 8: Kategorija stene**

| Kategorija | Trenje | Material |
|---|---|---|
| **D1** | Nizko | Hladno oblikovano nerjaveče jeklo,<br>polirano nerjaveče jeklo,<br>površina zaščitena s premazom. |
| **D2** | Srednje | Gladko jeklo z visoko vsebnostjo ogljika,<br>nepolirano nerjaveče jeklo,<br>galvanizirano jeklo. |
| **D3** | Visoko | Postarano (korodirano) jeklo,<br>jeklo odporno na abrazijo. |
| **D4** | Neenakomerno | Vodoravno nagubane stene,<br>profilirana pločevina,<br>nestandardne oblike sten. |

Če je kategorija stene enaka *D4*, potem je potrebno koeficient $\mu_m$ določiti v skladu z *EN 1991-4, Priloga D*.

### 3.1.1.2 Korak 2: Geometrijski parametri



**Slika 3: Geometrijski parametri osno simetričnega silosa**

**Neodvisni parametri**

Višina cilindra: $h_b$

Radij cilindra: $r$

Kot lijaka glede na simetrijsko os: $\beta$

Debelina stene: $t$

**Odvisni parametri**

Višina lijaka:
$$h_h = \frac{r}{tan\,\beta} \tag{I.1}$$

Višina zgornjega kupa:
$$h_{tp} = r \cdot tan\,\phi_r \tag{I.2}$$

Globina pod ekvivalentno površino:
$$h_0 = \frac{1}{3} \cdot h_{tp} \tag{I.3}$$

Ekvivalentna višina cilindra:
$$h_c = h_b - h_{tp} + h_0 \tag{I.4}$$

Ekvivalentna višina shranjenega materiala:
$$h_s = h_h + h_c < 100m \tag{I.5}$$

Premer cilindra:
$$d_c = 2 \cdot r < 60m \tag{I.6}$$

Dodatna geometrijska omejitev:
$$h_s / d_c < 10 \tag{I.7}$$

Obseg silosa:
$$U = \pi \cdot d_c \tag{I.8}$$

Prečni prerez silosa:
$$A = \pi \cdot \frac{{d_c}^2}{4} \tag{I.9}$$

Volumen shranjenega materiala:
$$V_m = A \cdot \left( h_c - h_0 + \frac{1}{3}\left(h_h + h_{tp}\right)\right) \tag{I.10}$$

Teža shranjenega materiala:
$$G_m = \gamma_{max} \cdot V_m \tag{I.11}$$

Teža lupine silosa:
$$G_s = 2\pi t \left( r + \frac{t}{2} \right) \cdot \left( h_b + \frac{1}{3} \cdot h_h \right) \cdot \gamma_{stena} \tag{I.12}$$

Težišče silosa:
$$h_m = \frac{1 + \frac{1}{18} \cdot \left( \frac{h_{tp}}{h_c} \right)^2 - \frac{1}{6} \cdot \left( \frac{h_h}{h_c} \right)^2}{1 + \frac{1}{3} \cdot \frac{h_h}{h_c}} \cdot \frac{h_c}{2} \tag{I.13}$$

Vztrajnostni moment cilindra (okoli vodoravne osi):

$$I = \frac{1}{8}\pi t \cdot (d_c + t) \cdot \left( d_c^{\,2} + 2 d_c t + 2t^2 \right) \qquad \text{(I.14)}$$

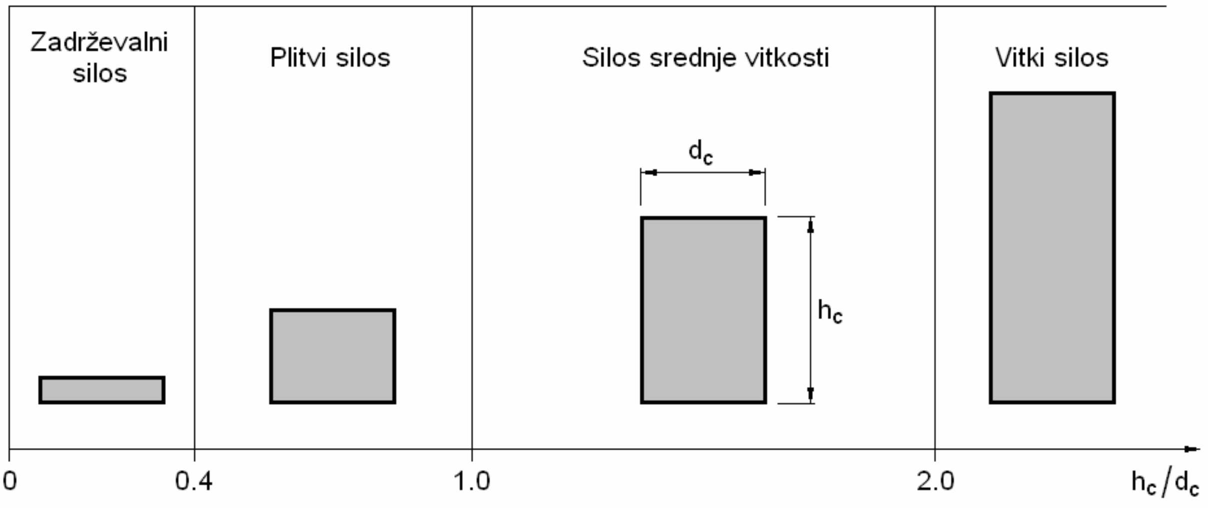
Če je stena silosa sestavljena iz več delov različne debeline $t$, se lahko pri računu teže shranjenega materiala $G_m$ in vztrajnostnega momenta cilindra $I$, upošteva povprečna debelina stene $\bar{t}$.

### 3.1.1.3 Korak 3: Tip cilindra in lijaka

Cilindri in lijaki, ki predstavljajo glavne sestavne dele lupine silosa, se po svoji obliki, oz. naklonu stene, delijo na več tipov.

**Tipi cilindrov:**

Tipi cilindrov so predstavljeni na spodnji sliki. Ko govorimo o klasifikaciji celotnega silosa, poimenujemo celoten silos kar glede na tip cilindra, saj je v cilindru shranjena večina materiala.



Slika 4: Različni tipi cilindrov oz. silosov

**OPOMBA:** V zgornjem kontekstu se izraz »vitkost« ne nanaša na lupino, ampak na globalno geometrijo cilindra.

Ker v času pisanja naloge, še nobeden od EN standardov, ki se nanašajo na silose, ni bil preveden v slovenščino, so navedeni še angleški izrazi za tipe silosov:

- zadrževalni silos – ang. »retaining silo«,
- plitvi silos – ang. »squat silo«,
- silos srednje vitkosti – ang. »intermediate slenderness silo«,
- vitki silos – ang. »slender silo«.

**Tipi lijakov:**

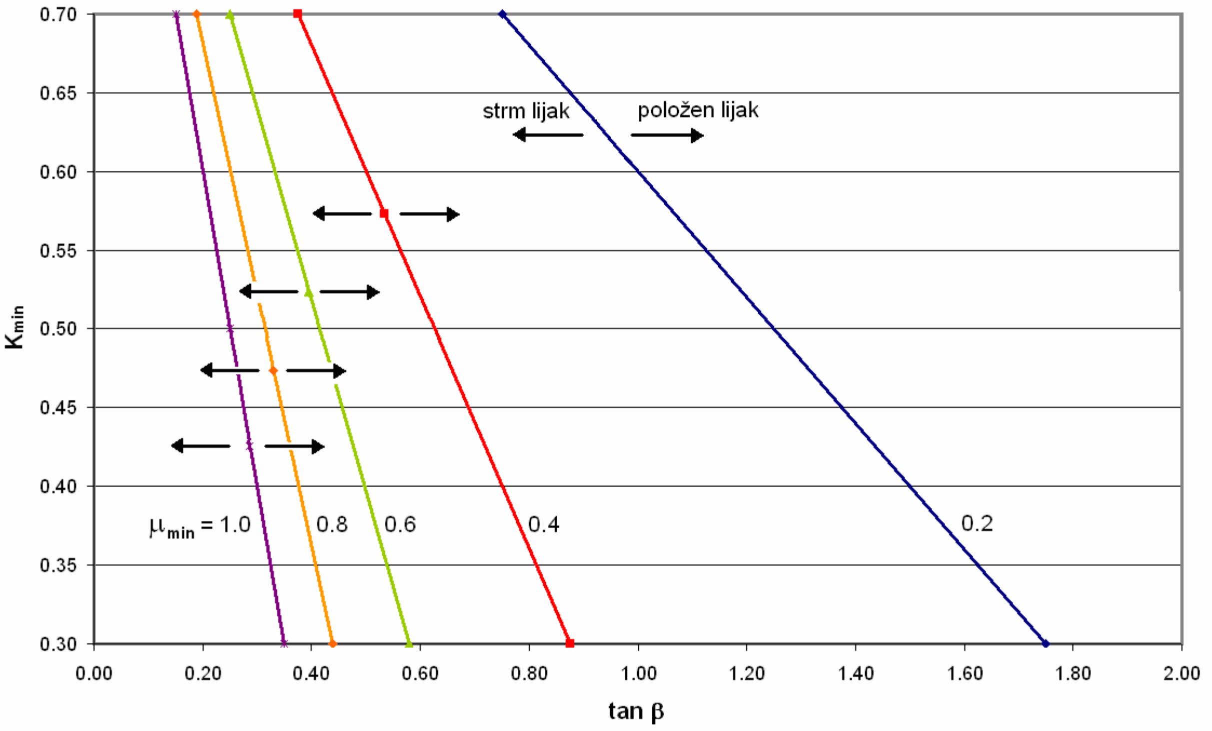
Lijaki se, glede na naklonski kot $\beta$, minimalni koeficient bočnega pritiska $K$ in minimalni koeficient trenja med steno in materialom $\mu_m$, delijo v tri tipe, ki so predstavljeni v spodnji tabeli.

**Preglednica 9: Tipi lijakov**

| Tip lijaka | Pogoj | |
|---|---|---|
| Strm lijak | $K_{\min} <$ | $1-2\mu_{\min}\tan\beta$ (*glej Slika 5*) |
| Polóžen lijak | $K_{\min} \geq$ | |
| Ravno dno | $\beta \geq 85^\circ$ | |

Angleški izrazi za zgoraj omenjene tipe lijakov so:

- strm lijak – ang. »steep hopper«,
- polóžen lijak – ang. »shallow hopper«,
- ravno dno – ang. »flat bottom«.



**Slika 5: Tipi lijakov**

Zgornja slika služi lažji določitvi tipa lijaka. Uporabi se tako, da se najprej izbere ustrezno premico, glede na podan minimalni koeficient trenja med steno in materialom $\mu_m$. Če leži točka, ki jo določata minimalni koeficient bočnega pritiska $K$ in tangens naklonskega kota lijaka $tan\beta$, levo od premice, potem je lijak strm. V nasprotnem primeru je lijak položen.

### 3.1.1.4 Korak 4: Razred obremenitve

Glede na kapaciteto se silosi delijo v tri razrede obremenitve. Delitev je pomembna pri nadaljnjem računu obremenitev, saj pri silosih z majhno kapaciteto veljajo določene poenostavitve v računu.

Kapaciteta silosa je enaka masi shranjenega materiala:

$$m_{solid} = \frac{G_m}{g} \cong 0.1 \cdot G_m \tag{I.15}$$

Razred obremenitve (ang. »action assessment class«), ki ga glede na angleški izraz okrajšano imenujemo *AAC*, se nato določi glede na izračunano kapaciteto silosa. Posamezni razredi in njihove pripadajoče kapacitete so podane v spodnji preglednici.

**Preglednica 10: Definicija razredov obremenitve pri silosih**

| **Razred obremenitve (AAC)** | $m_{solid}$ |
|---|---|
| **1** | pod 100 ton |
| **2** | med 100 tonami in 10 000 tonami |
| **3** | več kot 10 000 ton |

### 3.1.1.5 Korak 5: Kombinacije materialnih parametrov

Materialni parametri, določeni v poglavju *3.1.1.1*, se uporabljajo za izračun obtežb zaradi shranjenega materiala na stene silosa. Maksimalne vrednosti teh obtežb dobimo z ustreznim kombiniranjem minimalnih, maksimalnih in srednjih vrednosti parametrov $\mu$, $K$ in $\phi_i$. Pri parametru $\gamma$ se vedno vzame maksimalne vrednost $\gamma_{max}$.

**Stena cilindra:**

Za izračun maksimalnih vrednosti pritiskov ($p_h$ in $p_w$) na stene cilindra med procesom polnjenja in praznjenja, je potrebno upoštevati kombinacije materialnih parametrov, ki so podane v spodnji preglednici.

**Preglednica 11: Kombinacije materialnih parametrov za določitev maksimalnih obtežb na stene cilindra**

| Kombinacija | AAC | Namen | $\mu$ | K | $\phi_i$ |
|---|---|---|---|---|---|
| 1 | 1 | maksimalni normalni pritisk ($p_h$) in maksimalno trenje ($p_w$) | MEAN | MEAN | MEAN |
| 2 | 2 in 3 | maksimalni normalni pritisk ($p_h$) | MIN | MAX | MIN |
| 3 |  | maksimalno trenje ($p_w$) | MAX | MAX | MIN |

**Opomba:** oznake *mean*, *min* in *max* pod posameznim parametrom določajo katero vrednost parametra (srednjo, minimalno ali maksimalno) je potrebno upoštevati v posamezni kombinaciji.

## Stena lijaka:

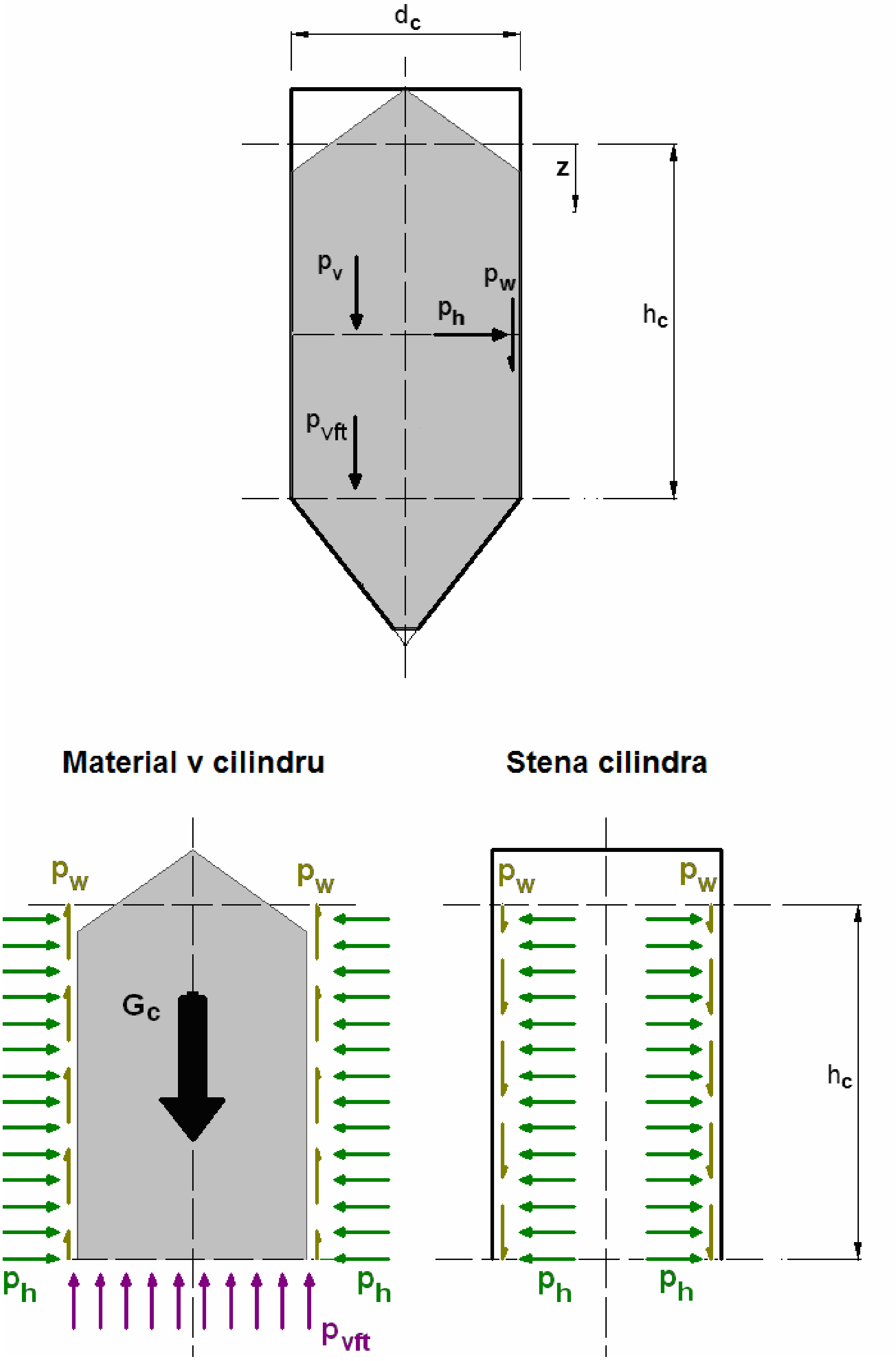
Pri računi maksimalnih obtežb na stene lijaka, je potrebno upoštevati kombinacije materialnih parametrov, ki povzročajo maksimalne obtežbe med procesom polnjenja in praznjenja. Tukaj torej ni potrebno ločeno upoštevati kombinacij za maksimalni horizontalni pritisk oz. trenje, saj se pri določeni kombinaciji maksimalni vrednosti pritiskov pojavita sočasno (kombinacija za maksimalni normalni pritisk $p_n$ povzroči tudi maksimalno trenje na stenah lijaka $p_t$).

Ker se vertikalni pritisk v materialu, ki je shranjen v cilindru ($p_v$), prenese v lijak, ima torej vpliv na normalen pritisk $p_n$ in trenje po steni lijaka $p_t$. Zato je potrebno v kombinaciji za maksimalne obtežbe v lijaku med procesom polnjenja in praznjenja upoštevati tudi kombinacijo za določitev maksimalnega vertikalnega pritiska v materialu, ki je shranjen v cilindru ($p_v$).

**Preglednica 12: Kombinacije materialnih parametrov za določitev maksimalnih obtežb na stene lijaka**

| Kombinacija | AAC | Velja za | Namen | $\mu$ | K | $\phi_i$ |
|---|---|---|---|---|---|---|
| 4 | 1, 2 in 3 | steno cilindra | maksimalni vertikalni pritisk ($p_v$) | MIN | MIN | MAX |
|  |  | steno lijaka | maksimalni pritisk ob polnjenju | MIN | MIN | MIN |
| 5 |  | steno cilindra | maksimalni vertikalni pritisk ($p_v$) | MIN | MIN | MAX |
|  |  | steno lijaka | maksimalni pritisk ob praznjenju | MIN | MAX | MAX |

### 3.1.1.6 Korak 6: Obtežbe na stene cilindra



**Slika 6: Pritiski, ki jih povzroča shranjen material na stene cilindra**

Za izračun pritiskov na stene cilindra, ki nastanejo pri procesu polnjenja in praznjenja silosa s sipkim materialom, je potrebno uporabiti proceduro opisano na *Diagramu 1*. Proceduro je potrebno opraviti za vsako kombinacijo opisano v *Preglednici 11*, ob upoštevanju ustreznega razreda obremenitve.

**Diagram 1: Določitev obtežbe na steno cilindra**

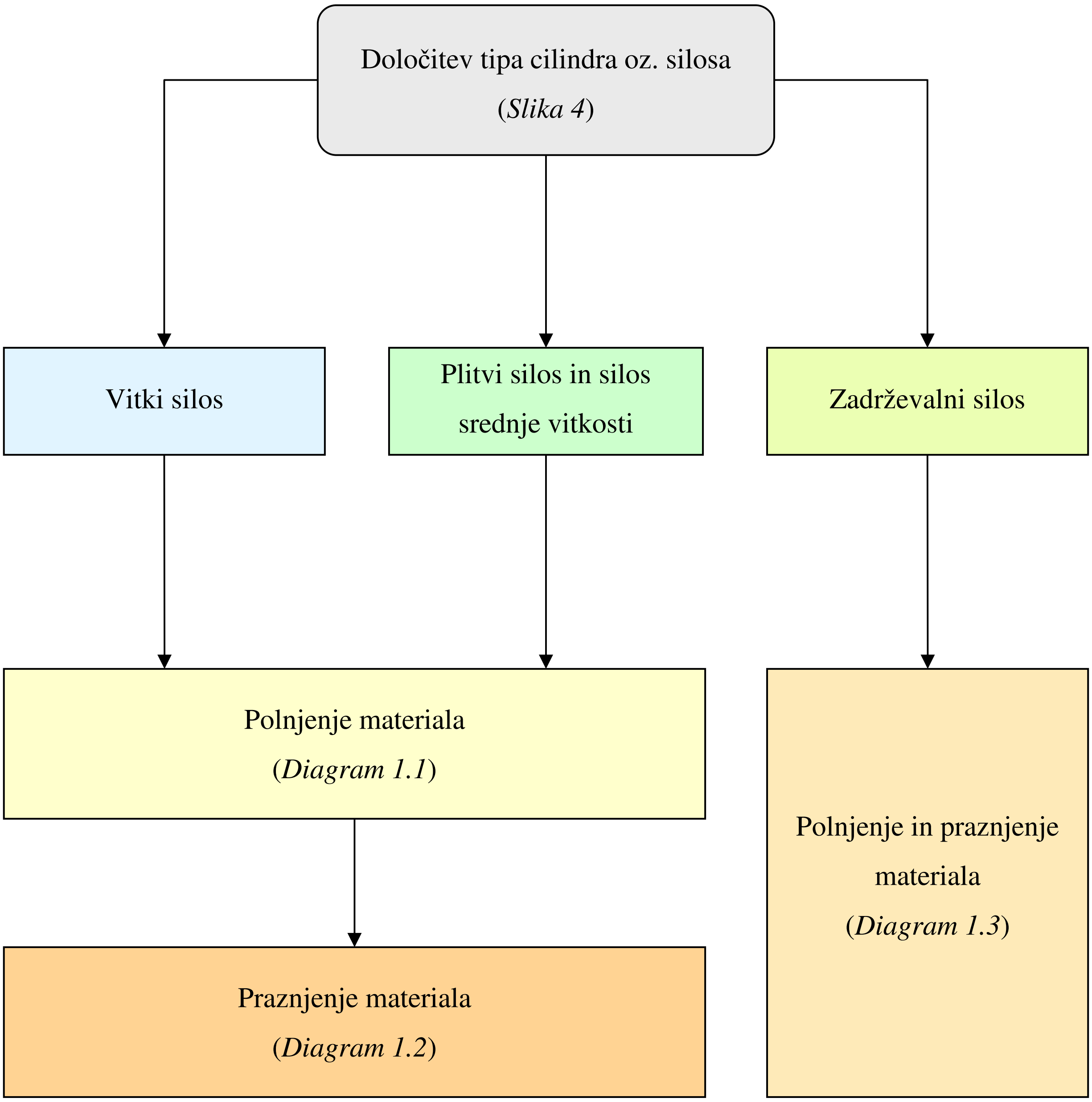


**Diagram 1.1**

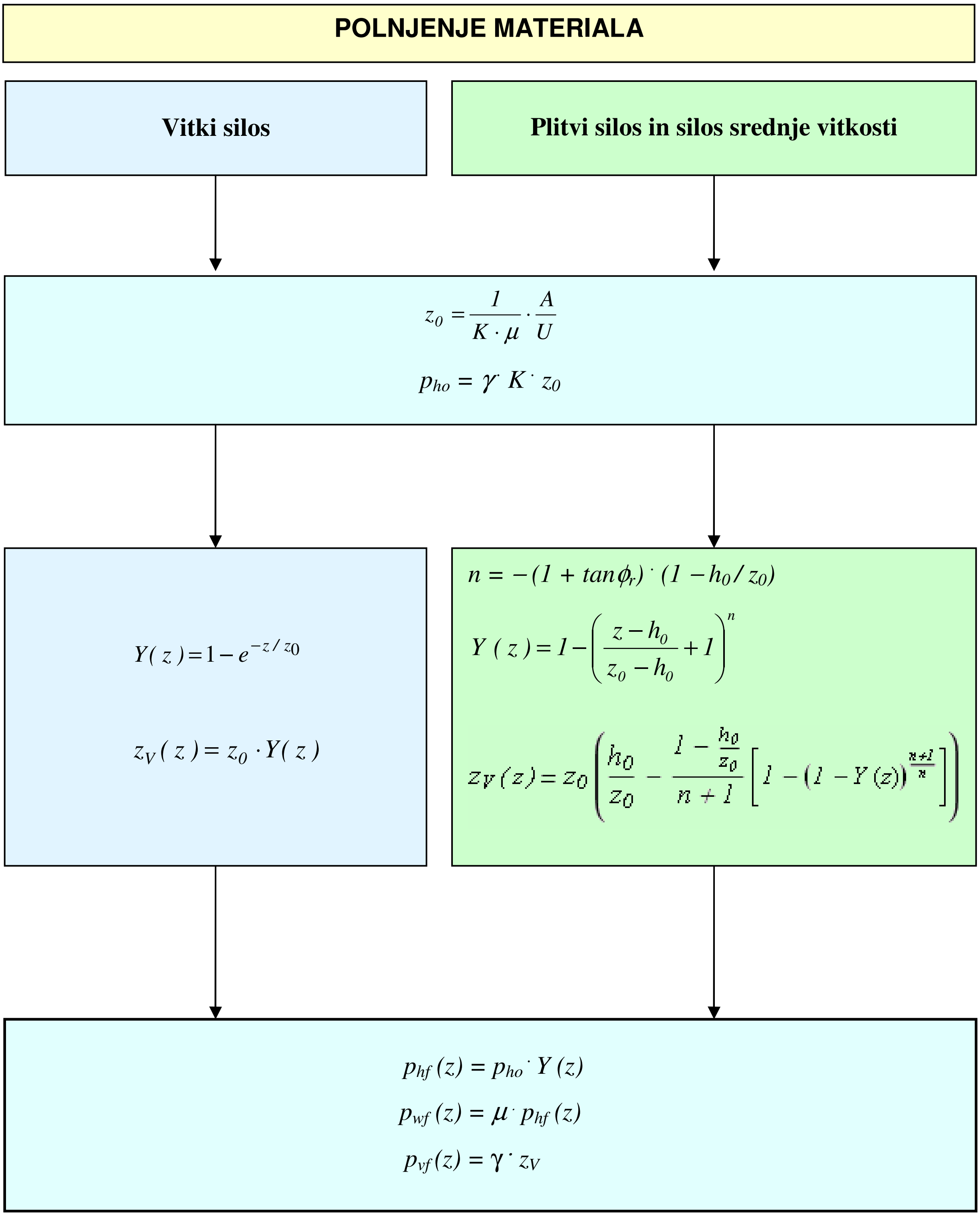

POLNJENJE MATERIALA
Vitki silos
Plitvi silos in silos srednje vitkosti
$z_0 = \frac{1}{K \cdot \mu} \cdot \frac{A}{U}$
$p_{ho} = \gamma \cdot K \cdot z_0$
$Y(z) = 1 - e^{-z/z_0}$
$z_V(z) = z_0 \cdot Y(z)$
$n = -(1 + \tan\phi_r) \cdot (1 - h_0/z_0)$
$Y(z) = 1 - \left( \frac{z - h_0}{z_0 - h_0} + 1 \right)^n$
$z_V(z) = z_0 \left( \frac{h_0}{z_0} - \frac{1 - \frac{h_0}{z_0}}{n + 1} \left[ 1 - (1 - Y(z))^{\frac{n+1}{n}} \right] \right)$
$p_{hf}(z) = p_{ho} \cdot Y(z)$
$p_{wf}(z) = \mu \cdot p_{hf}(z)$
$p_{vf}(z) = \gamma \cdot z_V$

**Diagram 1.2**

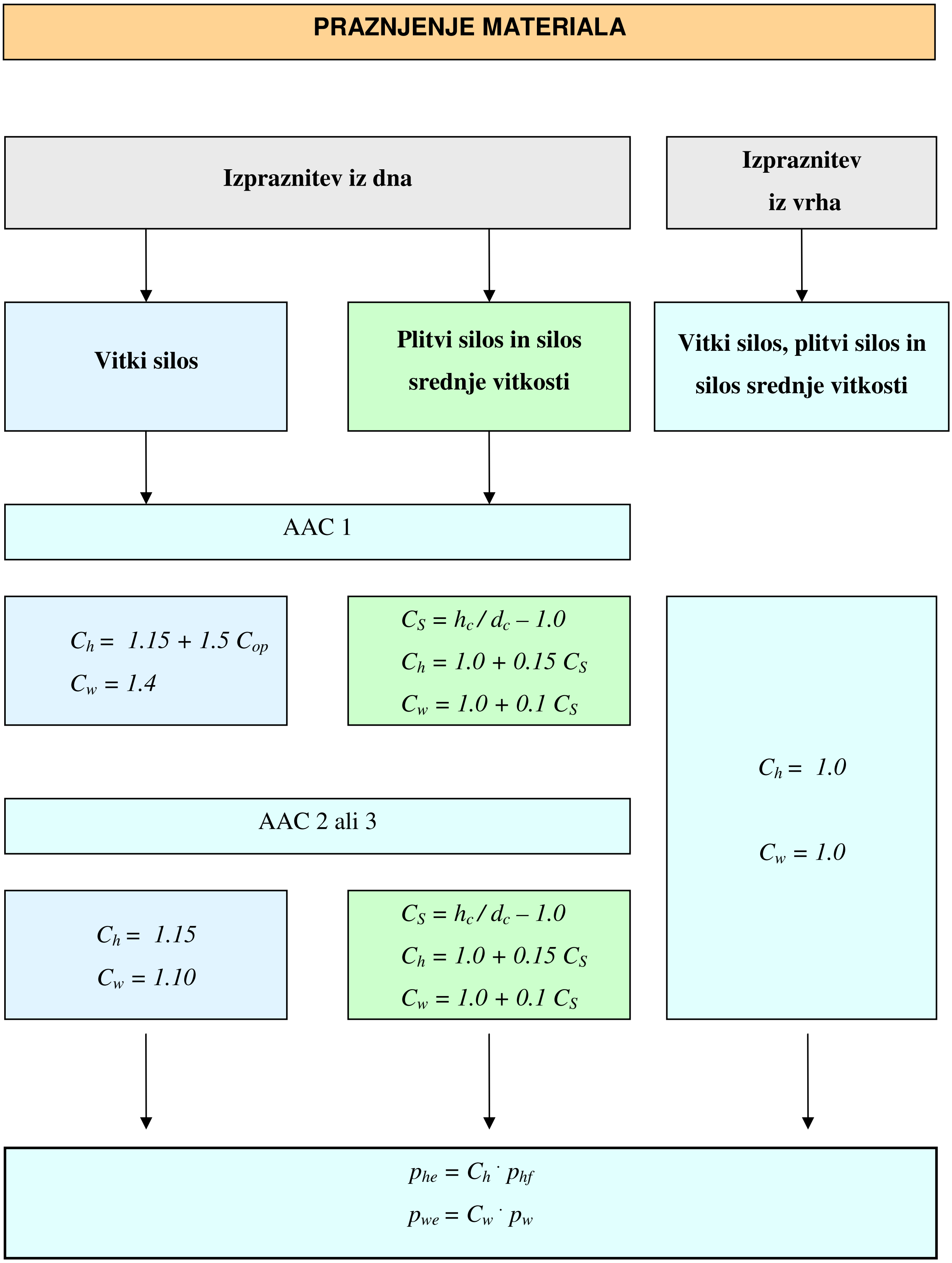
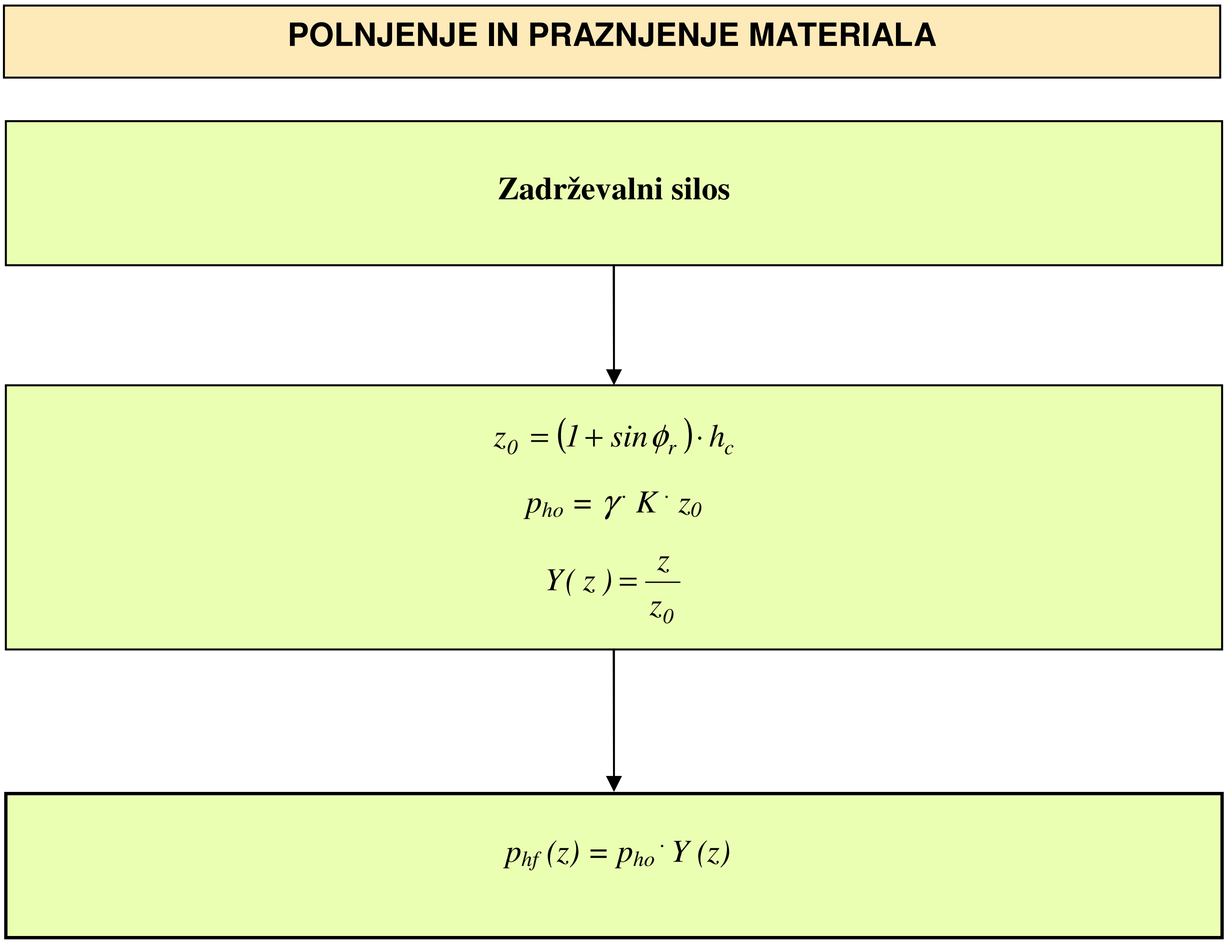

PRAZNJENJE MATERIALA
Izpraznitev iz dna
Izpraznitev iz vrha
Vitki silos
Plitvi silos in silos srednje vitkosti
Vitki silos, plitvi silos in silos srednje vitkosti
AAC 1
$C_h = 1.15 + 1.5\,C_{op}$
$C_w = 1.4$
$C_S = h_c / d_c - 1.0$
$C_h = 1.0 + 0.15\,C_S$
$C_w = 1.0 + 0.1\,C_S$
$C_h = 1.0$
$C_w = 1.0$
AAC 2 ali 3
$C_h = 1.15$
$C_w = 1.10$
$C_S = h_c / d_c - 1.0$
$C_h = 1.0 + 0.15\,C_S$
$C_w = 1.0 + 0.1\,C_S$
$p_{he} = C_h \cdot p_{hf}$
$p_{we} = C_w \cdot p_w$

**Diagram 1.3**



**POLNJENJE IN PRAZNJENJE MATERIALA**

**Zadrževalni silos**

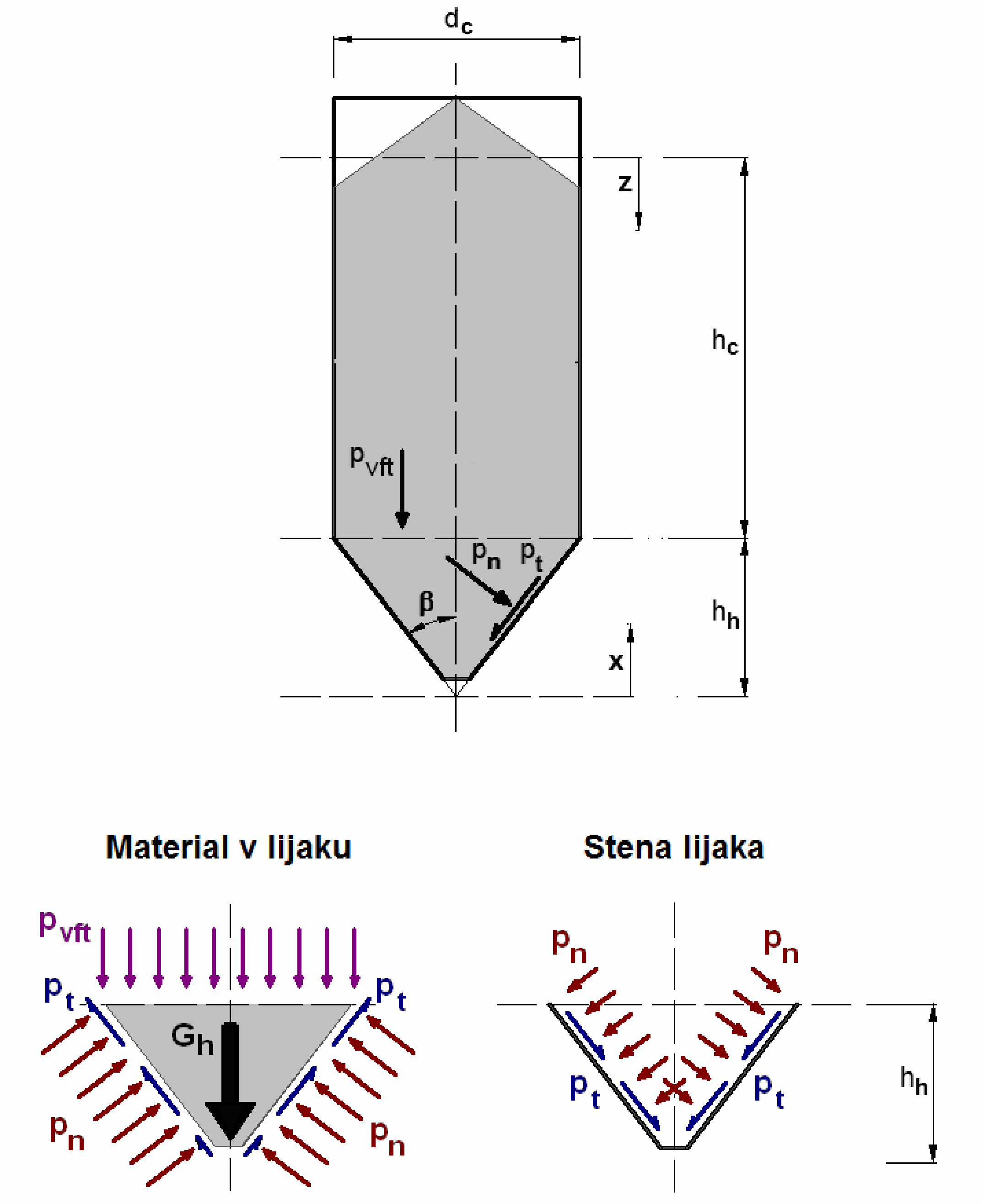
$$z_0 = (1 + \sin\phi_r) \cdot h_c$$

$$p_{ho} = \gamma \cdot K \cdot z_0$$

$$Y(z) = \frac{z}{z_0}$$

$$p_{hf}(z) = p_{ho} \cdot Y(z)$$

### 3.1.1.7 Korak 7: Obtežbe na stene lijaka



**Slika 7: Pritiski, ki jih povzroča shranjen material na stene lijaka**

Podobno kot pri računu obtežb na steno cilindra, je pri izračunu pritiskov na stene lijaka potrebno uporabiti proceduro opisano na *Diagramu 2*. Proceduro je potrebno opraviti za vsako kombinacijo parametrov, ki je podana v *Preglednici 12*.

Spodnja preglednica je namenjena izračunu faktorja dodatne povečave vertikalnega pritiska $C_b$, ki deluje na vertikalni pritisk v materialu, na prehodu iz cilindra v lijak.

**Preglednica 13: Faktor povečave pritiska ($C_b$)**

| | **AAC** | **$C_b$** |
|---|---|---|
| **STATIČEN oz. Standarden** | 1 | 1.3 |
| | 2 and 3 | 1.0 |
| **DINAMIČEN** | 1 | 1.6 |
| | 2 and 3 | 1.2 |

Dinamičen faktor povečave je potrebno uporabiti, kadar obstaja velika verjetnost, da se bodo v shranjenem materialu pojavili vplivi dinamičnega obremenjevanja. Takšni vplivi se lahko pojavijo če:

- v *vitkem* silosu shranjujemo materiale, ki nimajo nizke kohezije (več o tem najdemo v standardu *EN 1991-4,* poglavju *1.5.24*),
- v silosu shranjujemo material, v katerem se zrna lahko mehansko zaklinijo (npr. cementni klinker).

**Diagram 2: Določitev obtežbe na steno lijaka**

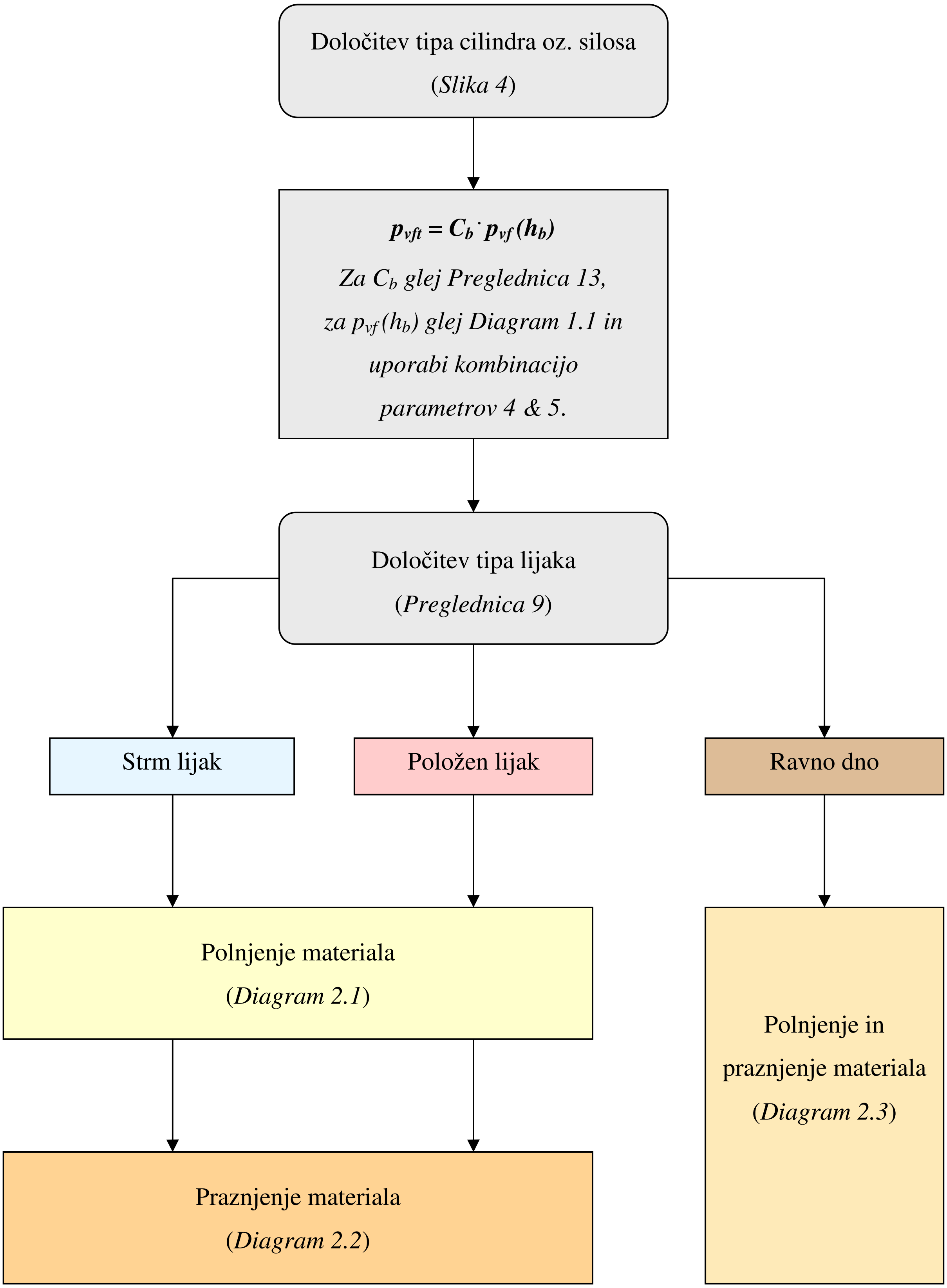


**Diagram 2.1**

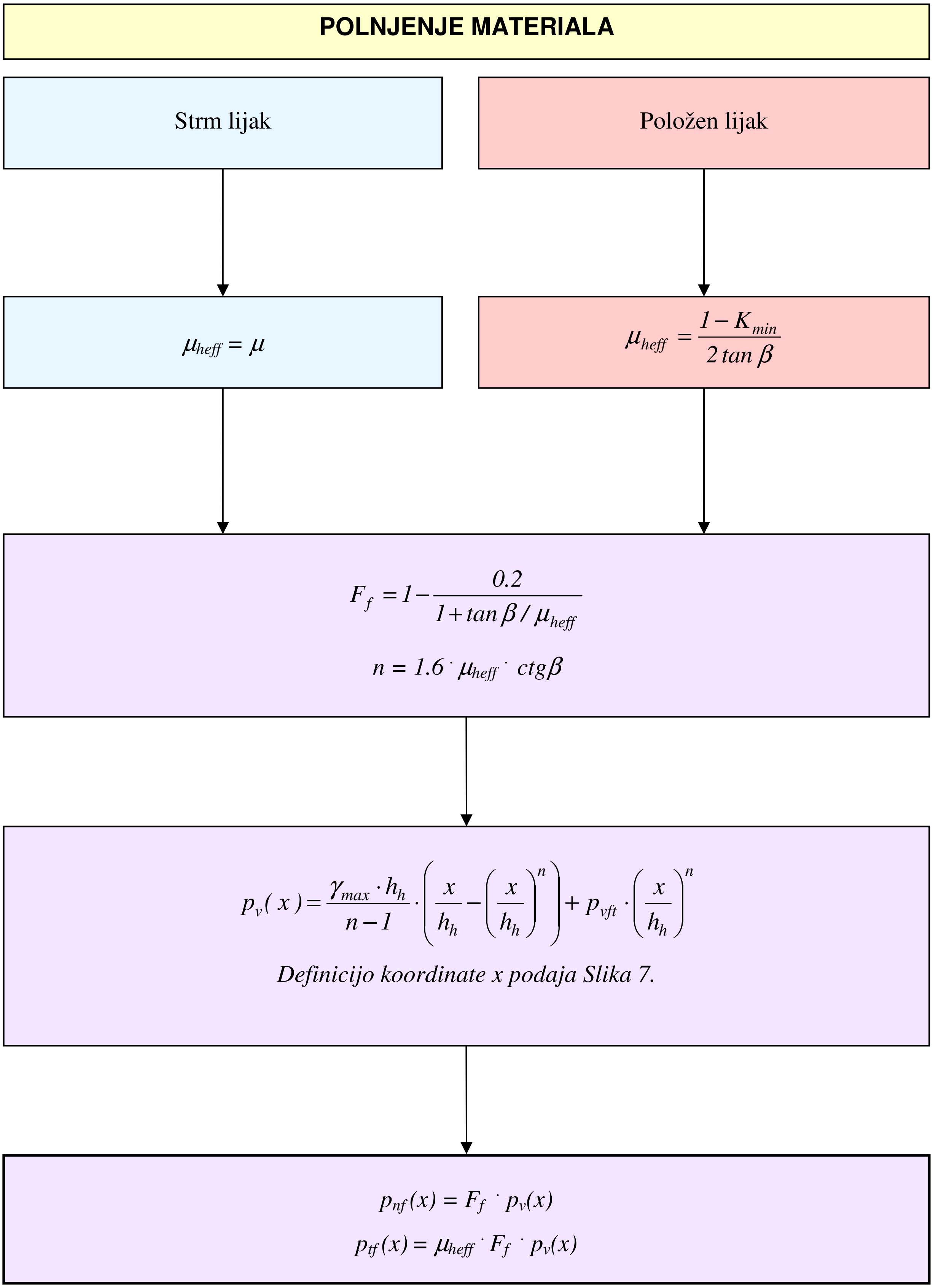
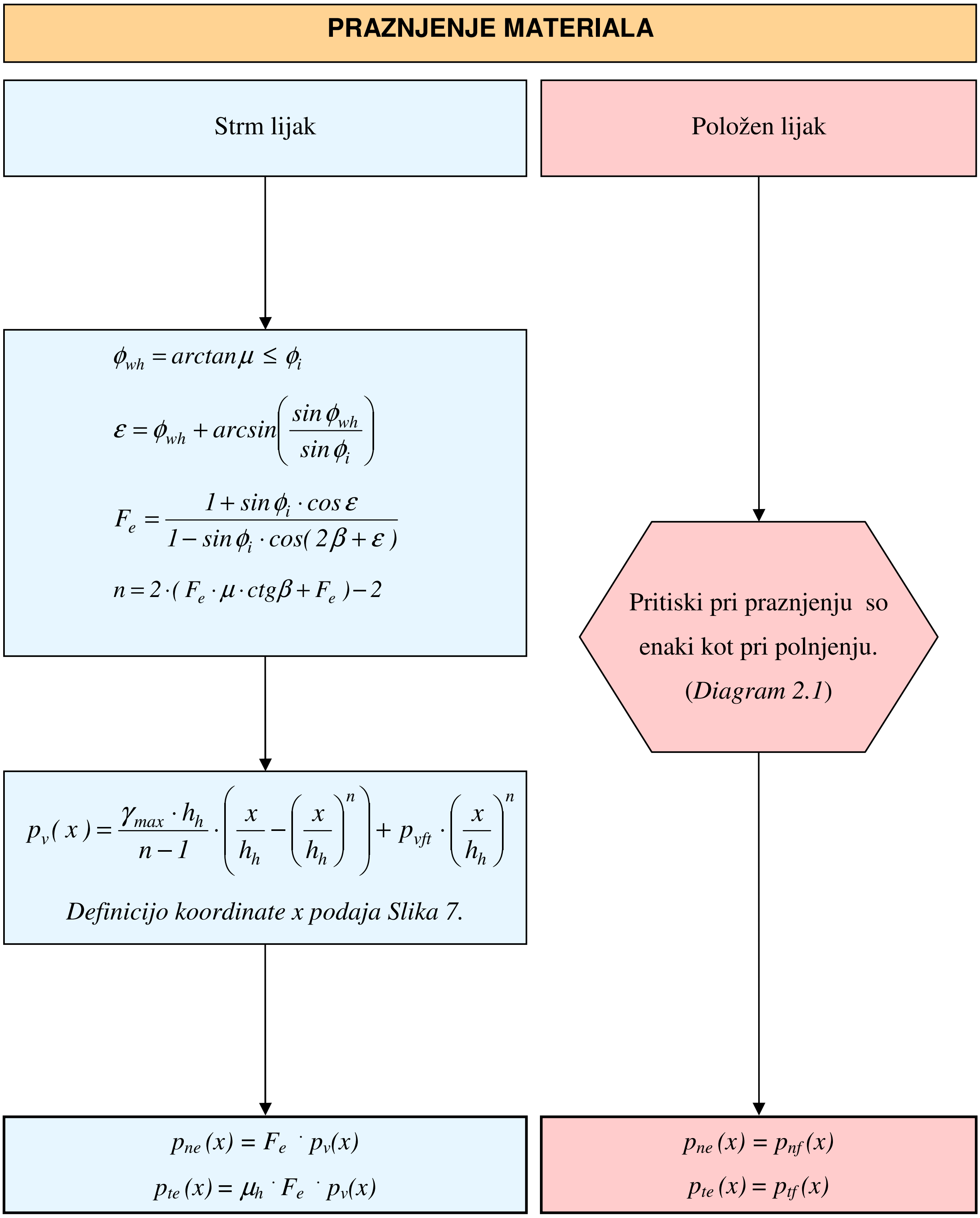

POLNJENJE MATERIALA
Strm lijak
Položen lijak
$\mu_{heff} = \mu$
$\mu_{heff} = \frac{1-K_{min}}{2\tan\beta}$
$F_f = 1 - \frac{0.2}{1+\tan\beta/\mu_{heff}}$
$n = 1.6 \cdot \mu_{heff} \cdot ctg\beta$
$p_v(x) = \frac{\gamma_{max} \cdot h_h}{n-1} \cdot \left( \frac{x}{h_h} - \left( \frac{x}{h_h} \right)^n \right) + p_{vft} \cdot \left( \frac{x}{h_h} \right)^n$
Definicijo koordinate x podaja Slika 7.
$p_{nf}(x) = F_f \cdot p_v(x)$
$p_{tf}(x) = \mu_{heff} \cdot F_f \cdot p_v(x)$

**Diagram 2.2**



**PRAZNJENJE MATERIALA**

Strm lijak

$$\phi_{wh} = arctan\,\mu \le \phi_i$$

$$\varepsilon = \phi_{wh} + arcsin\left(\frac{sin\,\phi_{wh}}{sin\,\phi_i}\right)$$

$$F_e = \frac{1 + sin\,\phi_i \cdot cos\,\varepsilon}{1 - sin\,\phi_i \cdot cos(\,2\beta + \varepsilon\,)}$$

$$n = 2 \cdot (\,F_e \cdot \mu \cdot ctg\beta + F_e\,) - 2$$

$$p_v(\,x\,) = \frac{\gamma_{max} \cdot h_h}{n-1} \cdot \left(\frac{x}{h_h} - \left(\frac{x}{h_h}\right)^n\right) + p_{vft} \cdot \left(\frac{x}{h_h}\right)^n$$

*Definicijo koordinate x podaja Slika 7.*

$$p_{ne}(x) = F_e \cdot p_v(x)$$

$$p_{te}(x) = \mu_h \cdot F_e \cdot p_v(x)$$

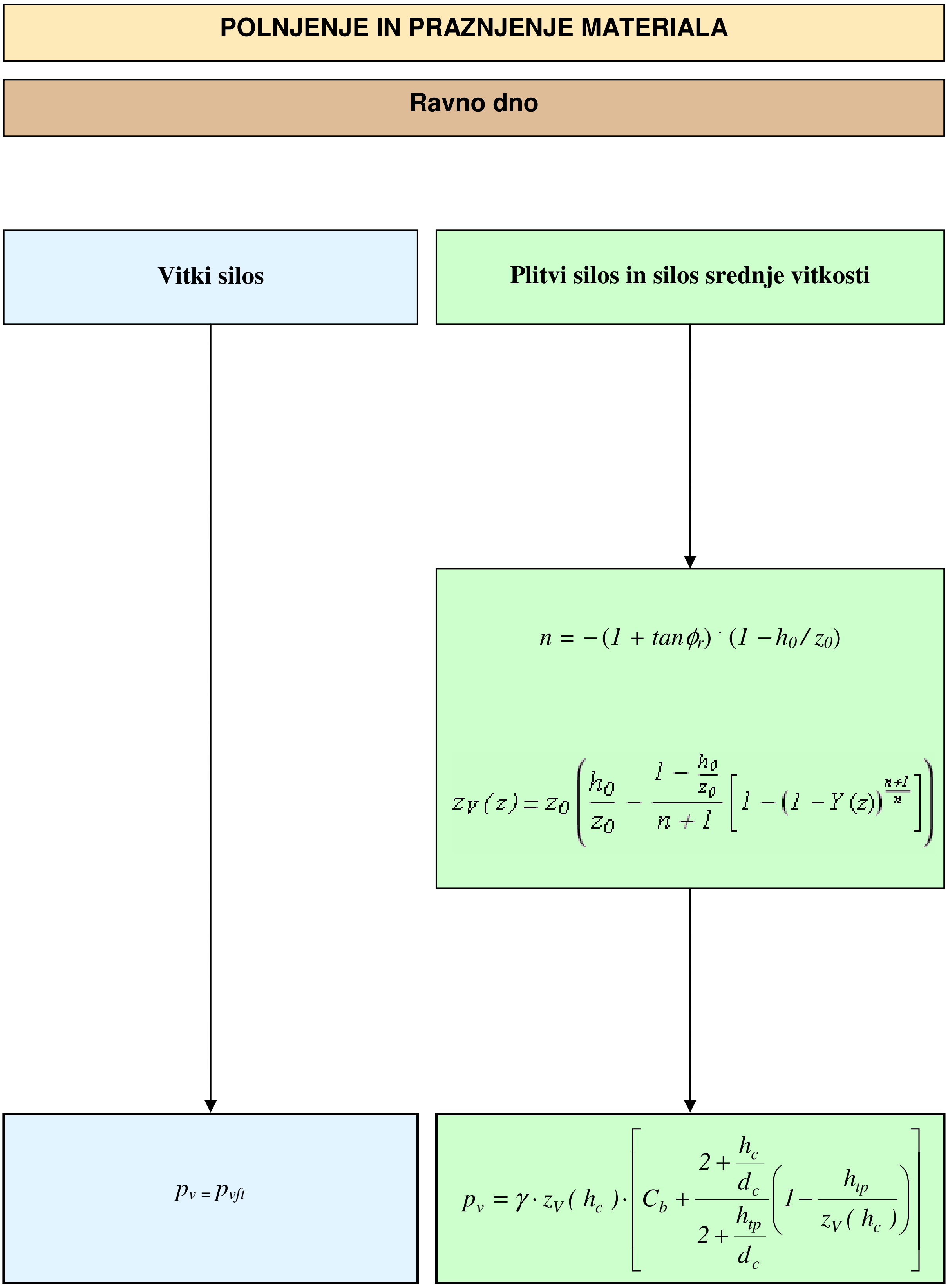
Položen lijak

Pritiski pri praznjenju so enaki kot pri polnjenju. (*Diagram 2.1*)

$$p_{ne}(x) = p_{nf}(x)$$

$$p_{te}(x) = p_{tf}(x)$$

**Diagram 2.3**


POLNJENJE IN PRAZNJENJE MATERIALA
Ravno dno
Vitki silos
Plitvi silos in silos srednje vitkosti
$n = -(1 + \tan\phi_r) \cdot (1 - h_0 / z_0)$
$z_V(z) = z_0 \left( \frac{h_0}{z_0} - \frac{1 - \frac{h_0}{z_0}}{n+1} \left[ 1 - (1 - Y(z))^{\frac{n+1}{n}} \right] \right)$
$p_v = p_{vft}$
$p_v = \gamma \cdot z_V(h_c) \cdot \left[ C_b + \frac{2 + \frac{h_c}{d_c}}{2 + \frac{h_{tp}}{d_c}} \left( 1 - \frac{h_{tp}}{z_V(h_c)} \right) \right]$

## 3.1.2 Grafična predstavitev obtežb na stene silosa

### 3.1.2.1 Obtežba na steno cilindra

V tem poglavju je predstavljen grafični potek pritiskov ($p_h$ in $p_w$), s katerima ponazorimo obtežbo na steno cilindra zaradi shranjenega materiala. Predstavljen je tudi potek vertikalnega pritiska $p_v$, ki deluje v shranjenem materialu. Da bi bili grafi veljavni splošno, torej za vse tipe silosov, so vsi pritiski izrisani v brezdimenzijski obliki.

Kot lahko vidimo iz *Diagrama 1.1*, sta pritiska $p_h$ in $p_w$ na steno cilindra enaka produktu funkcije variacije horizontalnega pritiska *Y(z)*, ki predstavlja »obliko« razporeditve pritiskov po globini, in konstantnih faktorjev, ki zajemajo različne lastnosti shranjenega materiala. Funkcija variacije horizontalnega pritiska je različna za različne tipe cilindrov.

Vertikalni pritisk $p_v$ v materialu je splošnem dan kot produkt funkcije variacije vertikalnega pritiska $z_V(z)$ in materialnih parametrov.

#### *3.1.2.1.1 Vitki silos*

V vitkih silosih se funkcija variacije horizontalnega pritiska imenuje *Janssenova funkcija* in je enaka:

$$Y(z) = 1 - e^{-z/z_0} \tag{I.16}$$

Parameter $z_0$ predstavlja karakteristično globino in je dan z enačbo:

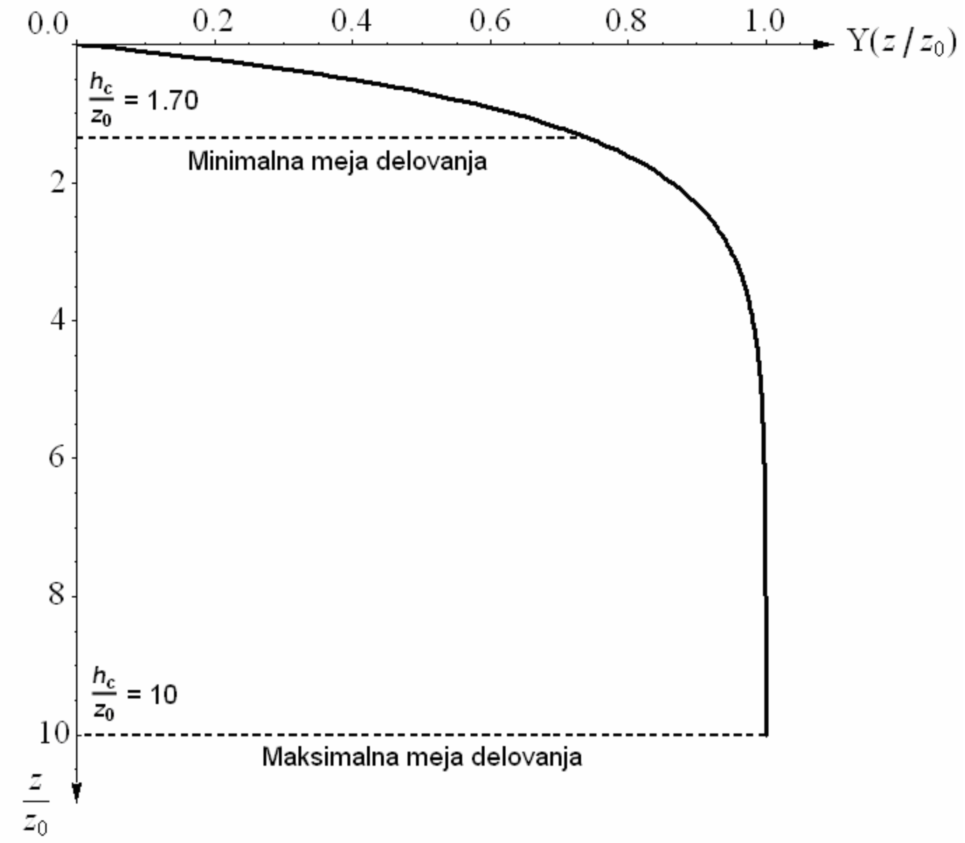
$$z_0 = \frac{1}{K \cdot \mu} \cdot \frac{A}{U} \tag{I.17}$$

Funkcija vertikalnega pritiska $z_V(z)$ je dana kot:

$$z_V(z) = z_0 \cdot Y(z) \tag{I.18}$$

Vidimo, da je v primeru vitkih silosov pomembna samo *Janssenova funkcija* $Y(z)$, saj z njo določimo potek vseh pritiskov v cilindru (pritiskov in trenja po steni cilindra in vertikalnega pritiska v materialu).

Spodnja slika prikazuje potek funkcije $Y(z/z_0)$. Ker želimo imeti graf v brezdimenzijski obliki, je funkcija izrisana za argument $z / z_0$. Koordinata $z$ poteka od ekvivalentne ravnine ($z = 0$) do ekvivalentne višine cilindra ($z = h_c$) (*Slika 3*).



**Slika 8: Brezdimenzijska oblika funkcije variacije horizontalnega pritiska *Y(z)* pri vitkih silosih**

Izraz »meja delovanja« se nanaša na mejno vrednost funkcije $Y(z)$ za dan cilinder. Mejna vrednost je dosežena, ko koordinata $z$ doseže vrednost $h_c$. Minimalna in maksimalna meja delovanja pri vitkih silosih znašata:

$$h_{c,min} = 1.7 \cdot z_0 \tag{I.19}$$

$$h_{c,max} = 10 \cdot z_0 \tag{I.20}$$

Določimo ju ob upoštevanju minimalne oz. maksimalne vrednosti razmerja višine cilindra proti premeru cilindra (*Slika 4*) in ob upoštevanju različnih tipov materialov, ki jih pogosto hranimo v silosih.

### 3.1.2.1.2 Plitvi silosi in silosi srednje vitkosti

Pri plitvih in srednje vitkih silosih je funkcija variacije horizontalnega pritiska enaka:

$$Y(z) = 1 - \left( \frac{z - h_0}{z_0 - h_0} + 1 \right)^n \qquad \text{(I.21)}$$

Parameter $z_0$ je enak kot pri vitkih silosih in je določen z enačbo (I.17). Parameter $h_0$ je določen z enačbo (I.3) in predstavlja razdaljo med ekvivalentno površino silosa in skrajno točko, pri kateri je material še v stiku s steno (*Slika 3*).

Parameter $n$ predstavlja eksponent v funkciji pritiska in je definiran s spodnjo enačbo:

$$n = -(1 + \tan\phi_r) \cdot (1 - h_0 / z_0) \qquad \text{(I.22)}$$

Funkcija variacije vertikalnega pritiska $z_V(z)$ je pri plitvih in srednje vitkih silosih je definirana kot:

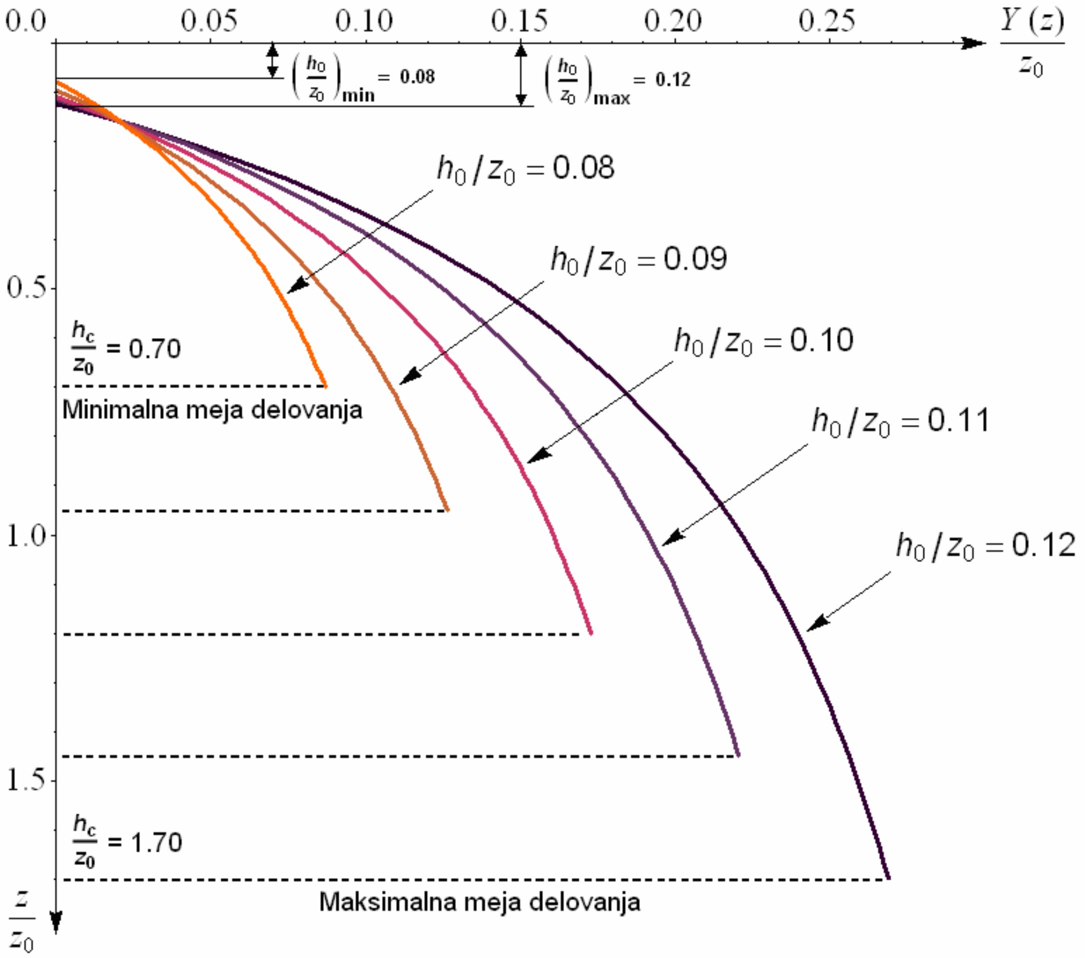
$$z_V(z) = z_0 \left( \frac{h_0}{z_0} - \frac{1 - \frac{h_0}{z_0}}{n + 1} \left[ 1 - (1 - Y(z))^{\frac{n+1}{n}} \right] \right) \qquad \text{(I.23)}$$

Vsi trije parametri $z_0$, $h_0$ in $n$ so odvisni od shranjenega materiala. Izberimo pet različnih razmerij $h_0 / z_0$. Razpon izbranih razmerij je takšen, da zajema različne tipe materialov, ki jih pogosto hranimo v silosih. Za vsakega od izbranih razmerij je potrebno upoštevati še pripadajoče vrednosti parametrov $h_c$ in $n$, ki so podane v spodnji preglednici.

**Preglednica 14: Izbrana vrednosti za razmerje $z_0 / h_c$ ter pripadajoče vrednosti parametrov $h_c$ in $n$**

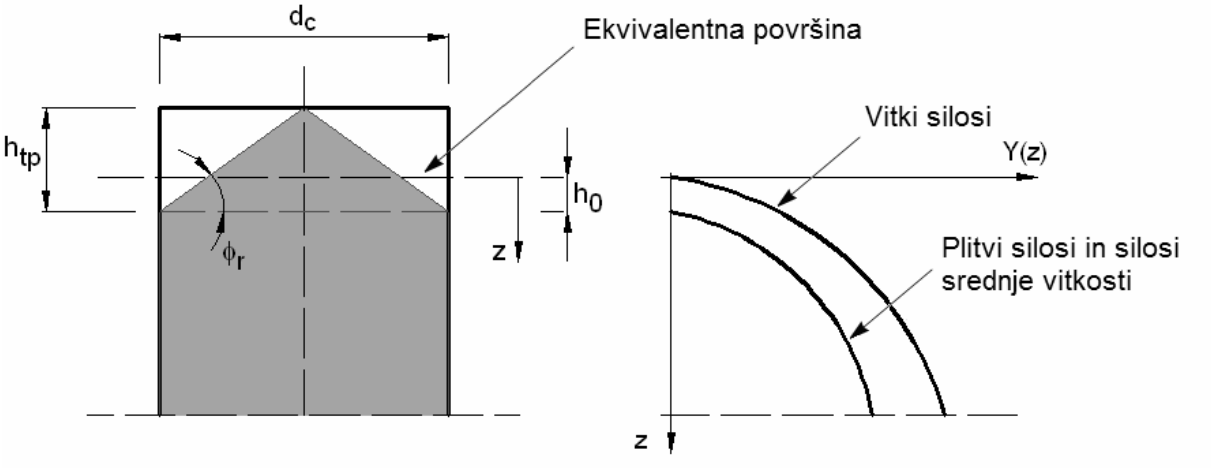
| $h_0 / z_0$ | **0.08** | **0.09** | **0.10** | **0.11** | **0.12** |
|---|---|---|---|---|---|
| $h_c / z_0$ | 0.70 | 0.95 | 1.2 | 1.45 | 1.70 |
| n | -1.52 | -1.59 | -1.64 | -1.80 | -1.88 |

Spodnja slika prikazuje potek funkcije $Y(z)/z_0$ za pet izbranih razmerij $z_0 / h_0$. Funkcija je bila deljena z $z_0$, da smo dobili brezdimenzijsko obliko.



**Slika 9: Brezdimenzijska oblika funkcije variacije horizontalnega pritiska $Y(z)$ pri plitvih silosih in silosih srednje vitkosti**

Na sliki so narisane samo pozitivne vrednosti. Negativne vrednosti niso obravnavane, saj shranjen material ne more povzročati negativnih pritiskov (srka) na steni cilindra. Iz slike vidimo, da je $Y(z) > 0$, ko je $z > h_0$. To pomeni, da so, v primeru plitvih in srednje vitkih silosov, pritiski, ki delujejo na stene cilindra, različni od nič takrat, ko se material dotika stene cilindra. Za razliko, so pri vitkih silosih te pritiski različni od nič od ekvivalentne površine dalje (*Slika 10*).



**Slika 10: Razlika med potekom pritiskov na steni cilindra pri vitkih silosih ter pri plitvih in silosih srednje vitkosti**

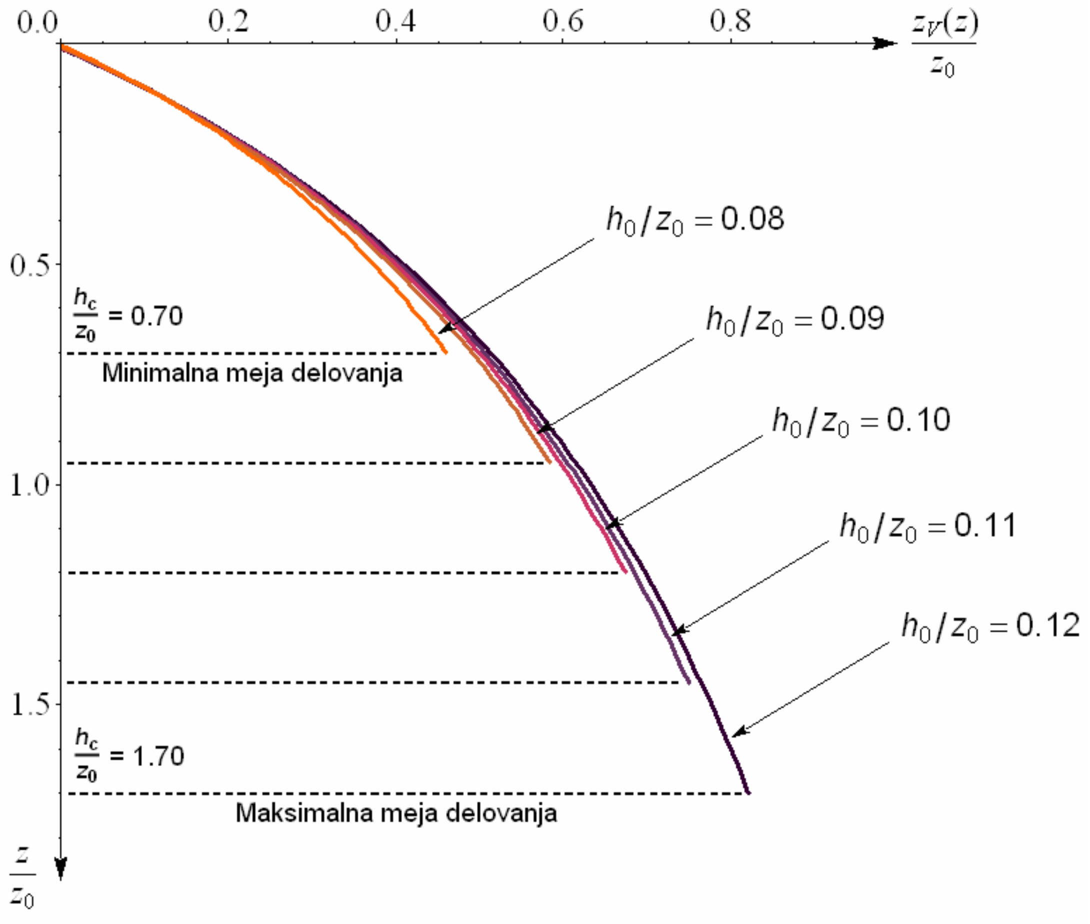
Kot je bilo že omenjeno, predstavlja parameter $h_0$ razdaljo med ekvivalentno površino in vrhnjo točko, kjer je material še v stiku s steno. *Slika 9* nam pove tudi, da lahko vrednost parametra $h_0$ predstavlja od *8%* do *12%* ekvivalentne višine $z_0$.

Kot prikazuje *Preglednica 14*, znaša minimalni nivo prekinitve pri plitvih in srednje vitkih silosih:

$$h_{c,min} = 0.7 \cdot z_0 \tag{I.24}$$

*Preglednica 14* podaja tudi maksimalni nivo prekinitve pri plitvih in srednje vitkih silosih. Ta pa je enak minimalnemu nivoju prekinitve pri vitkih silosih.

Poglejmo si sedaj še potek funkcije vertikalnega pritiska $z_V(z)$, ki se uporablja za določitev vertikalnih pritiskov v materialu. Tudi tukaj je funkcija $z_V(z)$ deljena z $z_0$ zato, da dobimo brezdimenzijsko obliko. Uporabljene so vrednosti parametrov, ki jih podaja *Preglednica 14*.



**Slika 11: Brezdimenzijska oblika funkcije variacije vertikalnega pritiska $z_V(z)$ pri plitvih silosih in silosih srednje vitkosti**

Če primerjamo grafa funkcij $z_V(z)$ (*Slika 11*) in $Y(z)$ (*Slika 9*), vidimo, da se z večanjem razmerja $h_0 / z_0$ vrednost funkcije $z_V(z)$ povečuje veliko manj kot se povečuje vrednost funkcije $Y(z)$. Iz tega lahko sklepamo, da ima razmerje $h_0 / z_0$ velik vpliv na pritiske, ki deluje na steno cilindra, na vertikalni pritisk v materialu, pa to razmerje nima velikega vpliva.

### 3.1.2.2 Obtežba na steno lijaka

V tem poglavju je predstavljen grafični potek vertikalnega pritiska $p_v(x)$, ki deluje na material, shranjen v lijaku. Pritisk je izrisan v brezdimenzijski obliki in je tako veljaven za poljubno geometrijo lijaka.

Vertikalni pritisk $p_v(x)$ podaja naslednja enačba:

$$p_v(x) = \frac{\gamma_{max} \cdot h_h}{n-1} \cdot \left( \frac{x}{h_h} - \left( \frac{x}{h_h} \right)^n \right) + p_{vft} \cdot \left( \frac{x}{h_h} \right)^n \tag{I.25}$$

Pri tem moramo biti pozorni, da je $p_v(x)$ funkcija koordinate $x$ (*Slika 7*). Vrednosti eksponenta $n$ so dane v *Diagramih 2.1*, *2.2* in *2.3*, odvisno od tipa lijaka in od tega ali obravnavamo proces polnjenja oz. praznjenja materiala.

Brezdimenzijsko obliko enačbe za vertikalni pritisk dobimo, če zgornjo enačbo delimo z $p_{vft}$:

$$\frac{p_v(x)}{p_{vft}} = \frac{\gamma_{max} \cdot h_h}{(n-1) \cdot p_{vft}} \cdot \left( \frac{x}{h_h} - \left( \frac{x}{h_h} \right)^n \right) + \left( \frac{x}{h_h} \right)^n \tag{I.26}$$

Novo enačbo lahko poenostavimo, če uvedemo:

$$\alpha(n) = \frac{\gamma_{max} \cdot h_h}{(n-1) \cdot p_{vft}} \tag{I.27}$$

$$\text{in} \quad \xi = \frac{x}{h_h}. \tag{I.28}$$

Enačba (I.26) se sedaj zapiše kot:

$$\frac{p_v(\xi)}{p_{vft}} = \alpha(n) \cdot \left( \xi - \xi^n \right) + \xi^n \tag{I.29}$$

Zgornja enačba predstavlja brezdimenzijsko obliko enačbe za srednjo vrednost vertikalnega pritiska na material, ki je shranjen v lijaku. Vidimo, da se lahko ta enačba zapiše kot vsota dveh ločenih delov. Levi del enačbe predstavlja prispevek teže materiala, ki je shranjen v lijaku, desni del pa prispevek vertikalnega pritiska materiala, ki je shranjen v cilindru.

Označimo levi del enačbe (I.29) z novo funkcijo $\kappa_{hop}(\xi)$, desni del pa z funkcijo $\kappa_{cyl}(\xi)$:

$$\kappa_{hop}(\xi) = \alpha(n) \cdot \left(\xi - \xi^n\right) \tag{I.30}$$

$$\kappa_{cyl}(\xi) = \xi^n \tag{I.31}$$

Enačbo (I.29) lahko tako zapišemo kot:

$$\frac{p_v(\xi)}{p_{vft}} = \kappa_{hop}(\xi) + \kappa_{cyl}(\xi) \tag{I.32}$$

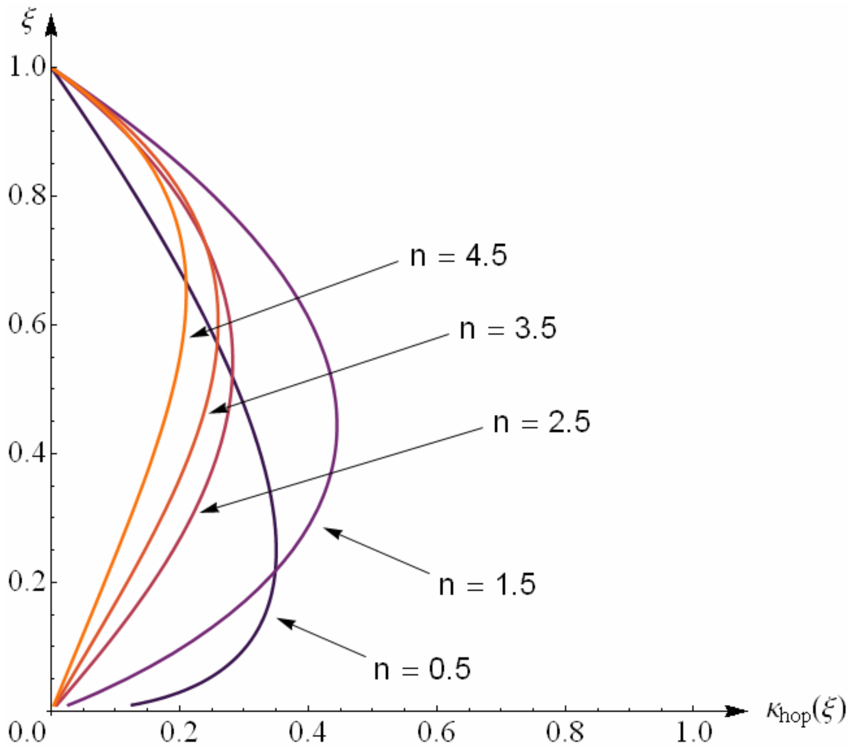
Poglejmo si sedaj grafični potek funkcij $\kappa_{hop}(\xi)$, $\kappa_{cyl}(\xi)$ in kombinirane funkcije $p_v(\xi)/p_{vft}$, da vidimo do kakšne mere vpliva teža materiala shranjenega v cilindru na vertikalni pritisk $p_v(\xi)$.

Uporabimo splošni material, ki je dan v *Prilogi C*, pod oznako »default material«. Parameter *n* je vzet v območju od *0.5* do *4.5*. Dano območje je takšno, da zajema različne tipe lijakov (večji *n* pomeni bolj strm lijak). Ker je tudi $\alpha(n)$ funkcija parametra *n*, je potrebno upoštevati pripadajoče vrednosti te funkcije za posamezen *n*. Obravnavane vrednosti za *n* ter pripadajoče vrednosti kota lijaka $\beta$ in funkcije $\alpha(n)$, so podane v spodnji preglednici.

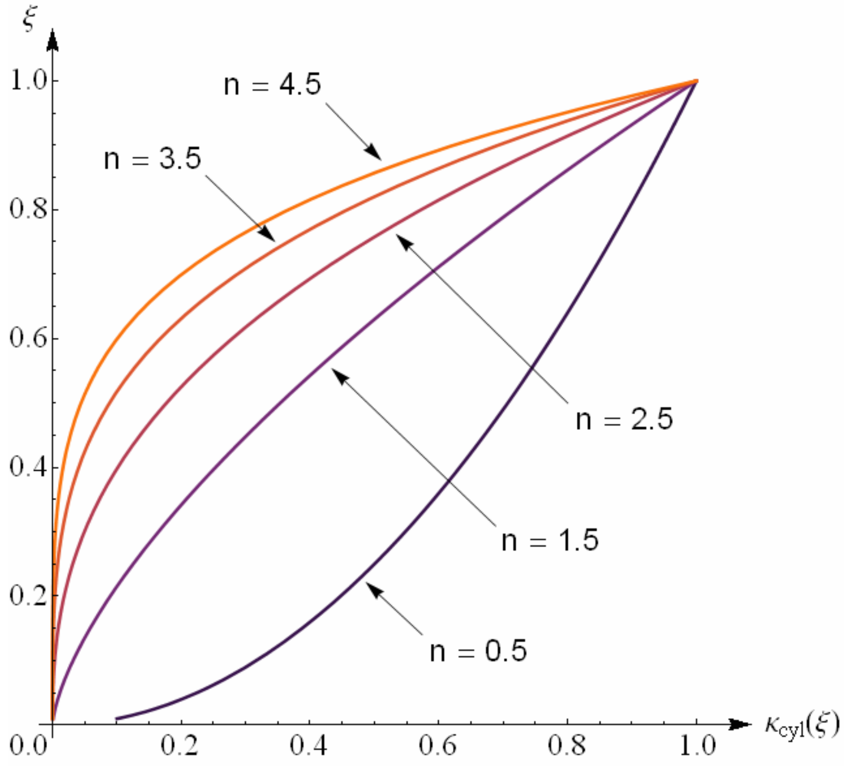
**Preglednica 15: Vrednosti kota lijaka $\beta$ in funkcije $\alpha(n)$ za izbran $n$**

| n | Približna vrednost kota $\beta$ | $\alpha$(n) |
|---|---|---|
| 0.5 | 70° | -1.40 |
| 1.5 | 45° | 3.00 |
| 2.5 | 30° | 0.85 |
| 3.5 | 15° | 0.60 |
| 4.5 | 15° | 0.40 |

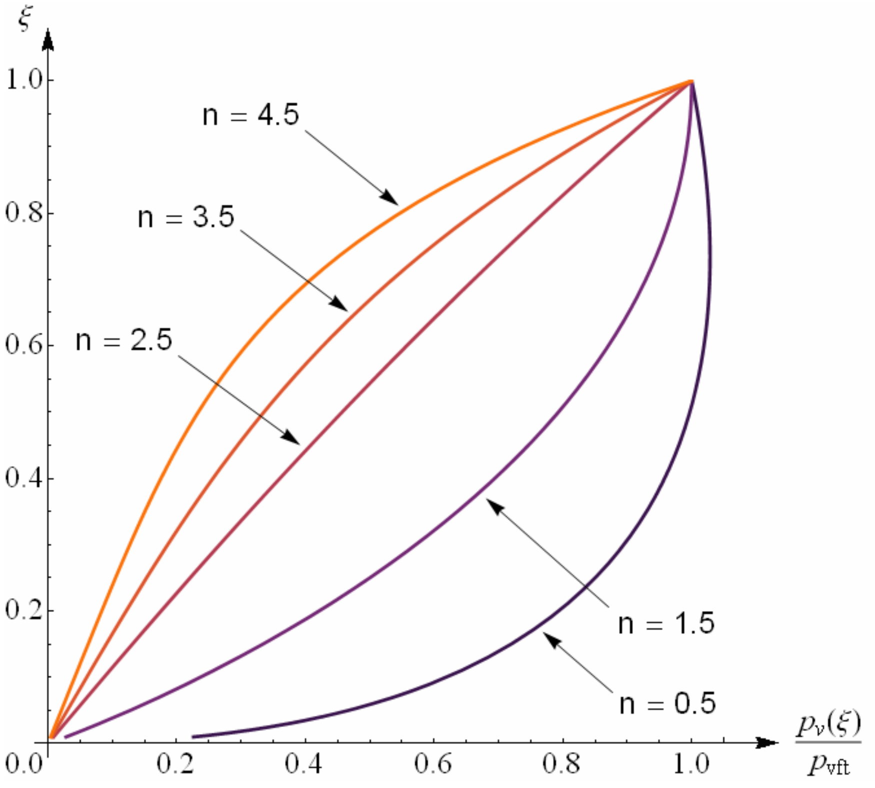
Za izbrane vrednosti parametra $n$ lahko sedaj določimo grafični potek vseh treh prej omenjenih funkcij.



Slika 12: Prispevek materiala shranjenega v lijaku na vertikalni pritisk $p_v(\xi)$



**Slika 13: Prispevek materiala shranjenega v cilindru na vertikalni pritisk $p_v(\xi)$**



**Slika 14: Potek vertikalnega pritiska $p_v(\xi)$ v lijaku**

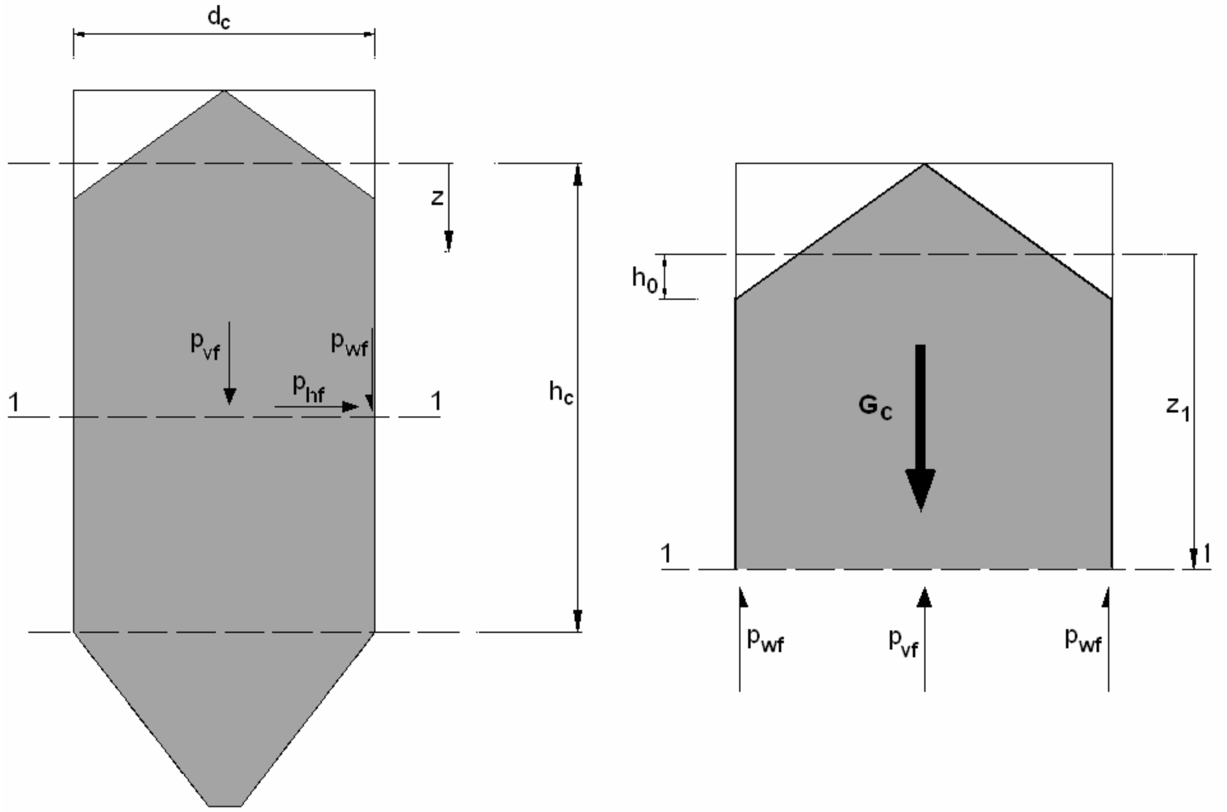
Vidimo lahko (*Slika 12*), da je največji prispevek materiala v lijaku k vertikalnemu pritisku $p_v(\xi)$ v območju, ko teče $\xi$ od *0.2* do *0.7* (odvisno od parametra *n*). Prispevek je največji, ko je $n = 1.5$. V tem primeru prispeva material v lijaku okoli *65%* k celotnega razmerja $p_v(\xi)/p_{vft}$. Ta prispevek je manjši, ko je *n* večji. Najmanjšo vrednost doseže, ko je $n = 4.5$. Takrat prispeva okoli *35%* k celotnemu razmerju $p_v(\xi)/p_{vft}$.

Zaključimo lahko, da ima v primeru ravnega dna oz. položnega lijaka, material v cilindru velik vpliv na vertikalni pritisk v lijaku (več kot *50%*). Če pa je lijak strm, pa je ta vpliv manjši (manj kot *50%*).

## 3.1.3 Kontrole ravnotežja v silosu

### 3.1.3.1 Cilinder

Vrednosti vertikalnega pritiska v materialu $p_{vf}(z)$ in trenja po steni cilindra $p_{wf}(z)$ na poljubni globini $z_1$, po oz. med polnitvijo silosa, morajo biti v skladu s težo shranjene snovi v cilindru $G_c$ na tej globini.



(a) (b)

Slika 15: Pritiski med procesom polnjenja (a) in ravnotežni pogoj (b)

#### *3.1.3.1.1 Vitki silos*

Vrednosti pritiskov na stene cilindra v vitkih silosih znašajo:

$$p_{wf}(z) = \mu \cdot \gamma \cdot K \cdot z_o \cdot Y_J(z) \tag{I.33}$$

$$p_{vf}(z) = \gamma \cdot z_o \cdot Y_J(z) \tag{I.34}$$

Funkciji $Y_J(z)$ and $z_0$ sta definirani v enačbah (I.16) in (I.17).

Celotna sila trenja $P_w$ na globini $z_1$ je enaka:

$$P_w = \int_0^{2\pi} \int_0^{z_1} p_{wf}(z)\, dz\, r\, d\phi \tag{I.35}$$

Če upoštevamo enačbo (I.33) z (I.35), dobimo:

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \int_0^{2\pi} \int_0^{z_1} (1 - e^{-z/z_0})\, dz\, r\, d\phi$$

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \cdot 2 \cdot \pi \cdot r \int_0^{z_1} (1 - e^{-z/z_0})\, dz$$

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \cdot U \cdot \left(z_1 - z_0 + z_0 \cdot e^{-z_1/z_0}\right) \tag{I.36}$$

Celotna vertikalna sila $P_v$ na globini $z_1$ je enaka:

$$P_v = A \cdot p_{vf}$$

$$P_v = A \cdot \gamma \cdot z_0 \cdot \left(1 - e^{-z_1/z_0}\right) \tag{I.37}$$

Teža shranjenega materiala v cilindru $G_c$ na globini $z_1$ je enaka:

$$G_c = A \cdot \gamma \cdot z_1 \tag{I.38}$$

Sistem je v ravnotežju, ko je teža $G_c$ enaka vsoti vertikalne sile $P_v$ in sile trenja $P_w$.

**Ravnotežni pogoj:** $$P_v + P_w = G_c \tag{I.39}$$

Če upoštevamo enačbe (I.36), (I.37) in (I.38) z ravnotežnim pogojem, dobimo:

$$\mu \cdot \gamma \cdot K \cdot z_0 \cdot U \cdot \left( z_1 - z_0 + z_0 \cdot e^{-z_1/z_0} \right) + A \cdot \gamma \cdot z_0 \cdot \left( 1 - e^{-z_1/z_0} \right) = A \cdot \gamma \cdot z_1 \tag{I.40}$$

Produkt $K \cdot z_0$ se poenostavi v:

$$K \cdot z_0 = \frac{1}{\mu} \cdot \frac{A}{U} \tag{I.41}$$

Enačba (I.40) se nato poenostavi:

$$\mu \cdot \gamma \cdot \frac{1}{\mu} \cdot \frac{A}{U} \cdot U \cdot \left( z_1 - z_0 + z_0 \cdot e^{-z_1/z_0} \right) + A \cdot \gamma \cdot z_0 \cdot \left( 1 - e^{-z_1/z_0} \right) = A \cdot \gamma \cdot z_1$$

$$z_1 - z_0 + z_0 \cdot e^{-z_1/z_0} + z_0 - z_0 \cdot e^{-z_1/z_0} = z_1$$

$$\boxed{z_1 = z_1} \tag{I.42}$$

To pomeni, da je ravnotežni pogoj izpolnjen za poljubno globino $z$ znotraj cilindra.

#### *3.1.3.1.2 Plitvi silos in silos srednje vitkosti*

Vrednosti vertikalnega pritiska $p_{vf}(z)$ in trenja $p_{wf}(z)$ na poljubni globini $z$ sta dani z enačbama:

$$p_{vf}(z) = \gamma\, z_V(z) \tag{I.43}$$

$$p_{wf}(z) = \mu \cdot p_{hf} \tag{I.44}$$

Funkcija $z_V(z)$ je definirana v enačbi (I.23).

Horizontalni pritisk $p_{hf}(z)$ določa spodnja enačba:

$$p_{hf}(z) = p_{ho} \cdot Y_J(z), \tag{I.45}$$

kjer sta funkciji $Y_J(z)$ in $z_0$ definirani v enačbah (I.21) in (I.17), pritisk $p_{ho}$ pa je enak:

$$p_{ho} = \gamma \cdot \frac{1}{\mu} \cdot \frac{A}{U}. \tag{I.46}$$

Celotna sila trenja $P_w$ na globini $z_1$ se izračuna po enačbi (I.35). Tukaj moramo integrirati od $h_0$, saj se pri plitvih in srednje vitkih silosih pritiski računajo od globine $h_0$ dalje (*Slika 10*).

$$P_w = \int_0^{2\pi} \int_{h_0}^{z_1} p_{wf}(z)\, dz\; r\, d\phi$$

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \cdot 2\pi \cdot r \int_{h_0}^{z_1} 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n dz$$

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \cdot U \int_{h_0}^{z_1} Y_R(z)\, dz \tag{I.47}$$

Celotna vertikalna sila $P_v$ izračunana na globini $z_1$ je enaka:

$$P_v = A \cdot p_{vf}$$

$$P_v = A \cdot \gamma \cdot z_V \tag{I.48}$$

Teža shranjenega materiala v cilindru $G_c$, na globini je dana $z_1$ z enačbo (I.38).

Sistem je v ravnotežju, ko je teža $G_c$ enaka vsoti vertikalne sile $P_v$ in sile trenja na določeni višini $P_w$. Ravnotežni pogoj (I.39) se tako glasi:

$$P_v + P_w = G_c$$

$$A \cdot \gamma \cdot z_V + \mu \cdot \gamma \cdot K \cdot z_0 \cdot U \int_{h_0}^{z_1} Y_R(z)\, dz = A \cdot \gamma \cdot z_1 \tag{I.49}$$

Če upoštevamo enačbo (I.41), se ravnotežni pogoj poenostavi:

$$A \cdot \gamma \cdot z_V + \mu \cdot \gamma \cdot \frac{1}{\mu} \cdot \frac{A}{U} \cdot U \int_{h_0}^{z_1} Y_R(z)\, dz = A \cdot \gamma \cdot z_1$$

$$z_V + \int_{h_0}^{z_1} Y_R(z)\, dz = z_1 \tag{I.50}$$

Če gornjo enačbo sedaj delimo z $z_0$ in jo izrazimo z $\frac{z_V}{z_0}$, dobimo:

$$\frac{z_V}{z_0} + \int_{h_0}^{z_1} \frac{Y_R(z)}{z_0}\, dz = \frac{z_1}{z_0}$$

$$\frac{z_V}{z_0} = \frac{z_1}{z_0} - \int_{h_0}^{z_1} \frac{Y_R(z)}{z_0}\, dz \tag{I.51}$$

Označimo sedaj s črko $L$ levo stran in s črko $R$ pa desno stran gornje enačbe. Če je $L = R$, potem je naš ravnotežni pogoj izpolnjen.

$$L=\frac{z_V}{z_0}=\frac{h_0-\dfrac{1}{n+1}\left(z_0-h_0-\dfrac{(z_1+z_0-2h_0)^{n+1}}{(z_0-h_0)^n}\right)}{z_0}$$

$$L=\frac{h_0}{z_0}-\frac{1}{n+1}\left(1-\frac{h_0}{z_0}-\left[\frac{z_1/z_0-h_0/z_0}{1-h_0/z_0}+1\right]^{n+1}\right)$$

$$L=\frac{h_0}{z_0}-\frac{1-h_0/z_0}{n+1}\left(1-\frac{z_0}{z_0-h_0}\left[\frac{z_1/z_0-h_0/z_0}{1-h_0/z_0}+1\right]^{n+1}\right) \qquad \text{(I.52)}$$

$$R=\frac{z_1}{z_0}-\int_{h_0}^{z_1}\frac{Y_R(z)}{z_0}\,dz=\frac{z_1}{z_0}-\int_{h_0}^{z_1}\frac{1-\left(\dfrac{z-h_0}{z_0-h_0}+1\right)^n}{z_0}\,dz$$

$$R=\frac{z_1}{z_0}-\left(\frac{z_1-h_0}{z_0}-\frac{1}{n+1}\left(1-\frac{h_0}{z_0}-\left[\frac{z_1/z_0-h_0/z_0}{1-h_0/z_0}+1\right]^{n+1}\right)\right)$$

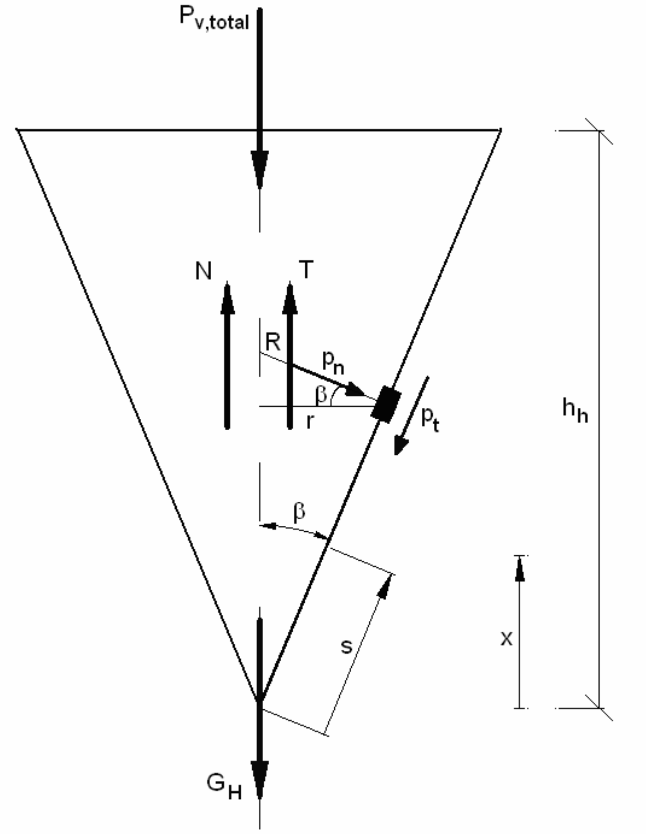
$$R=\frac{z_1}{z_0}-\int_{h_0}^{z_1}\frac{1}{z_0}-\left(\frac{z/z_0-h_0/z_0}{1-h_0/z_0}+\frac{1}{z_0}\right)^n dz$$

$$R=\frac{h_0}{z_0}-\frac{1-h_0/z_0}{n+1}\left(1-\frac{z_0}{z_0-h_0}\left[\frac{z_1/z_0-h_0/z_0}{1-h_0/z_0}+1\right]^{n+1}\right)$$

$$\boxed{R=L} \qquad \text{(I.53)}$$

Vidimo, da je ravnotežni pogoj izpolnjen za poljubno globini *z*.

### 3.1.3.2 Lijak



$$\xi = \frac{x}{h_h} \tag{I.54}$$

$$r = x \cdot tan\,\beta \tag{I.55}$$

$$s = x \cdot cos\,\beta \tag{I.56}$$

**Slika 16: Ravnotežni pogoj v lijaku**

Celotna teža materiala shranjenega v lijaku $G_{h,total}$ mora biti v ravnotežju z vsoto vertikalne komponente rezultirajoče normalne sile na steno lijaka $N$ in vertikalno komponento sile trenja na steno lijaka $T$ po polnjenju.

**Ravnotežni pogoj:** $$G_{h,total} = N + T \tag{I.57}$$

Po drugi strani pa mora biti celotna teža materiala shranjenega v lijaku $G_{h,total}$ enaka vsoti rezultante vertikalnih pritiskov na dnu cilindra $P_{v,total}$ in teži materiala, shranjenega v lijaku, $G_h$.

$$G_{h,total} = P_{v,total} + G_h \tag{I.58}$$

Sili $P_{v,total}$ in $G_h$ sta enaki:

$$P_{v,total} = \pi \cdot r^2 \cdot p_{vft} \tag{I.59}$$

$$G_h = \frac{1}{3}\pi \cdot r^2 \cdot \gamma_U \cdot h_h \tag{I.60}$$

Simbol $p_{vft}$ označuje vrednost vertikalnega pritiska na prehodu med cilindrom in lijakom. Izračunamo ga z enačbo:

$$p_{vft} = C_b \cdot p_{vf}(z = h_c) \tag{I.61}$$

Simbol $C_b$ označuje faktor dodatne povečave pritiska (*Preglednica 13*).

Celotna sila teže materiala, ki je shranjen v lijaku, je torej enaka:

$$G_{h,total} = \pi \cdot r^2 \cdot p_{vft} + \frac{1}{3}\pi \cdot r^2 \cdot \gamma \cdot h_h$$

$$G_{h,total} = \pi \cdot r^2 \cdot \left( p_{vft} + \frac{1}{3} \cdot \gamma \cdot h_h \right) \tag{I.62}$$

Vertikalni komponenti rezultirajoče normalne sile $N$ in sile trenja $T$ se izračunata z integracijo vertikalnih komponent normalnega pritiska $p_n$ in trenja $p_t$ po površini stene lijaka:

$$N = \iint_A p_n(x) \cdot \sin\beta \, dA \tag{I.63}$$

$$T = \iint_A p_t(x) \cdot \cos\beta \, dA \tag{I.64}$$

Normalni pritisk $p_n$ in trenje $p_t$ na, poljubni višini $x$, v lijaku sta enaki:

$$p_n(x) = F_f \cdot p_v(x) \tag{I.65}$$

$$p_t(x) = \mu_{heff} \cdot F \cdot p_v(x) \tag{I.66}$$

Parameter $F_f$ predstavlja karakteristično vrednost razmerja pritiskov v lijaku po polnjenju, parameter $\mu_{heff}$ pa predstavlja efektrivni oz. mobilizirani koeficient trenja na steni lijaka. Enačbe za njun izračun se nahajajo na *Diagramu 2.1.*

Vertikalni pritisk $p_v(x)$ v shranjenem materialu na globini $x$ določa enačba (I.25).

Vsota sil $N$ in $T$ predstavlja reakcijo stene lijaka zaradi teže shranjenega materiala. Označimo jo z $R_w$ in jo razširimo z uporabo enačb (I.63) do (I.66).

$$R_w = N + T$$

$$R_w = \iint_A \left(p_n \cdot sin\,\beta + p_t \cdot cos\,\beta\right) dA \qquad \text{(I.67)}$$

Dvojni integral po površini stene lijaka se prevede na dvakratni integral, eden po obsegu cilindra z diferencialom $d\phi$ in drugi v smeri koordinate $s$ z diferencialom $ds$.

$$R_w = F \cdot \int_0^{2\pi} \int_0^{h_h'} \left(sin\,\beta + \mu_{heff} \cdot cos\,\beta\right) p_v \; ds \;\; r\, d\phi \qquad \text{(I.68)}$$

Spremenimo sedaj integracijo notranjega integrala z integracijo po koordinati $s$, na integral z integracijo po koordinati $x$ in upoštevajmo izraze podane v enačbah (I.54) do (I.56):

$$R_w = F_f \cdot sin\,\beta \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right) \cdot 2\,\pi \cdot tan\,\beta \cdot \frac{1}{cos\,\beta} \int_0^{h_h} p_v \cdot x\, dx$$

$$R_w = F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right) \cdot 2\,\pi \cdot tan^2\,\beta \cdot {h_h}^2 \int_0^1 p_v \cdot \xi\, d\xi$$

$$R_w = F_f \cdot \left(sin\,\beta + \mu_{heff} \cdot cos\,\beta\right) \cdot 2\,\pi \int_0^{h_h} p_v \cdot x \cdot tan\,\beta \cdot \frac{1}{cos\,\beta}\, dx$$

$$R_w = \pi \cdot r^2 \cdot 2\, F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right) \int_0^1 p_v \cdot \xi\, d\xi \qquad \text{(I.69)}$$

Integral $\int_0^1 p_v \cdot \xi \, d\xi$ izračunamo posebej z uporabo definicije za vertikalni pritisk $p_v(x)$, ki je dana v enačbi (I.25).

$$\int_0^1 p_v \cdot \xi \, d\xi = \int_0^1 \left[ \frac{\gamma \cdot h_h}{n-1} \left( \xi - \xi^n \right) + C_b \cdot p_{vft} \cdot \xi^n \right] \xi \, d\xi$$

$$\int_0^1 p_v \cdot \xi \, d\xi = \int_0^1 \frac{\gamma \cdot h_h}{n-1} \left( \xi - \xi^n \right) \xi \, d\xi + C_b \cdot p_{vft} \int_0^1 \xi^n \xi \, d\xi$$

$$\int_0^1 p_v \cdot \xi \, d\xi = \frac{\gamma \cdot h_h}{n-1} \cdot \left( \left[ \frac{\xi^3}{3} \right]_0^1 + \left[ \frac{\xi^{n+2}}{n+2} \right]_0^1 \right) + C_b \cdot p_{vft} \left[ \frac{\xi^{n+2}}{n+2} \right]_0^1$$

$$\int_0^1 p_v \cdot \xi \, d\xi = \frac{\gamma \cdot h_h}{n-1} \cdot \left( \frac{1}{3} - \frac{1}{n+2} \right) + C_b \cdot p_{vft} \cdot \frac{1}{n+2}$$

$$\int_0^1 p_v \cdot \xi \, d\xi = \frac{\gamma \cdot h_h}{n-1} \cdot \frac{n-1}{3(n+2)} + C_b \cdot p_{vft} \cdot \frac{1}{n+2}$$

$$\int_0^1 p_v \cdot \xi \, d\xi = \frac{1}{n+2} \cdot \left( \frac{\gamma \cdot h_h}{3} + C_b \cdot p_{vft} \right) \tag{I.70}$$

Reakcija $R_w$ se sedaj lahko zapiše kot:

$$R_w = \frac{\pi \cdot r^2 \cdot 2\, F_f \cdot \left( 1 + \mu_{heff} \cdot ctg\beta \right)}{n+2} \cdot \left( \frac{\gamma \cdot h_h}{3} + C_b \cdot p_{vft} \right) \tag{I.71}$$

Če upoštevamo ravnotežni pogoj iz enačbe (I.57), dobimo:

$$W_s = R_w$$

$$\pi \cdot r^2 \cdot \left( C_b \cdot p_{vf} + \frac{1}{3} \cdot \gamma \cdot h_h \right) = \frac{\pi \cdot r^2 \cdot 2\, F_f \cdot \left( 1 + \mu_{heff} \cdot ctg\beta \right)}{n+2} \cdot \left( \frac{\gamma \cdot h_h}{3} + C_b \cdot p_{vft} \right)$$

$$1 = \frac{2\, F_f \cdot \left( 1 + \mu_{heff} \cdot ctg\beta \right)}{n+2} \tag{I.72}$$

Preverimo sedaj, če je desna stran prejšnje enačbe res enaka *1*. Najprej si poglejmo definiciji za simbola $F_f$ in $n$.

Kot že omenjeno, označuje parameter $F_f$ karakteristično vrednost razmerja pritiskov v lijaku po polnjenju. Izračunamo ga po izrazu:

$$F_f = 1 - \frac{0.2}{1 + \frac{\tan\beta}{\mu_{heff}}} \tag{I.73}$$

Parameter $n$ predstavlja eksponent v funkciji za vertikalni pritisk $p_v(x)$. Izračunamo ga po izrazu:

$$n = 1.6 \cdot \frac{\mu_{heff}}{\tan\beta} \tag{I.74}$$

Če uporabimo definiciji za oba parametra na desni strani enačbe (I.72), dobimo:

$$\begin{aligned} \frac{2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n+2} &= \frac{2 \cdot \left[1 - 0.2 \Big/ \left(1 + \frac{\tan\beta}{\mu_{heff}}\right)\right] \cdot \left[1 + \mu_{heff} \cdot ctg\beta\right]}{1.6 \cdot \frac{\mu_{heff}}{\tan\beta} + 2} \\ \frac{2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n+2} &= \frac{\left[1 - 0.2 \Big/ \left(1 + \frac{\tan\beta}{\mu_{heff}}\right)\right] \cdot \left[1 + \mu_{heff} \cdot ctg\beta\right]}{0.8 \cdot \frac{\mu_{heff}}{\tan\beta} + 1} \end{aligned} \tag{I.75}$$

Označimo števec na desni strani enačbe (I.75) z $U$ in ga obravnavajmo posebej:

$$U = \left[1 - 0.2 \Big/ \left(1 + \frac{\tan\beta}{\mu}\right)\right] \cdot \left[1 + \mu_{heff} \cdot ctg\beta\right]$$

$$U = 1 + \mu_{heff} \cdot ctg\beta - \frac{0.2}{1 + \frac{\tan\beta}{\mu_{heff}}} - \frac{0.2 \cdot \mu_{heff} \cdot ctg\beta}{1 + \frac{\tan\beta}{\mu_{heff}}}$$

$$U = \frac{1.8\,\mu_{heff} + 0.8\mu_{heff}^{\;2}\, ctg\beta + tan\,\beta}{\mu_{heff} + tan\,\beta} = \frac{\left(1 + \mu_{heff} \cdot ctg\beta\right) \cdot \left(0.8\mu_{heff} + tan\,\beta\right)}{\mu_{heff} + tan\,\beta}$$

$$U = \frac{\left(1 + \mu_{heff} \cdot ctg\beta\right) \cdot tan\,\beta \cdot \left(0.8\mu_{heff} \cdot ctg\beta + 1\right)}{\mu_{heff} + tan\,\beta}$$

$$U = \frac{\left(tan\,\beta + \mu_{heff}\right) \cdot \left(0.8\mu_{heff} \cdot ctg\beta + 1\right)}{\mu_{heff} + tan\,\beta}$$

$$U = 0.8\mu_{heff} \cdot ctg\beta + 1 \tag{I.76}$$

Če nesemo enačbo (I.76) nazaj v enačbo (I.75), dobimo:

$$\frac{2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n + 2} = \frac{U}{0.8 \cdot \dfrac{\mu_{heff}}{tan\,\beta} + 1}$$

$$\frac{2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n + 2} = \frac{0.8\mu_{heff} \cdot ctg\beta + 1}{0.8 \cdot \dfrac{\mu_{heff}}{tan\,\beta} + 1}$$

$$\frac{2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n + 2} = \frac{0.8 \cdot \dfrac{\mu_{heff}}{tan\,\beta} + 1}{0.8 \cdot \dfrac{\mu_{heff}}{tan\,\beta} + 1}$$

$$\frac{2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n + 2} = 1 \tag{I.77}$$

Vidimo, da je desna stran enačbe (I.72) res enaka *1*. To pomeni, da je $G_{h,\ total} = R_w$, in da je ravnotežni pogoj izpolnjen.

Ta kontrola ravnotežja je veljavna za oba tipa lijakov – strme in položne, saj je razlika med njima samo v definiciji mobiliziranega (efektivnega) koeficient trenja na steni lijaka $\mu_{heff}$.

Pri **strmih lijakih** je $\mu_{heff}$ enak minimalni karakteristični vrednosti parametra $\mu$:

$$\mu_{heff} = \mu_{min} \tag{I.78}$$

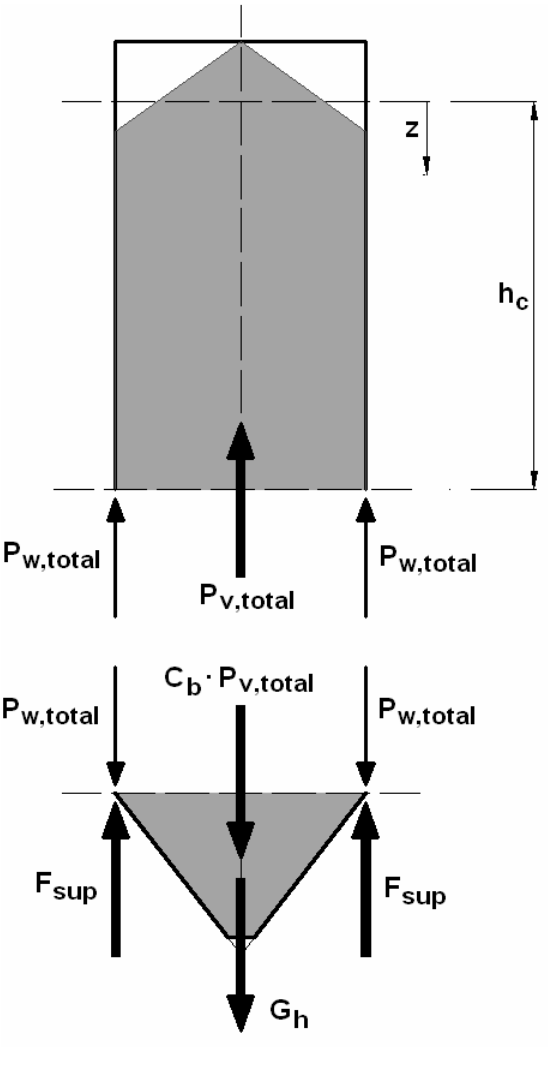
Pri **položnih lijakih** je $\mu_{heff}$ enak:

$$\mu_{heff} = \frac{1 - K_{min}}{2 \tan \beta} \tag{I.79}$$

Ker se parameter $\mu_{heff}$ v enačbi (I.77) izniči, to pomeni, da je ravnotežni pogoj neodvisen od tega parametra in je tako veljaven za oba tipa lijakov.

### 3.1.3.3 Neujemanje celotne vertikalne sile v lijaku z reakcijami v podporah

Kot je bilo predstavljeno že v prejšnjem poglavju, v enačbi (I.63), je potrebno vertikalne pritiske v shranjenem materialu, pri prehodu iz cilindra v lijak, pomnožiti s faktorjem dodatne povečave pritiska $C_b$ (*Preglednica 13*).



**Slika 17: Neujemanje vertikalnih sil z reakcijami v podporah**

Reakcija v podporah silosa zaradi delovanja shranjenega materiala tako znaša:

$$F_{sup} = P_{w,total} + C_b \cdot P_{v,total} + G_h \tag{I.80}$$

Sili $P_{w,total}$ in $P_{v,total}$ označujeta celotno trenjsko in vertikalno silo izračunano na globini $z = h_c$. Sila $G_h$ predstavlja silo teže materiala, ki je shranjen v lijaku (I.60).

Ravnotežni pogoj iz enačbe (I.39) se tako glasi:

$$P_{v,total} + P_{w,total} = G_c \, , \tag{I.81}$$

kjer $G_c$ označuje težo materiala, shranjenega v cilindru, na globini $z = h_c$.

Če z $G_m$ označimo celotno težo materiala, ki je shranjen v silosu, ($G_m = G_c + G_h$), potem lahko zapišemo enačbo (I.80) kot:

$$F_{sup} = G_c + G_h + (1 - C_b) \cdot P_{v,total}$$

$$F_{sup} = G_m + (1 - C_b) \cdot P_{v,total} \tag{I.82}$$

Vrednosti faktorja nam podaja $C_b$ *Preglednica 13*. Vidimo lahko, da za silose v *razredu obremenitve* (*AAC*) *2* oz. 3 znaša statični faktor dodatne povečave pritiska $C_b = 1.0$.

Reakcija v podporah je v tem primeru enaka:

$$F_{sup} = G_m \tag{I.83}$$

Pri silosih v *razredu obremenitve 1*, znaša statični faktor dodatne povečave pritiska $C_b = 1.3$. Reakcija v podporah tedaj znaša:

$$F_{sup} = G_m + 0.3 \cdot P_{v,total} \tag{I.84}$$

Sila, ki jo morajo podpore prevzeti, je $\dfrac{0.3 \cdot P_{v,total}}{G_m}$ krat večja od $G_m$. S tem pogoj ravnotežja ni izpolnjen.

Vprašanje torej ostaja, ali za silose v *razredu obremenitve 1* upoštevamo reakcijo v podporah, ki je enaka $G_m$, ali pa upoštevamo njeno povečano vrednost, ki ustreza faktorju povečave pritiskov $C_b = 1.3$.

**Predlagana rešitev:**

- v statičnih pogojih naj bo reakcijo v podporah enaka celotni teži shranjenega materiala ($F_{sup} = G_m$), saj je faktor $C_b$ pomemben samo pri dimenzioniranju lijaka,
- če pa imamo opravka z dinamičnimi pogoji, potem je za reakcijo v podporah potrebno upoštevati povečane vrednostmi.

### 3.1.4 Računalniški program

Da bi bil račun obtežb in notranjih sil na stene cilindra in lijaka hitrejši in preglednejši, je bil izdelan računalniški program, ki omogoča izračun pritiskov in notranjih sil, ki se pojavijo v silosu med polnjenjem in praznjenjem shranjenega materiala.

Program se nahaja na zgoščenki, ki je priložena nalogi in se nahaja na zadnji platnici. Za zagon programa je potrebno odpreti datoteko »*Stored Solid Load on Cylindrical Silos.xls*«.

Za pravilno delovanje mora biti na računalniku nameščen *Microsoft Excel 2003* (ali novejši). Makroji morajo biti omogočeni, njihova varnost pa nastavljena na *Srednjo* oz. *Nizko* stopnjo.

Podrobnejša navodila se nahajajo v *III. delu*, v poglavju *12*.

## 3.2 Nesimetrična obtežba

Nesimetrična obtežba se uporablja za ponazoritev slučajnih nesimetrij, ki nastanejo zaradi nepopolnosti pri procesu polnjenja in praznjenja. Pojavi se samo v cilindru silosa.

Enako vrsto nesimetrične obtežbe je možno uporabiti za tri tipe silosov:

- vitki silosi,
- silosi srednje vitkosti,
- plitvi silosi.

Nesimetrične obtežbe ne potrebno upoštevati kadar:

- je *razred obremenitve* enak *1* (*AAC 1*),
- imamo opravka z zadrževalnim silosom.

### 3.2.1 Postopka določitve nesimetrične obtežbe

Za *AAC 2* in *AAC 3* veljata različna postopka:

### 3.2.1.1 Razred obremenitve 2 (AAC 2)

Ta postopek se sme uporabiti, kadar je silos na vrhu pokrit s streho oz. je na dnu in vrhu dodan ojačitveni obroč (»ring stiffener«). Če temu ni tako, potem je potrebno uporabiti postopek za *AAC 3*.

V tem postopku lahko nesimetrično obtežbo nadomestimo z enakomernim prirastkom pritiska pravokotnega pritiska $p_h$ in trenja $p_w$ med procesom polnjenja in praznjenja.

**Polnjenje:** $p_{hf,u} = p_{hf} \cdot (1 + 0.5 \cdot C_{pf})$ (I.85)

$p_{wf,u} = p_{wf} \cdot (1 + C_{pf})$ (I.86)

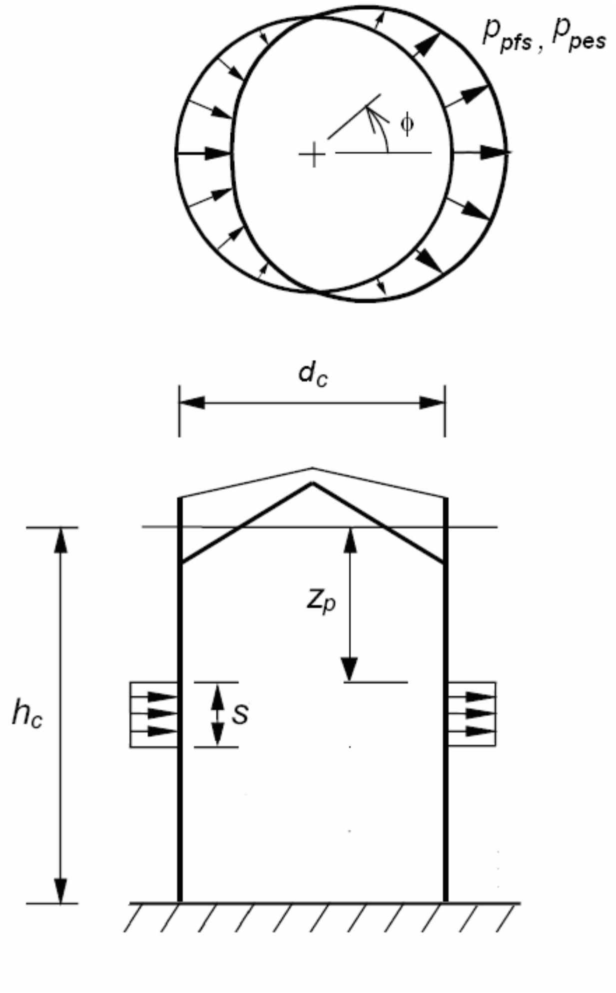
**Praznjenje:** $p_{he,u} = p_{he} \cdot (1 + 0.5 \cdot C_{pe})$ (I.87)

$p_{we,u} = p_{we} \cdot (1 + C_{pe})$ (I.88)

Vrednosti za koeficienta $C_{pf}$ in $C_{pe}$ sta določena v proceduri za *AAC 3*.

### 3.2.1.2 Razred obremenitve 3 (AAC 3)

Potek nesimetrične obtežbe prikazuje spodnja slika. Postopek za določitev te obtežbe se nahaja na *Diagramu 3*.



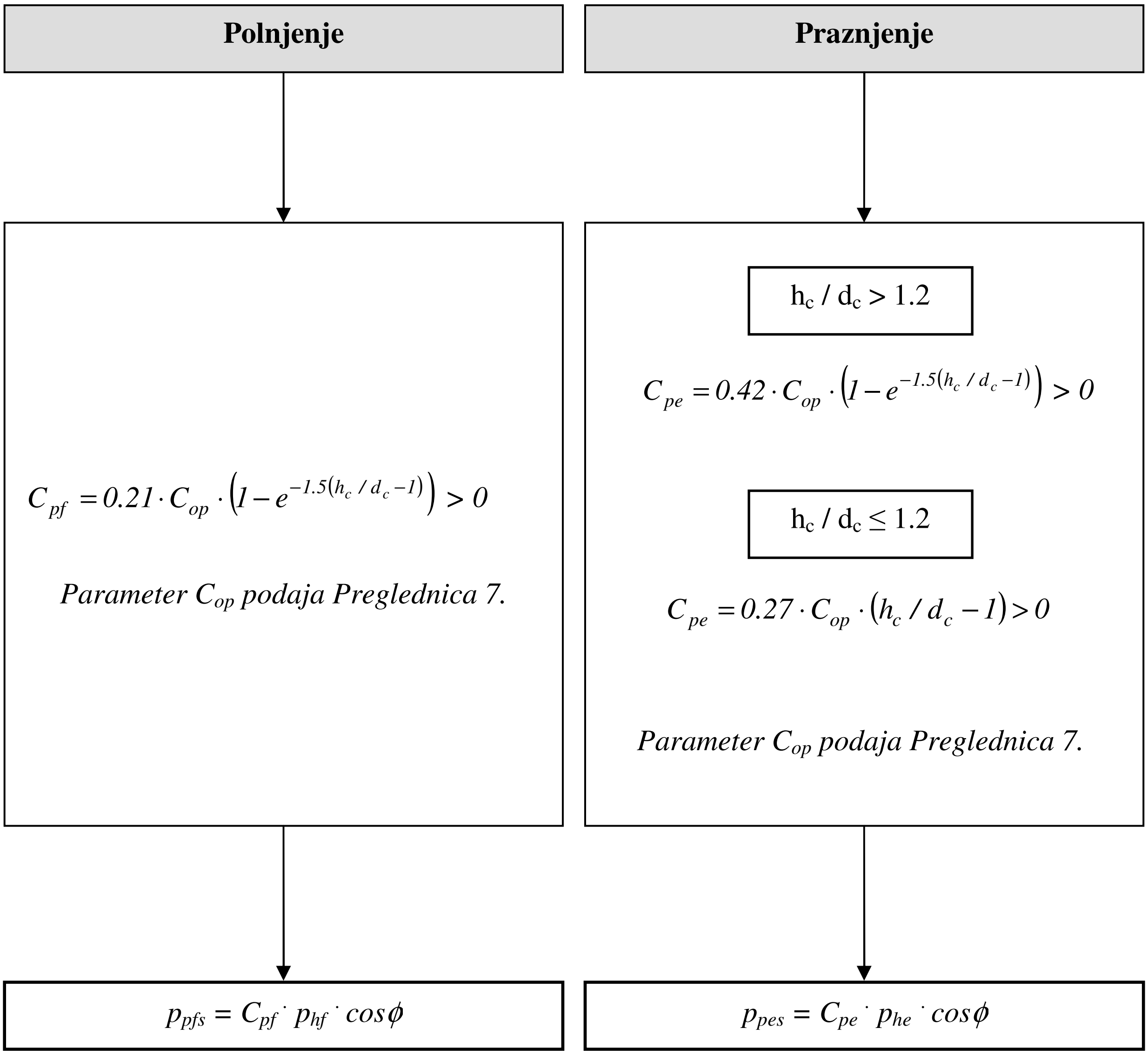
**Slika 18: Parametri za določitev nesimetrične obtežbe pri silosih v razredu obremenitve 3**

Višina $s$ in $z_p$ na zgornji sliki sta enaki:

$$s \cong 0.2 \cdot d_c \tag{I.89}$$

$$z_p \cong 0.6 \cdot h_c \tag{I.90}$$

**Diagram 3: Določitev nesimetrične obtežbe pri silosih v razredu obremenitve 3**



$$C_{pf} = 0.21 \cdot C_{op} \cdot \left(1 - e^{-1.5(h_c / d_c - 1)}\right) > 0$$

*Parameter $C_{op}$ podaja Preglednica 7.*

$$C_{pe} = 0.42 \cdot C_{op} \cdot \left(1 - e^{-1.5(h_c / d_c - 1)}\right) > 0$$

$$C_{pe} = 0.27 \cdot C_{op} \cdot (h_c / d_c - 1) > 0$$

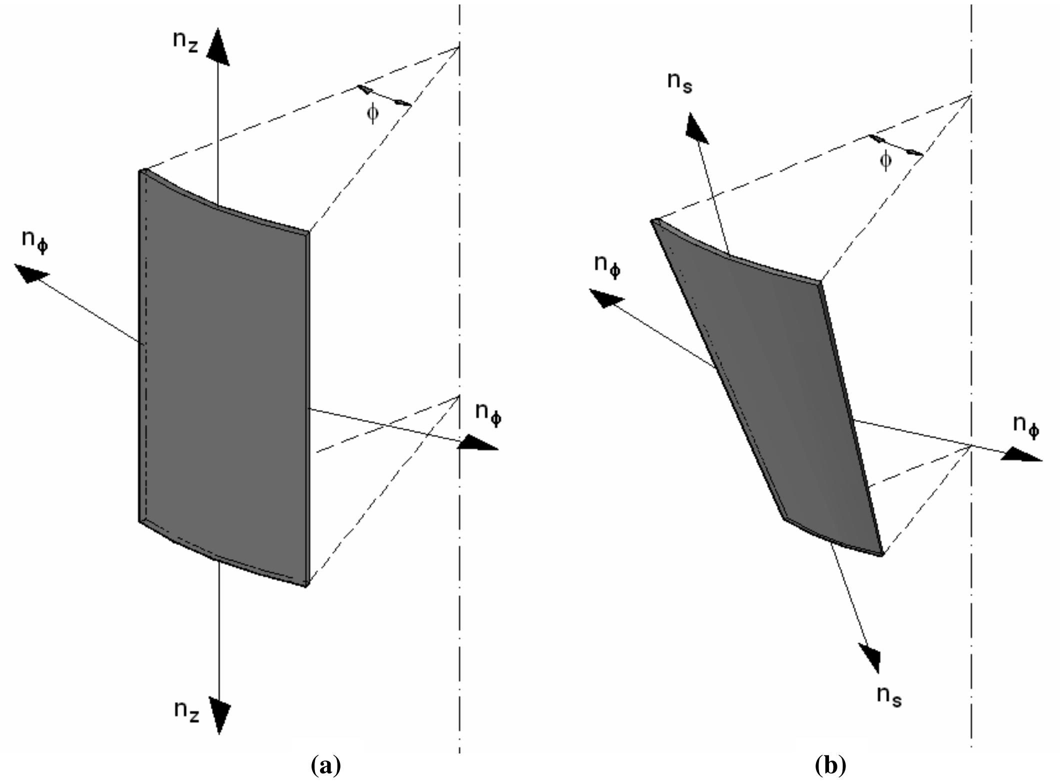
*Parameter $C_{op}$ podaja Preglednica 7.*

$$p_{pfs} = C_{pf} \cdot p_{hf} \cdot \cos\phi$$

$$p_{pes} = C_{pe} \cdot p_{he} \cdot \cos\phi$$

## 3.3 Membranske sile

Določeni so izrazi za notranje sile v steni cilindra in lijaka, ki so izpeljani po membranski teoriji lupin. Za notranje sile so uporabljene oznake, ki so prikazane na spodnji sliki (prikazane so pozitivne smeri sil).



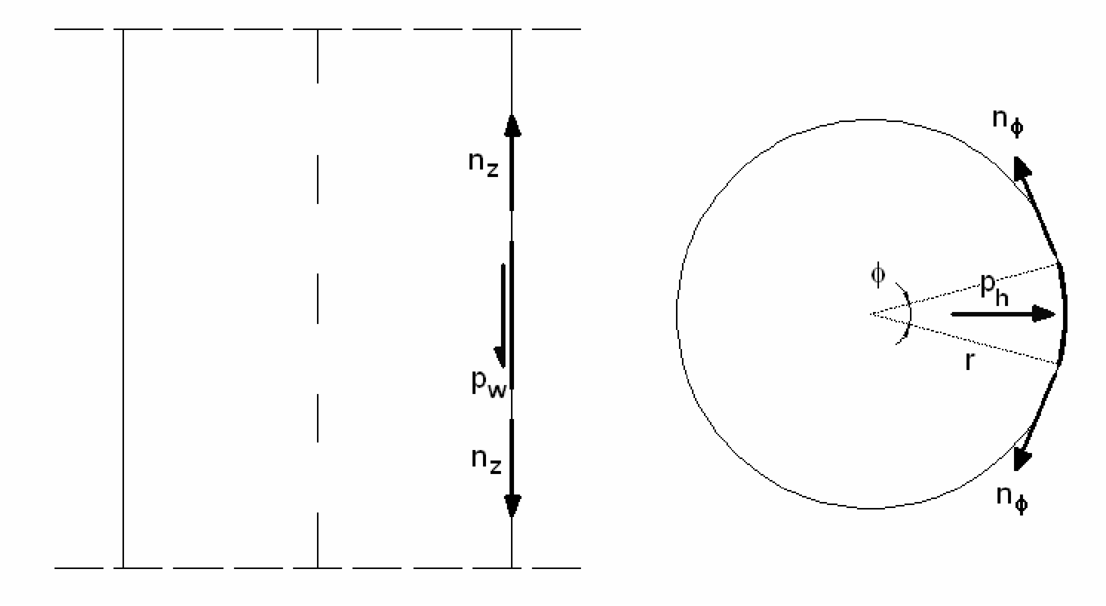
**(a)** **(b)**

**Slika 19: Membranske sile v cilindru (a) in lijaku (b)**

Strižne sile se pojavijo samo v primeru nesimetričnih obtežb in zaradi preglednosti niso prikazane na sliki.

## 3.3.1 Osno simetrična obtežba

### 3.3.1.1 Stena cilindra



**Slika 20: Pritiski in membranske sile na steno cilindra**

**Vertikalna sila** $n_z$ na neki izbrani globini $z = z_1$ is se izračuna z integracijo trenja po steni ($p_w$) vzdolž celotne višine cilindra. V procesu polnjenja in praznjenja silosa je ta sila tlačna (negativna).

$$n_z(z = z_1) = \int_0^{z_1} p_w(z)dz \tag{I.91}$$

Oznaka za trenje po steni $p_w$ se tukaj nanaša na polnjenje ( $p_{wf}$ ) ali praznjenje ( $p_{we}$ ) silosa.

**Obročna sila** $n_\phi$ se izračuna po kotelni formuli, kot produkt horizontalnega pritiska $p_h$ in polmera $r$. Je enaka za poljuben kot $\phi$. V procesu polnjenja in praznjenja silosa je ta sila natezna (pozitivna).

$$n_\phi(z) = p_h(z) \cdot r \tag{I.92}$$

Podobno kot oznaka za trenje po steni, se tudi oznaka za horizontalni pritisk $p_h$ nanaša na polnjenje ($p_{hf}$) ali praznjenje ($p_{he}$) silosa. Pri določitvi notranjih sil je potrebno upoštevati ustrezne enačbe za pritiske, ki so odvisne od tipa cilindra oz. silosa (poglavje 3.1.1.3).

#### *3.3.1.1.1 Vitki silos*

**Polnjenje:**

Pritisk trenja na steno cilindra med polnjenjem $p_{wf}$ je v vitkem silosu enaka (*Diagram 1.1*):

$$p_{wf}(z) = \mu \cdot p_{h0} \cdot Y_J(z) = \mu \cdot p_{h0} \cdot \left(1 - e^{-z/z_0}\right) \tag{I.93}$$

Izraz za vertikalno silo med polnjenjem na globini $z = z_1$ se glasi:

$$n_{zf}(z_1) = \int_0^{z_1} p_{wf}(z)dz = \int_0^{z_1} \mu \cdot p_{h0} \cdot \left(1 - e^{-z/z_0}\right) dz$$

$$n_{zf}(z_1) = \mu \cdot p_{h0} \cdot \left( [z]_0^{z_1} - z_0 \left(1 - e^{-z/z_0}\right)_0^{z_1} \right)$$

$$n_{zf}(z_1) = \mu \cdot p_{h0} \cdot \left(z_1 - z_0 \left(1 - e^{-z_1/z_0}\right)\right) \tag{I.94}$$

Vertikalna sila $n_{zf}$ na poljubni globini $z$ je torej enaka:

$$\boxed{n_{zf}(z) = \mu \cdot p_{h0} \cdot \left(z - z_0 \left(1 - e^{-z/z_0}\right)\right)} \tag{I.95}$$

Horizontalni pritisk na steno cilindra med polnjenjem $p_{hf}$ je v vitkem silosu enak:

$$p_{hf}(z) = p_{h0} \cdot Y_J(z) = p_{h0} \cdot \left(1 - e^{-z/z_0}\right) \tag{I.96}$$

Obročna sila na $n_\phi$ poljubni globini $z$ je enaka:

$$\boxed{n_{\phi f}(z) = p_{h0} \cdot \left(1 - e^{-z/z_0}\right) \cdot r} \tag{I.97}$$

Pritisk $p_{h0}$ je dan v enačbi (I.46), globina $z_0$ pa ve enačbi (I.17).

**Praznjenje:**

Pritiske med procesom praznjenja določimo tako, da pritiske iz procesa polnjenja pomnožimo z dodatnimi faktorji za primer praznjenja $C_w$ in $C_h$ (*Diagram 1.2*).

$$p_{we} = C_w \cdot p_{wf} \tag{I.98}$$

$$p_{he} = C_h \cdot p_{hf} \tag{I.99}$$

Izraz za vertikalno silo med praznjenjem na poljubni globini $z$ se glasi:

$$\boxed{n_{ze}(z) = C_w \cdot \mu \cdot p_{h0} \cdot \left( z - z_0 \left( 1 - e^{-z/z_0} \right) \right)} \tag{I.100}$$

Obročna sila na poljubni globini $z$ je enaka:

$$\boxed{n_{\phi e}(z) = C_h \cdot p_{h0} \cdot \left( 1 - e^{-z/z_0} \right) \cdot r} \tag{I.101}$$

### *3.3.1.1.2 Plitvi silos in silos srednje vitkosti*

**Polnjenje:**

Pritisk trenja na steno cilindra med polnjenjem $p_{wf}$ je v plitvem silosu ter silosu srednje vitkosti enak (*Diagram 1.1*):

$$p_{wf}(z) = \mu \cdot p_{h0} \cdot Y_R(z) = \mu \cdot p_{h0} \cdot \left[ 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n \right] \tag{I.102}$$

Izraz za vertikalno silo med polnjenjem na globini $z = z_1$ se glasi:

$$n_{zf}(z_1) = \int_0^{z_1} p_{wf}(z) dz = \int_0^{z_1} \mu \cdot p_{h0} \cdot \left[ 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n \right] dz$$

$$n_{zf}(z_1) = \mu \cdot p_{h0} \cdot \int_0^{z_1} 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n dz$$

$$n_{zf}(z_1) = \mu \cdot p_{h0} \cdot \left[ z_1 - h_0 + \frac{1}{n+1}\left( z_0 - h_0 - \frac{(z_1 + z_0 - 2h_0)^{n+1}}{(z_0 - h_0)^n} \right) \right] \tag{I.103}$$

Če označimo:

$$z_V(z) = h_0 - \frac{1}{n+1}\left( z_0 - h_0 - \frac{(z + z_0 - 2h_0)^{n+1}}{(z_0 - h_0)^n} \right), \tag{I.104}$$

lahko enačbo (I.103) za silo $n_{zf}$ na poljubni globini $z$ zapišemo kot:

$$\boxed{n_{zf}(z) = \mu \cdot p_{h0} \cdot \left( z - z_V(z) \right)} \tag{I.105}$$

Pritisk $p_{h0}$ je dan v enačbi (I.46), globina $z_0$ pa ve enačbi (I.17).

Horizontalni pritisk na steno cilindra med polnjenjem $p_{hf}$ je v plitvem silosu ter silosu srednje vitkosti enak:

$$p_{hf}(z) = p_{h0} \cdot Y_R(z) = p_{h0} \cdot \left[ 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n \right] \tag{I.106}$$

Obročna sila na poljubni globini $z$ je enaka:

$$\boxed{n_{\phi f}(z) = p_{h0} \cdot \left[ 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n \right] \cdot r} \tag{I.107}$$

**Praznjenje:**

Veljata enačbi (I.98) in (I.99) .

Izraz za vertikalno silo med praznjenjem na poljubni globini *z* se glasi:

$$\boxed{n_{ze}(z) = C_w \cdot \mu \cdot p_{h0} \cdot \left(z_1 - z_V(z_1)\right)} \qquad \text{(I.108)}$$

Obročna sila na poljubni globini *z* je enaka:

$$\boxed{n_{\phi e}(z) = C_h \cdot p_{h0} \cdot \left[1 - \left(\frac{z - h_o}{z_o - h_o} + 1\right)^n\right] \cdot r} \qquad \text{(I.109)}$$

### *3.3.1.1.3 Zadrževalni silosi*

Horizontalni pritisk na steno cilindra:

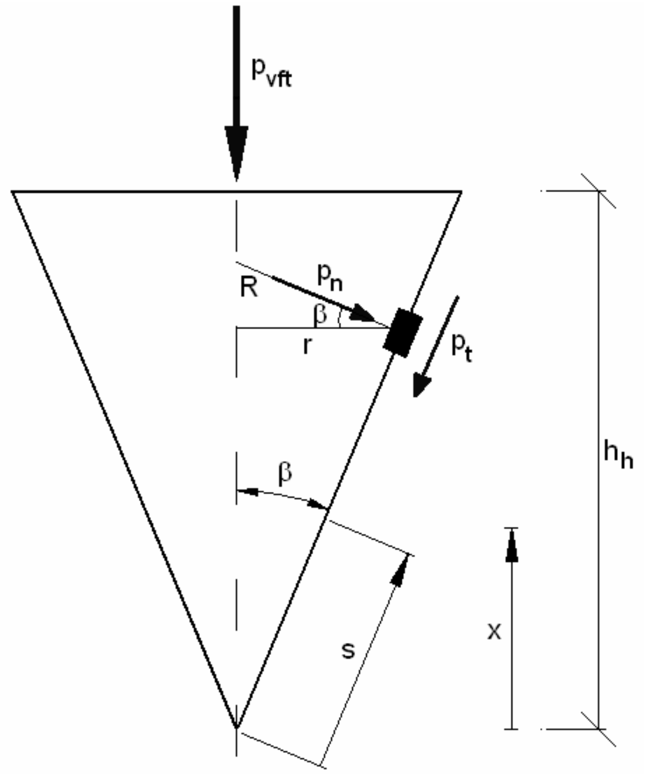
$$p_h(z) = \gamma K\,(1 + \sin\phi_r)\,z \qquad \text{(I.110)}$$

Membranski sili pri polnjenju in praznjenju znašata:

$$\boxed{n_z(z) = \gamma \cdot \frac{\mu \cdot K}{2} \cdot (1 + \sin\phi_r) \cdot z^2} \qquad \text{(I.111)}$$

$$\boxed{n_\phi(z) = \gamma \cdot K \cdot (1 + \sin\phi_r) \cdot z \cdot r} \qquad \text{(I.112)}$$

### 3.3.1.2 Stena lijaka



$$r = x \cdot \tan\beta \qquad (I.113)$$

$$R = r \cdot \cos\beta \qquad (I.114)$$

$$s = x \cdot \cos\beta \qquad (I.115)$$

$$\xi = \frac{x}{h_h} \qquad (I.116)$$

Slika 21: Pritiski na stene cilindra

Celotni pritisk v smeri koordinate *s* na poljubni višini *x* je enak:

$$p_s(x) = p_t(x) + \tan\beta \cdot p_n(x) \qquad (I.117)$$

Normalni pritisk $p_n\ (x)$ in trenje $p_t\ (x)$ na poljubni višini lijaka določata enačbi:

$$p_t(x) = \mu_{heff} \cdot F \cdot p_v(x) \qquad (I.118)$$

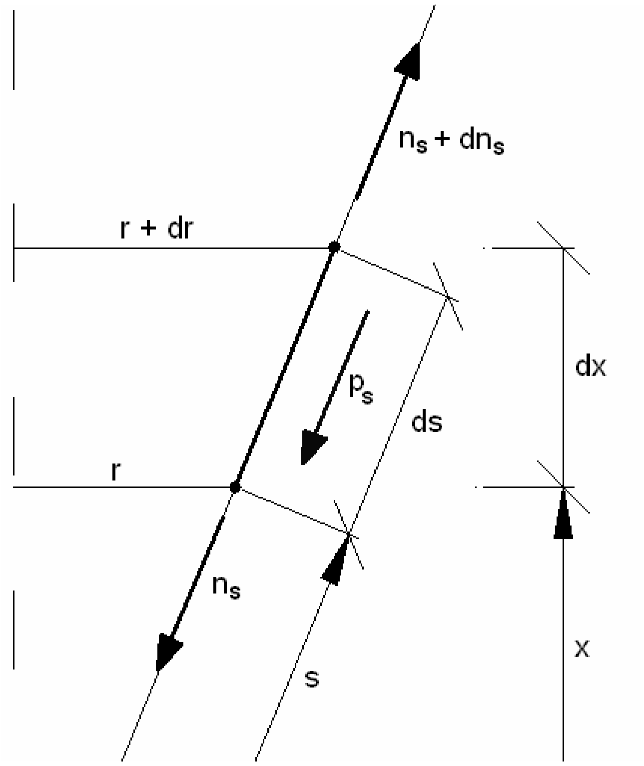
$$p_n(x) = F \cdot p_v(x) \qquad (I.119)$$

Parameter $F$ predstavlja karakteristično vrednost razmerja pritiskov v lijaku pri polnjenju ($F_f$) ali praznjenju ($F_e$), parameter $\mu_{heff}$ pa predstavlja efektivni oz. mobilizirani koeficient trenja na steni lijaka. Enačbe za njun izračun se nahajajo na *Diagramu 2.1.*

Pritisk $p_s$ se tako lahko zapiše kot:

$$p_s(x) = F \cdot (\mu_{heff} + \tan\beta) \cdot p_v(x) \qquad \text{(I.120)}$$

Vertikalni pritisk $p_v(x)$ v shranjenem materialu na globini $x$ določa enačba (I.25). Obravnavajmo sedaj delček stene lijaka dolžine $ds$.



**Slika 22: Delček stene lijaka dolžine *ds***

Ravnotežni pogoj za ta delček se glasi:

$$n_s \cdot r \cdot d\phi + p_s(x) \cdot r \cdot d\phi \cdot ds - (n_s + dn_s) \cdot (r + dr) \cdot d\phi = 0 \qquad \text{(I.121)}$$

Če gornjo enačbo poenostavimo, dobimo:

$$n_s \cdot r + p_s(x) \cdot r \cdot ds - n_s \cdot r - n_s \cdot dr - r \cdot dn_s - dr \cdot dn_s = 0$$

$$n_s \cdot dr + r \cdot dn_s = p_s(x) \cdot r \cdot ds$$

$$d\left(n_s \cdot r\right) = p_s(x) \cdot r \cdot ds \qquad \text{(I.122)}$$

Celotna sila $n_s$ na višini *s'* se izračuna po izrazu:

$$n_s = \frac{1}{r} \int_0^{s'} p_s(x) \cdot r \, ds \qquad \text{(I.123)}$$

Če upoštevamo enačbe (I.113) do (I.115) in preoblikujemo prejšnjo enačbo tako, da $n_s$ računamo do višine *x'* namesto *s'*, potem dobimo:

$$n_s = \frac{1}{x \cdot \tan\beta} \int_0^{x'} p_s(x) \cdot \frac{\tan\beta \cdot x}{\cos\beta} \, dx$$

$$n_s = \frac{1}{x \cdot \cos\beta} \int_0^{x'} p_s(x) \cdot x \, dx \qquad \text{(I.124)}$$

Če upoštevamo še enačbi (I.116) in (I.120), dobimo:

$$n_s = \frac{1}{\xi \cdot h_h \cdot \cos\beta} \int_0^{\xi'} F \cdot (\mu_{heff} + \tan\beta) \cdot p_v(\xi) \cdot h_h^{\,2} \cdot \xi \, d\xi$$

$$n_s = \frac{F \cdot (\mu_{heff} + \tan\beta)}{\cos\beta} h_h \cdot p_{vft} \cdot \frac{1}{\xi} \int_0^{\xi'} \frac{p_v(\xi)}{p_{vft}} \cdot \xi \, d\xi \qquad \text{(I.125)}$$

Označimo sedaj

$$I = \int_0^{\xi'} \frac{p_v(\xi)}{p_{vft}} \cdot \xi \, d\xi \qquad \text{(I.126)}$$

in

$$\boxed{\alpha = \frac{\gamma_{max} \cdot h_h}{(n-1) \cdot p_{vft}}} \tag{I.127}$$

Parameter *n* predstavlja eksponent v funkciji za vertikalni pritisk *p<sub>v</sub>(x)* in je določen v *Diagramu 2.1* in *2.2*, odvisno od tega ali obravnavamo polnjenje oz. praznjenje materiala.

Če upoštevamo sedaj enačbe (I.25) in (I.127) v (I.126), dobimo:

$$I = \int_0^{\xi'} \frac{p_v(\xi)}{p_{vft}} \cdot \xi \, d\xi$$

$$I = \int_0^{\xi'} \left[ \frac{\gamma_{max} \cdot h_h / (n-1)}{p_{vft}} \left( \xi - \xi^n \right) + \xi^n \right] \cdot \xi \, d\xi$$

$$I = \int_0^{\xi'} \left[ \alpha \cdot \left( \xi - \xi^n \right) + \xi^n \right] \cdot \xi \, d\xi$$

$$I = \alpha \int_0^{\xi'} \xi^2 \, d\xi - \alpha \int_0^{\xi'} \xi^{n+1} \, d\xi + \int_0^{\xi'} \xi^{n+1} \, d\xi \tag{I.128}$$

Če integriramo enačbo (I.128) in zapišemo $\xi' = \xi$, dobimo:

$$I = \alpha \frac{\xi^3}{3} + \frac{(1-\alpha) \cdot \xi^{n+2}}{n+2} \tag{I.129}$$

Označimo sedaj:

$$f_s(\xi) = \frac{I}{\xi}$$

$$\boxed{f_s(\xi)=\frac{\xi}{3}\left(\alpha\cdot\xi+\frac{3\cdot(1-\alpha)\cdot\xi^n}{n+2}\right)} \tag{I.130}$$

in

$$\boxed{n_{s0}=F\cdot\frac{(\mu_{heff}+\tan\beta)}{\cos\beta}h_h\cdot p_{vft}} \tag{I.131}$$

Celotna sila $n_s$ izražena z brezdimenzijsko koordinato $\xi$ se sedaj glasi:

$$\boxed{n_s(\xi)=n_{s0}\cdot f_s(\xi)} \tag{I.132}$$

Podoben razmislek uporabimo, ko računamo celotno obročno silo $n_\phi$ na višini $\xi$.

$$n_\phi=p_n(\xi)\cdot R \tag{I.133}$$

Ob upoštevanju enačb (I.133), (I.113), (I.114) in (I.116), dobimo:

$$n_\phi=p_n(\xi)\cdot\frac{r}{\cos\beta}=p_n\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot\xi \tag{I.134}$$

Če nadalje uporabimo še definicije, ki so dane v enačbah (I.25), (I.119) in (I.127), z enačbo (I.134), dobimo:

$$n_\phi=F\cdot p_v(\xi)\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot\xi$$

$$n_\phi=F\cdot p_{vft}\cdot\left[\alpha\cdot\left(\xi-\xi^n\right)+\xi^n\right]\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot\xi$$

$$n_\phi=F\cdot p_{vft}\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot\left[\alpha\cdot\xi^2+(1-\alpha)\cdot\xi^{n+1}\right] \tag{I.135}$$

Označimo:

$$\boxed{f_{\phi}(\xi) = \alpha \cdot \xi^{2} + (1-\alpha) \cdot \xi^{n+1}} \tag{I.136}$$

in

$$\boxed{n_{\phi 0} = F \cdot \frac{\tan\beta}{\cos\beta} h_{h} \cdot p_{vft}} \tag{I.137}$$

Celotna obročna sila $n_{\phi h}$ na višini $\xi$, se določi kot:

$$\boxed{n_{\phi}(\xi) = n_{\phi 0} \cdot f_{\phi}(\xi)} \tag{I.138}$$

Izrazi za izračun parametrov $p_{vft}$, $n$, $F$ so dani v poglavju *3.1.1.7*, *Diagramu 2.1* and *2.2*.

## 3.3.2 Nesimetrična obtežba

Dodatne membranske sile, ki se pojavijo na globini $z_p$, je potrebno upoštevati, ko je bil uporabljen postopek za določitev nesimetrične obtežbe za silose v *AAC 3* (poglavje *3.2.1.2*). V obeh spodnjih primeri teče koordinata $z$ od $z_p$ do $z_p + s$ (*Slika 18*).

**Polnjenje:**

Dodatna vertikalna sila $n_{z,pfs}$ in obročna sila $n_{\phi,pfs}$, ki se pojavita zaradi nesimetrične obtežbe v steni cilindra med polnjenjem, sta podani kot:

$$\boxed{n_{z,pfs}(z,\phi) = \frac{p_{pfs}(\phi)}{d_c} \cdot z^{2}} \tag{I.139}$$

$$\boxed{n_{\phi,pfs}(\phi) = -p_{pfs}(\phi) \cdot r} \tag{I.140}$$

Strižna sila $n_{z\phi,pfs}$ znaša:

$$\boxed{n_{z\phi,pfs}(z,\phi) = -p_{hf} \cdot \sin\phi \cdot z} \tag{I.141}$$

**Praznjenje:**

Dodatna vertikalna sila $n_{z,pes}$ in obročna sila $n_{\phi,pes}$ ki se pojavita zaradi nesimetrične obtežbe v steni cilindra med praznjenjem, sta podani kot:

$$\boxed{n_{z,pes}(z,\phi) = \frac{p_{pes}(\phi)}{d_c} \cdot z^2} \tag{I.142}$$

$$\boxed{n_{\phi,pes}(\phi) = -p_{pes}(\phi) \cdot r} \tag{I.143}$$

Strižna sila $n_{z\phi,pes}$ znaša:

$$\boxed{n_{z\phi,pes}(z,\phi) = -p_{he} \cdot \sin\phi \cdot z} \tag{I.144}$$

# 4 Obtežba vetra

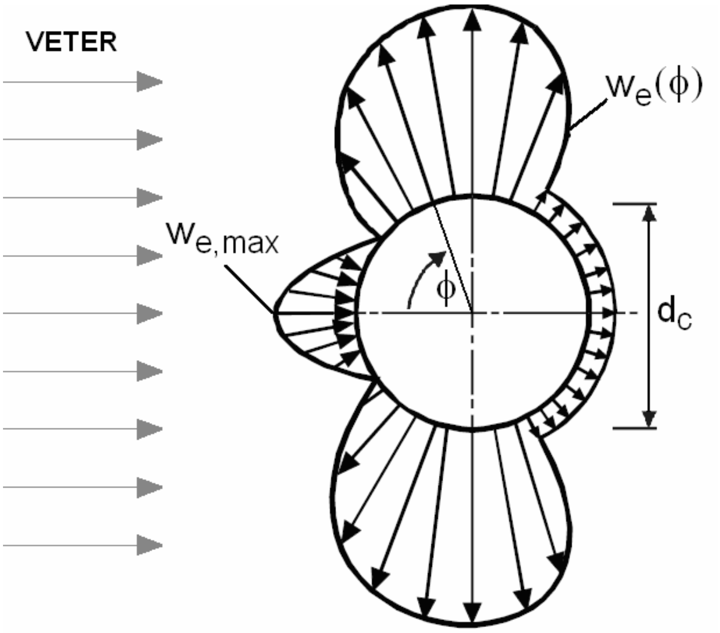
## 4.1 Uvod in pregled

V tem poglavju je predstavljeno določevanje obtežbe vetra na silose. Najprej je predstavljena razporeditev zunanjih pritiskov na stene cilindra. Izpeljani so končni izrazi za določitev tega pritiska pri tipičnih geometrijah silosov in predstavljene različna projektna stanja, ki jih je potrebno obravnavati pri dimenzioniranju.

Izdelan je bil tudi računalniški program, ki omogoča določitev obtežbe vetra vzdolž poljubnega silosa. Program je predstavljen v zadnjem delu tega poglavja.

## 4.2 Pritisk vetra na silose

Če je telo cilindrične oblike izpostavljeno zračnemu toku visoke hitrosti (vetru), se okoli cilindra pojavi sledeča porazdelitev pritiskov.



**Slika 23: Razporeditev pritiska vetra po obodu cilindra**

Iz slike vidimo, da povzroča pritisk vetra tlake samo na privetrni strani. Na zavetrni strani in na straneh cilindra se zaradi delovanja vetra pojavijo srki.

Zunanji pritisk vetra na krožni cilinder $w_e(\phi)$ se z višino cilindra ne spreminja. Podan je v *EN 1991-1-4*, z enačbo:

$$w_e(\phi) = q_p \cdot c_{p0}(\phi) \cdot \psi_{\lambda\phi}(\phi) \qquad \text{(I.145)}$$

Parameter $q_p$ predstavlja tlak pri konični hitrosti vetra in je neodvisen od tipa konstrukcije. Potrebno ga je določiti v skladu z določili, podanimi v *EN 1991-1-4*.

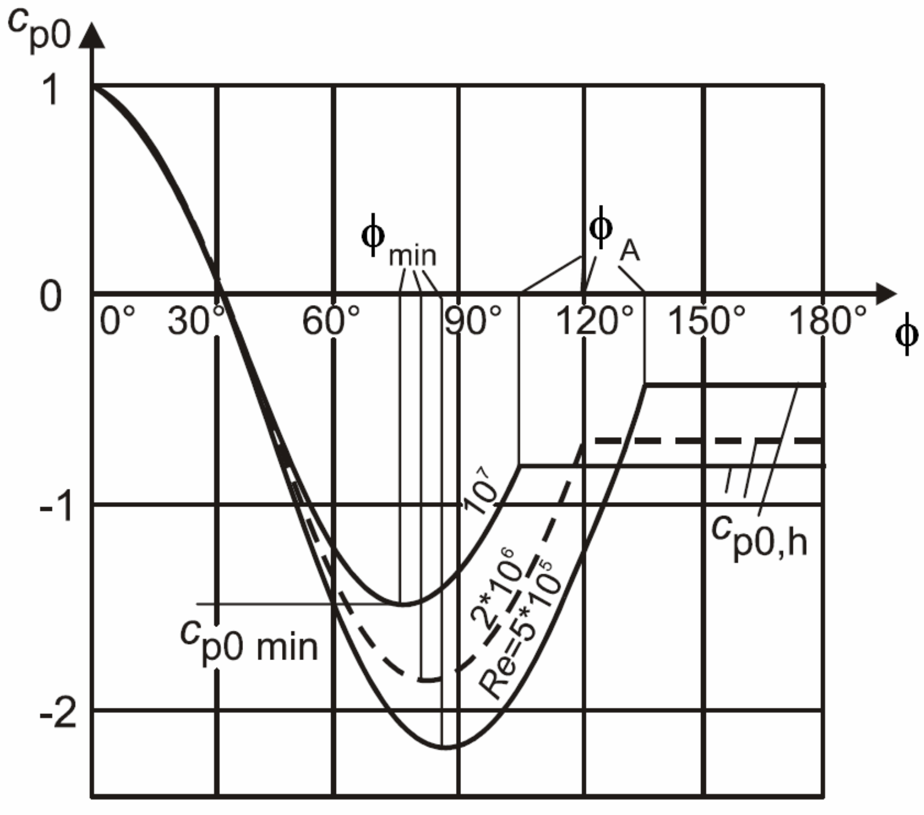
Funkcija $c_{p0}(\phi)$ se imenuje koeficient zunanjega pritiska vetra in predstavlja »obliko« razporeditve pritiska vetra okoli cilindra. Odvisna je od Reynoldsovega števila izbranega cilindra. Funkcija $\psi_{\lambda\phi}(\phi)$ pa predstavlja funkcijo vitkosti cilindra.

Oglejmo si sedaj funkciji $c_{p0}(\phi)$ in $\psi_{\lambda\phi}(\phi)$, značilni za tipične geometrije silosov (*Slika 4*).

### 4.2.1 Koeficient zunanjega pritiska vetra

V tem poglavju je izpeljana formula za funkcijo $c_{p0}(\phi)$, ki je značilna za tipične geometrije silosov, ki se uporabljajo v inženirski praksi.

Spodnja slika je vzeta iz standarda in predstavlja potek funkcije $c_{p0}(\phi)$ za tri različna Reynoldsova števila.



**(Povzeto po standardu EN 1991-1-4:2005, Slika 7.27)**

Silosi, ki se pogosto uporabljajo v inženirski praksi, imajo Reynoldsova števila enaka približno $Re = 5 \cdot 10^5$, zato obravnavamo samo krivuljo, ki ustreza tej vrednosti. Standard *EN 1991-1-4* ne podaja splošnih enačb za krivulje prikazane na zgornji sliki. Namesto tega so te krivulje podane zgolj z določenimi karakterističnimi točkami.

Karakteristične točke za krivuljo $c_{p0}(\phi)$, ki je značilna za tipične geometrije silosov, so podane v spodnji preglednici:

**Preglednica 16: Karakteristične točke za funkcijo $C_{p0}(\phi)$[2]**

| **$\phi$ [°]** | 0 | 33 | 60 | 85 | 135 |
|---|---|---|---|---|---|
| **$c_{p0}(\phi)$** | 1 | 0 | -1.45 | -2.2 | -0.4 |

---

[2] Vrednosti vzete iz EN 1991-1-4:2005, Slika 7.27 in Preglednica 7.12

Funkcijo $c_{p0}(\phi)$ zapišimo kot Fourierevo kosinusno vrsto:

$$c_{p0}(\phi)=\frac{c_0}{2}+c_1\cdot cos\phi+c_2\cdot cos2\phi+c_3\cdot cos3\phi+\dots$$

$$\boxed{c_{p0}(\phi)=\frac{c_0}{2}+\sum_{m=1}^{\infty}c_m\cdot cos\,m\phi} \qquad \text{(I.146)}$$

Kot lahko vidimo v sledečih poglavjih, je, zaradi ortogonalnosti kosinusne funkcije, ta zapis zelo ugoden.

Zaradi enostavnosti združimo koeficienta $c_0$ in $c_m$, kjer $m\in[1,\ 2,\dots,\infty]$, v en sam koeficient $c_n$, kjer $n\in[0,1,\ 2,\dots,\infty]$. Koeficiente $c_n$ se izračuna po metodi najmanjših kvadratov, z uporabo karakterističnih točk za dano krivuljo .

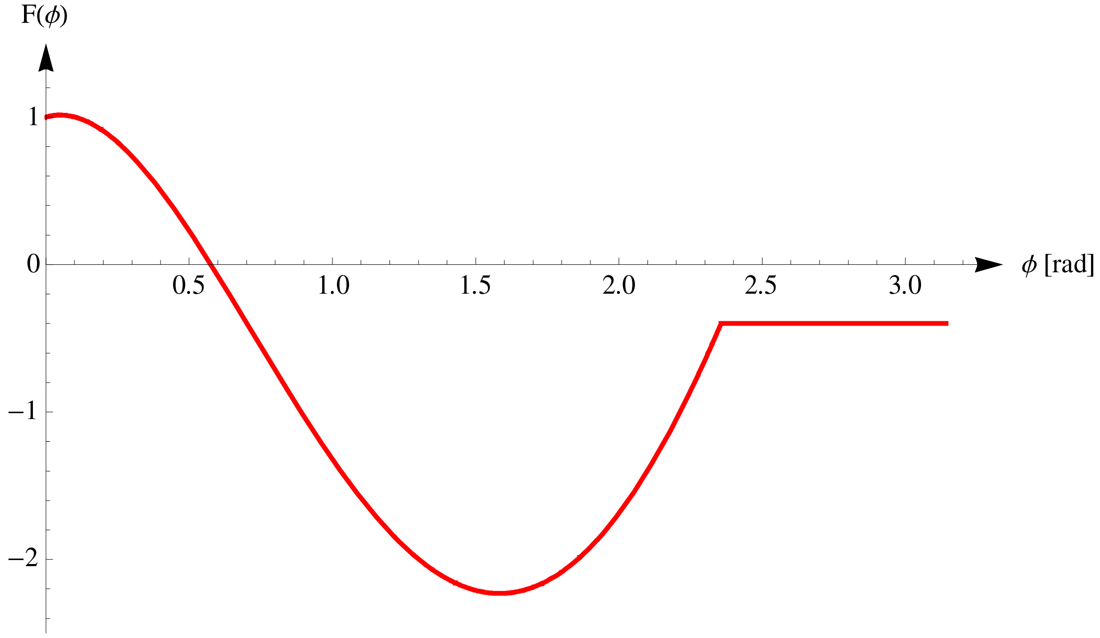
Najprej je potrebno aproksimirati funkcijo $c_{p0}(\phi)$, ki poteka skozi karakteristične točke (*Preglednica 13*). Nelinearni del te krivulje je aproksimiran z uporabo polinoma 4. stopnje[3], medtem ko je konstantni del funkcije enak – *0.4*. Združena aproksimacija *F(ϕ)* torej znaša:

$$F(\phi)=\left\langle\begin{array}{lcl}-0.462\,\phi^4+3.308\,\phi^3-5.711\,\phi^2+0.544\,\phi+1 & ; & \phi<135^\circ\\ -0.4 & ; & \phi\geq135^\circ\end{array}\right\rangle \qquad \text{(I.147)}$$

Na spodnji sliki je predstavljen grafični potek aproksimacije $F(\phi)$. Kot $\phi$ je podan v radianih.

---

[3] Za aproksimacijo je bil uporabljen računalniški program Wolfram Mathematica 6.0



**Slika 24: Aproksimacija koeficienta zunanjega pritiska**

Metoda najmanjših kvadratov temelji na izreku, da mora biti razlika oz. napaka vsote kvadratov razlike med aproksimirano in dejansko vrednostjo minimalna.

$$\varepsilon = \int_0^{\pi} \left( F(\phi) - c_{p0}(\phi) \right)^2 d\phi = min \qquad \text{(I.148)}$$

Napaka doseže svojo ekstremno vrednost (v našem primeru je to minimum), ko je njen prvi odvod enak nič:

$$\frac{\partial \varepsilon}{\partial c_n} = 0$$

$$\frac{\partial \varepsilon}{\partial c_n} = -2\int_0^{\pi} \left( F(\phi) - c_{p0}(\phi) \right) \cdot \frac{\partial c_{p0}(\phi)}{\partial c_n} d\phi = 0 \qquad \text{(I.149)}$$

Odvod funkcije $c_{p0}(\phi)$ po koeficientih $c_n$ je enak:

$$\frac{\partial c_{p0}(\phi)}{\partial c_n} = cos\, n\phi \qquad \text{(I.150)}$$

Enačba (I.149) se sedaj lahko zapiše kot:

$$\int_0^{\pi} F(\phi)\cdot cos\, n\phi\, d\phi - \int_0^{\pi}\left(\frac{c_0}{2} + \sum_{m=1}^{\infty} c_m \cdot cos\, m\phi\right)\cdot cos\, n\phi\, d\phi = 0 \qquad (I.151)$$

Posebej obravnavajmo integral *I*:

$$I = \int_0^{\pi}\left(\frac{c_0}{2} + \sum_{m=1}^{\infty} c_m \cdot cos\, m\phi\right)\cdot cos\, n\phi\, d\phi$$

$$I = c_0 \cdot \frac{sin\, n\pi}{2n} + \int_0^{\pi} \sum_{m=1}^{\infty} c_m \cdot cos\, m\phi \cdot cos\, n\phi\, d\phi \qquad (I.152)$$

Iz lastnosti ortogonalnosti funkcije kosinus[4] izhaja:

$$\int_0^{\pi} cos\, m\phi \cdot cos\, n\phi\, d\phi = \left\langle \begin{array}{l} 0,\ \check{c}e\ m \neq n \\ \frac{\pi}{2},\ \check{c}e\ m = n \end{array} \right\rangle \qquad (I.153)$$

Če upoštevamo zgornjo zvezo, lahko integral *I* zapišemo kot:

$$I = c_0 \cdot \frac{sin\, n\pi}{2n} + c_n \cdot \frac{\pi}{2} \qquad (I.154)$$

Enačba (I.151) se nato poenostavi:

$$\int_0^{\pi} F(\phi)\cdot cos\, n\phi\, d\phi - c_0 \cdot \frac{sin\, n\pi}{2n} - c_n \cdot \frac{\pi}{2} = 0 \qquad (I.155)$$

---

[4] Funkciji *i(x)* in *j(x)* sta ortogonalni na območju od *a* do *b*, če $\int_a^b i(x)\cdot j(x)\,dx = 0$ , kadar je $i(x) \neq j(x)$.

Koeficient $c_n$ se lahko sedaj izrazi kot:

$$c_n = \frac{2}{\pi}\left( \int_0^{\pi} F(\phi) \cdot \cos n\phi \, d\phi - c_0 \cdot \frac{\sin n\pi}{2n} \right) = 0, \qquad \text{(I.156)}$$

kjer $n \in [0, 1,\ 2, \ldots, \infty]$.

Ker je $n$ naravno število, je desni člen dela enačbe v oklepaju enak $0$:

$$c_0 \cdot \frac{\sin n\pi}{2n} = 0, \quad \forall n \in [0, 1,\ 2, \ldots, \infty]. \qquad \text{(I.157)}$$

Enačba (I.156) se tako lahko zapiše kot:

$$\boxed{c_n = \frac{2}{\pi}\left( \int_0^{\pi} F(\phi) \cdot \cos n\phi \, d\phi \right) = 0, \quad n \in [0, 1,\ 2, \ldots, \infty]} \qquad \text{(I.158)}$$

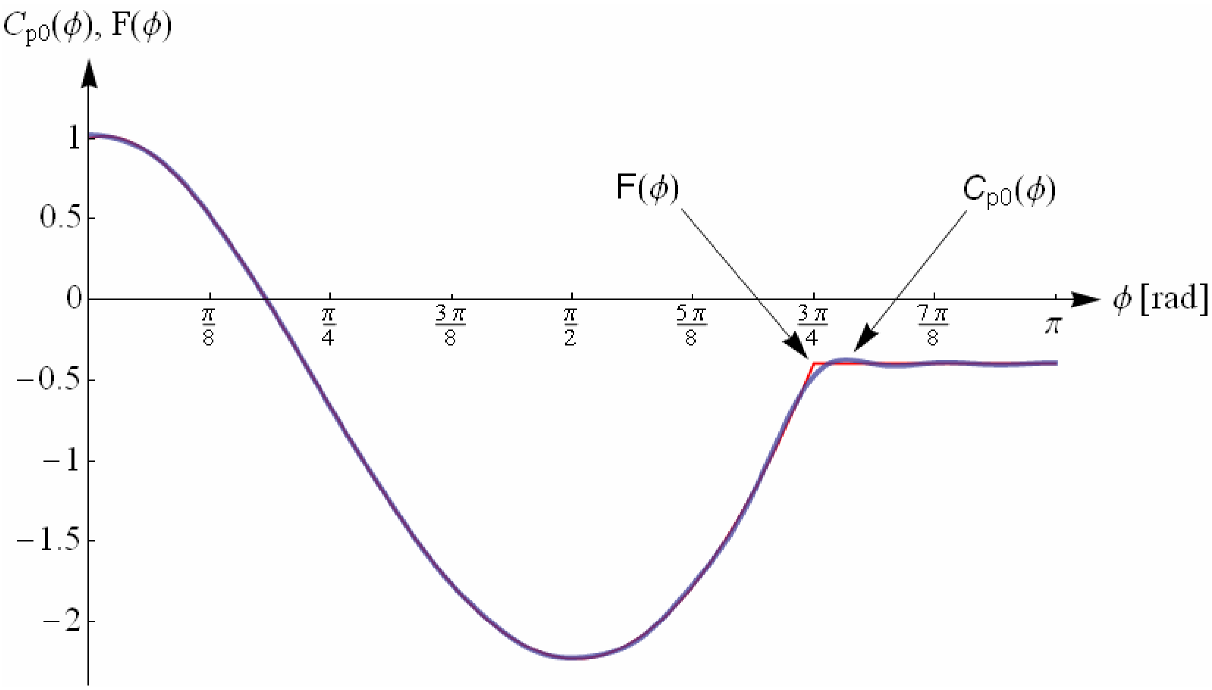
Izračunajmo sedaj prvih 20 členov Fouriereve vrste (I.146), ki predstavlja funkcijo $c_{p0}(\phi)$. Vrednosti za koeficiente $c_n$ so podane v spodnji preglednici.

**Preglednica 17: Prvih 20 členov funkcije $C_{p0}(\phi)$**

| n | $c_n$ |
|---|---|
| 0 | -1.6386 |
| 1 | 0.3720 |
| 2 | 1.2855 |
| 3 | 0.2832 |
| 4 | -0.1685 |
| 5 | 0.1106 |
| 6 | -0.0142 |
| 7 | -0.0384 |
| 8 | 0.0438 |
| 9 | -0.0309 |

| n | $c_n$ |
|---|---|
| 10 | 0.0008 |
| 11 | 0.0135 |
| 12 | -0.0220 |
| 13 | 0.0115 |
| 14 | -0.0025 |
| 15 | -0.0098 |
| 16 | 0.0102 |
| 17 | -0.0086 |
| 18 | -0.0004 |
| 19 | 0.0045 |

Na spodnji sliki je prikazana primerjava med funkcijo $c_{p0}(\phi)$, izračunano za prvih 20 členov, in njeno aproksimacijo $F(\phi)$.



**Slika 25: Funkcija koeficienta zunanjega pritiska vetra $C_{p0}(\phi)$ določena po metodi najmanjših kvadratov in njena aproksimacija $F(\phi)$**

Vidimo lahko, da se krivulji dobro ujemata. Največja razlika se nahaja v bližini kota $\phi = \frac{3\pi}{4}$, kjer znaša okoli *20%*. Manjšo razliko bi dobili, če bi povečali število členov za izračun $c_{p0}(\phi)$.

## 4.2.2 Funkcija vitkosti cilindra

Izračun zunanjega pritiska vetra je odvisen tudi od funkcije vitkosti cilindra $\psi_{\lambda\phi}(\phi)$, ki je prav tako določena kot funkcija kota $\phi$.

Za cilindrične silose je funkcija vitkosti definirana kot:

$$\psi_{\lambda\phi}(\phi)=\left\langle \begin{array}{ll} 1 & ;\ 0° \le \phi \le 85° \\ \psi_\lambda + (1-\psi_\lambda)\cdot cos\left(\frac{3}{20}(\pi+12\phi)\right) & ;\ 85° < \phi < 135° \\ \psi_\lambda & ;\ 135° \le \phi \le 180° \end{array} \right\rangle \qquad (I.159)$$

Parameter $\psi_\lambda$ predstavlja faktor vitkosti. Vrednosti faktorja vitkosti in funkcije vitkosti cilindra so za tipične silose podane v spodnji preglednici.

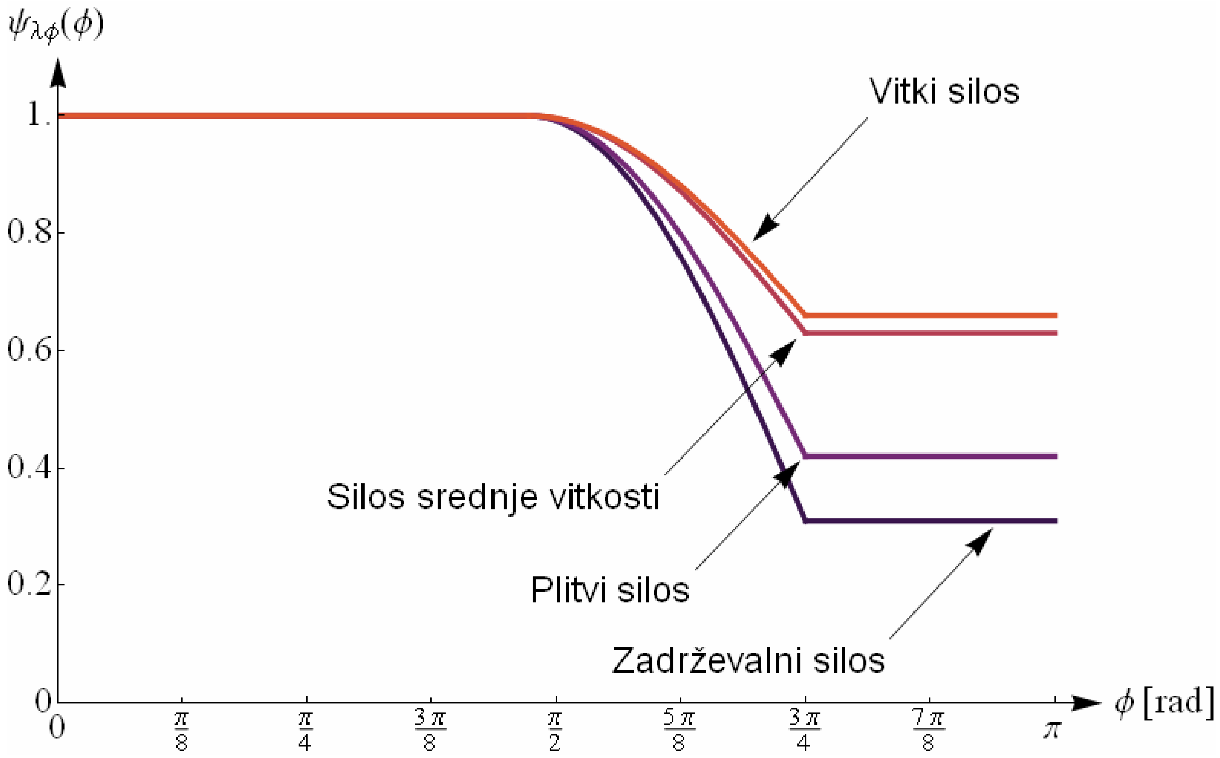
**Preglednica 18: Vrednosti faktorja vitkosti in funkcije vitkosti za tipične silose**

| **Tip silosa** | $\psi_\lambda$ | $\psi_{\lambda\phi}(\phi)$, **ko 85°<** $\phi$ **< 135°** |
|---|---|---|
| Zadrževalni silos | $0.31$ | $0.31-0.69\cdot\eta$ |
| Plitvi silos | $0.42$ | $0.42-0.58\cdot\eta$ |
| Silos srednje vitkosti | $0.63$ | $0.63-0.37\cdot\eta$ |
| Vitki silos | $0.66$ | $0.66-0.34\cdot\eta$ |

Parameter $\eta$ je enak:

$$\eta = cos\left(\frac{3}{20}(\pi+12\phi)\right) \quad ; \quad \phi \text{ je merjen v radianih.} \qquad (I.160)$$

Funkcija vitkosti je grafično prikazana na naslednji sliki.



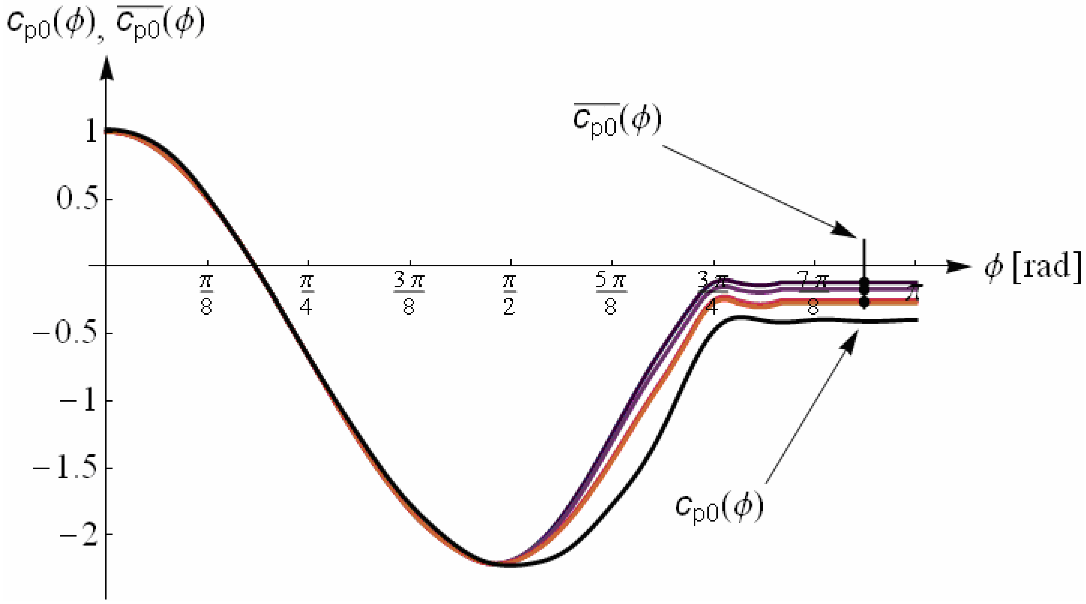
**Slika 26: Funkcija vitkosti za tipične silose**

### 4.2.3 Ponazoritev pritiska vetra za praktični račun

Označimo funkcijo, ki predstavlja produkt koeficienta zunanjega pritiska vetra in funkcije vitkosti, kot $\overline{c_{p0}}(\phi)$:

$$\overline{c_{p0}}(\phi) = c_{p0}(\phi) \cdot \psi_{\lambda\phi}(\phi) \tag{I.161}$$

Spodnja slika prikazuje primerjavo med dejansko funkcijo $c_{p0}(\phi)$ in modificirano funkcijo $\overline{c_{p0}}(\phi)$ za različne tipe cilindrov.



**Slika 27: Primerjava med dejansko in modificirano funkcijo zunanjega pritiska vetra**

Vidimo, da ima funkcija vitkosti $\psi_{\lambda\phi}(\phi)$ vpliv samo na zavetrno stran cilindra ($\phi > \frac{\pi}{2}$), kjer zmanjšuje srk vetra. To pomeni, da smo na varni strani, če predpostavimo, da je $\psi_{\lambda\phi}(\phi) = 1$, in uporabimo dejansko oz. nemodificirano funkcijo zunanjega pritiska vetra $c_{p0}(\phi)$.

Ob upoštevanju zgornje predpostavke lahko zapišemo enačbo za izračun zunanjega pritiska vetra kot:

$$\boxed{w_e(\phi) = q_p \cdot c_{p0}(\phi)} \qquad \text{(I.162)}$$

Funkcija $c_{p0}(\phi)$ je določena v poglavju *4.2.1*.

## *4.3 Projektna stanja*

Pri obtežbi vetra na silos je potrebno obravnavati dve projektni stanji:

- ko je silos poln,
- ko je silos prazen.

V prvem primeru je potrebno upoštevati celotno silo vetra na silos, ki jo uporabimo pri kontroli globalne prevrnitve silosa. V tem projektnem stanju je potrebno obravnavati tudi upogib silosa.

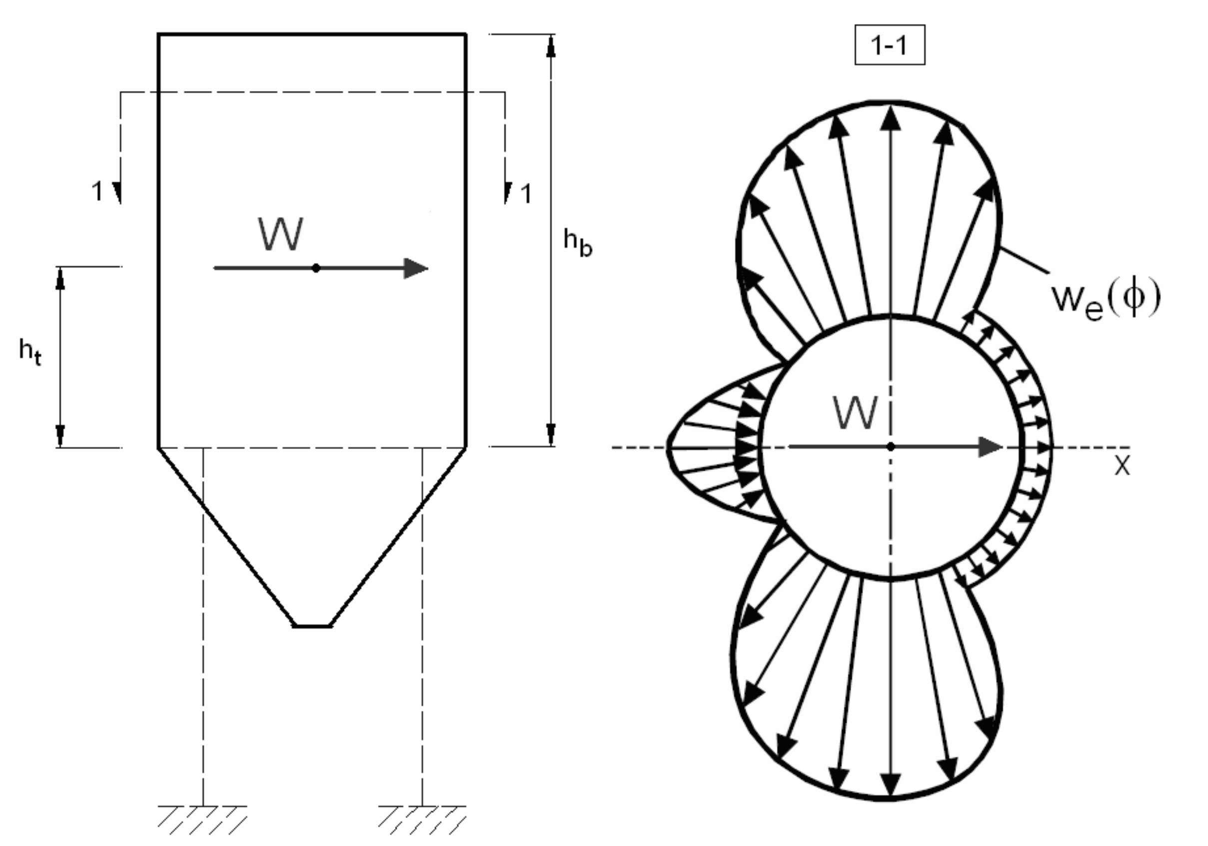
V drugem primeru, ko je silos prazen, mora biti zagotovljena stabilnost lupine. Tukaj je potrebno upoštevati celotni pritisk vetra, ki lahko povzroči uklon cilindra silosa.

### 4.3.1 Poln silos

V primeru polnega silosa mora biti zagotovljena zadostna nosilnost silosa in podporne konstrukcije. To pomeni da moramo upoštevati:

- povečanje sil v podporah zaradi delovanja celotne sile vetra, ki skuša prevrniti silos (kontrola prevrnitve),
- upogib cilindra zaradi celotne sile vetra.

V obeh primerih računamo z celotno silo vetra, ki deluje na silos.



**Slika 28: Celotna sila vetra na silos**

Celotna sila vetra $W$ predstavlja rezultanto zunanjega pritiska vetra $w_e\ (\phi)$, ki deluje na cilinder. Izračuna se jo po sledeči enačbi:

$$W = h_c \cdot \int_0^{2\pi} w_e(\phi) \cdot cos\ \phi \cdot r\ d\phi \tag{I.163}$$

Ta sila ima prijemališče v težišču cilindra, na višini $h_t$ od prehoda od lijaka k cilindru.

$$h_t = \frac{h_b}{2} \tag{I.164}$$

Iz enačbe (I.162) lahko vidimo, da je zunanji pritisk vetra $w_e(\phi)$ enak produktu tlaka pri konični hitrosti vetra $q_p$ in funkcije zunanjega pritiska vetra $c_{p0}(\phi)$.

Silo vetra lahko torej zapišemo kot:

$$W = h_b \cdot r \cdot q_p \cdot \int_0^{2\pi} c_{p0}(\phi) \cdot \cos\phi \, d\phi \tag{I.165}$$

Kot je bilo prikazano v poglavju *4.2.1*, predstavlja funkcija $c_{p0}(\phi)$ »obliko« pritiska vetra po obodu cilindra, ki smo jo definirali kot Fourierevo kosinusno vrtsto.

Označimo sedaj z *I* integral $I = \int_0^{2\pi} c_{p0}(\phi) \cdot \cos\phi \, d\phi$:

$$I = \int_0^{2\pi} c_{p0}(\phi) \cdot \cos\phi \, d\phi$$

$$I = \int_0^{2\pi} \left( \frac{c_0}{2} + \sum_{m=1}^{\infty} c_m \cdot \cos m\phi \right) \cdot \cos\phi \, d\phi$$

$$I = \int_0^{2\pi} \left( \frac{c_0}{2} \cdot \cos 0\phi + \sum_{m=1}^{\infty} c_m \cdot \cos m\phi \right) \cdot \cos\phi \, d\phi \tag{I.166}$$

Tako kot v enačbi (I.153), tudi tukaj ortogonalnost funkcije kosinus izhaja:

$$\int_0^{2\pi} \cos m\phi \cdot \cos n\phi \, d\phi = \left\langle \begin{matrix} 0, & if\ m \neq n \\ \pi, & if\ m = n \end{matrix} \right\rangle \tag{I.167}$$

V našem primeru je $n = 1$. To pomeni, da je integral *I* različen od *0* samo kadar je $m = 1$. Integral *I* je tako enak:

$$I = \int_0^{2\pi} c_1 \cdot \cos(\phi)^2 \, d\phi$$

$$I = c_1 \cdot \pi \tag{I.168}$$

Vrednost za celotno silo vetra pa je enaka:

$$W = \pi \cdot h_b \cdot r \cdot c_1 \cdot q_p \qquad \text{(I.169)}$$

Če upoštevamo še funkcijo koeficienta zunanjega pritiska vetra $c_{p0}(\phi)$, izračunano za 20 členov Fouriereve vrste, vidimo (*Preglednica 14*) da je:

$$c_1 = 0.372\,. \qquad \text{(I.170)}$$

Celotna sila vetra na silose tako znaša:

$$\boxed{W = 0.372 \cdot \pi \cdot r \cdot h_b \cdot q_p} \qquad \text{(I.171)}$$

### 4.3.2 Prazen silos

Ko obravnavamo obtežbo vetra na prazne silose, moramo upoštevati zunanji pritisk vetra za naslednji dve mejni stanji[5]:

- mejno stanje plastičnosti (LS1)
- mejno stanje uklona (LS3)

#### 4.3.2.1 Pritisk vetra pri kontroli mejnega stanja plastičnosti

Potrebno je upoštevati zunanji pritisk vetra $w_e(\phi)$, ki ga podaja enačba (I.162). Membranske sile za ta primer so dane v poglavju *4.4.2*.

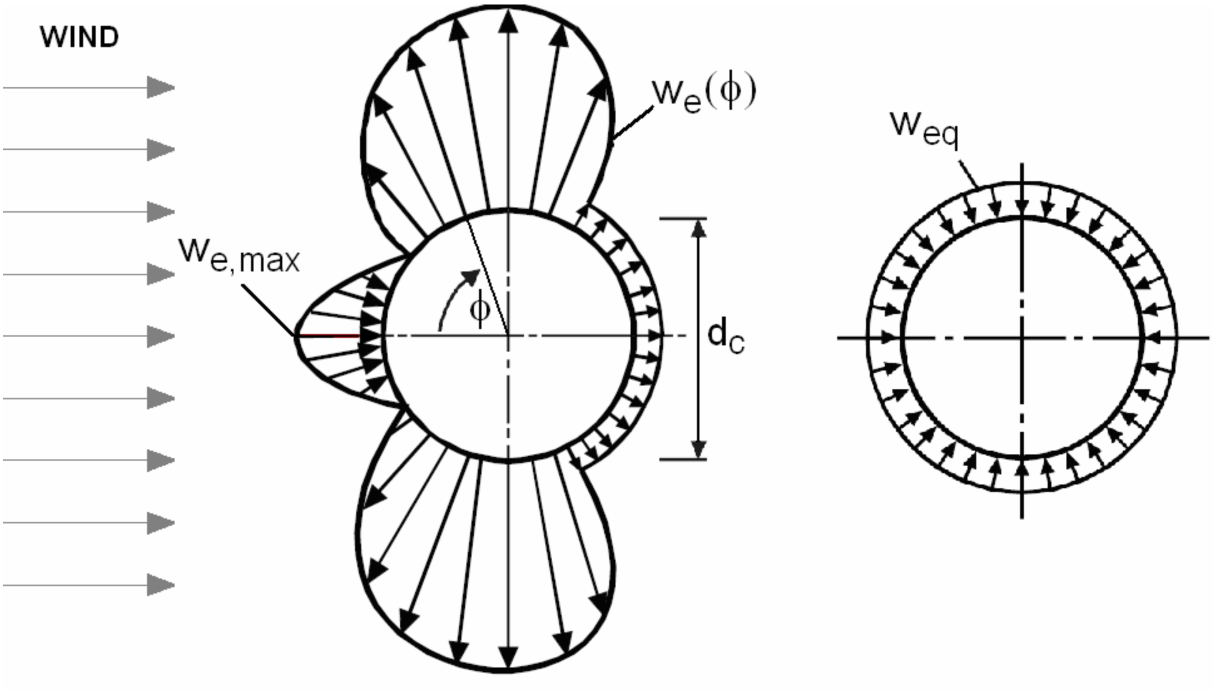
---

[5] Opisi mejnih stanj so podani v poglavju *7.1.2*.

### 4.3.2.2 Pritisk vetra pri kontroli mejnega stanja uklona

Pri kontroli mejnega stanja uklona nadomestimo zunanji pritisk vetra $w_e(\phi)$ z ekvivalentno osno simetrično obtežbo. Jakost obtežbe je takšna, da ustreza kontroli uklona, saj so z njo zajete imperfektnosti cilindra in drugi parametri, ki vplivajo na pojav uklona.

*Slika 29 (b)* spodaj ponazarja ekvivalentno osno simetrično obtežbo, ki deluje tlačno po obodu cilindra.



(a) (b)

**Slika 29: Dejanski zunanji pritisk vetra (a) in ekvivalentni osno simetrični pritisk (b)**

Funkcija, ki določa zunanji pritisk vetra $w_e(\phi)$, je dana v enačbi (I.162). Za izračun ekvivalentnega osno simetričnega pritiska $w_{eq}$ potrebujemo samo maksimalno vrednost tlaka na cilinder. To je vrednost $w_e(\phi)$ pri $\phi = 0°$

$$w_{e,max} = w_e(0)$$

$$w_{e,max} = q_p \cdot c_{p0}(0) \qquad \text{(I.172)}$$

Iz poglavja *4.2.1* (*Slika 25*) vidimo:

$$c_{p0}(0) = 1. \qquad \text{(I.173)}$$

Maksimalna tlačna vrednost vetra $w_{e,max}$ torej znaša:

$$w_{e,max} = q_p \qquad \text{(I.174)}$$

Ekvivalentni osno simetrični pritisk je enak:

$$w_{eq} = k_w \cdot w_{e,max}$$

$$\boxed{w_{eq} = k_w \cdot q_p} \qquad \text{(I.175)}$$

kjer

$$\boxed{k_w = 0.46 \cdot \left(1 + 0.1\sqrt{\frac{C_\Theta}{\omega} \cdot \frac{r}{t}}\right)} \qquad \text{(I.176)}$$

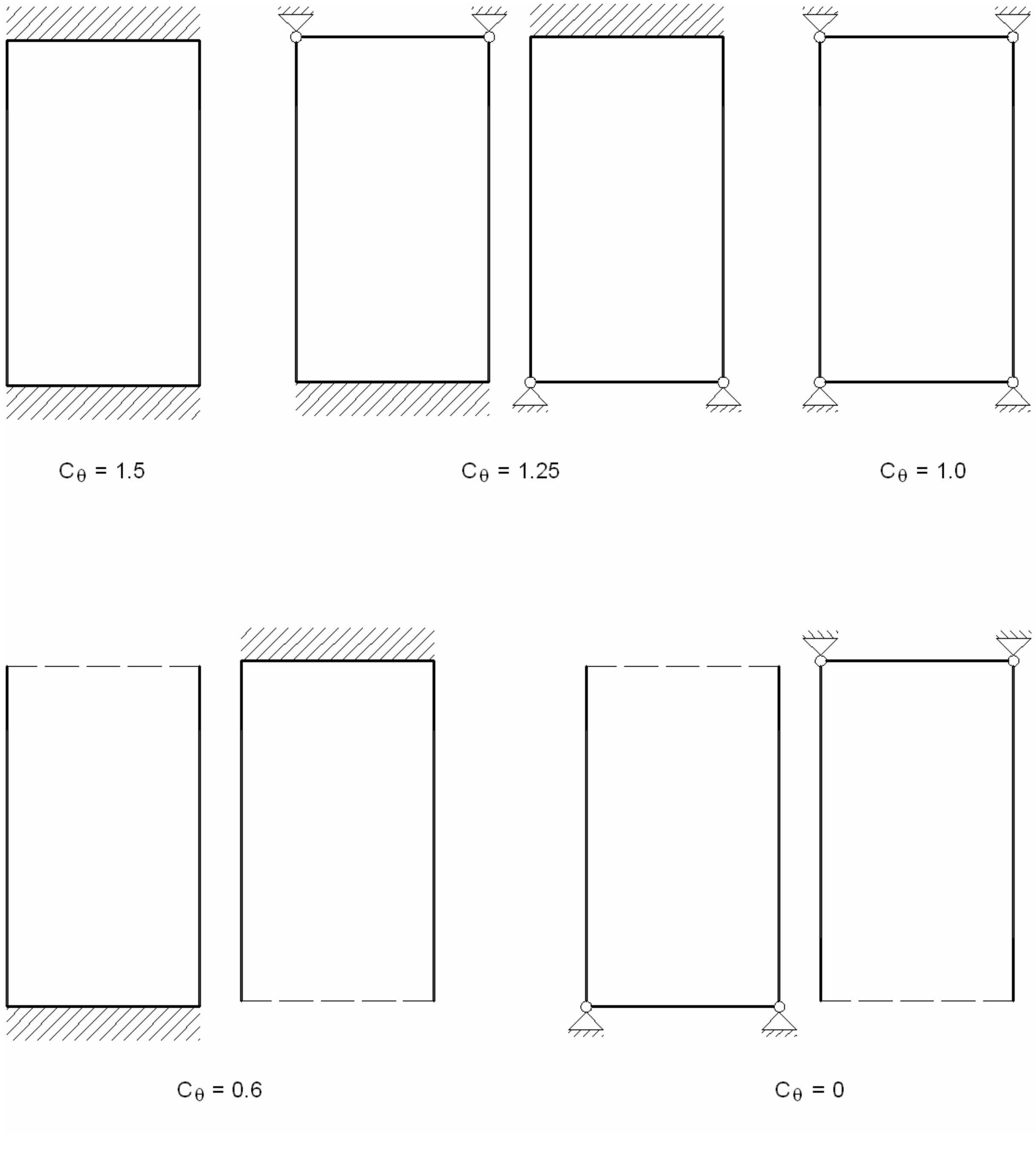
Dodatno velja še, da mora $k_w$ ležati znotraj območja :

$$\boxed{0.65 \leq k_w \leq 1.0} \qquad \text{(I.177)}$$

Parameter $\omega$ predstavlja brezdimenzijsko dolžino cilindra:

$$\omega = \frac{h_c}{\sqrt{rt}} \qquad \text{(I.178)}$$

Parameter $C_\Theta$ predstavlja uklonski faktor zunanjega pritiska. Določi se ga ob upoštevanju robnih pogojev za cilinder, ki so podani na spodnji sliki.

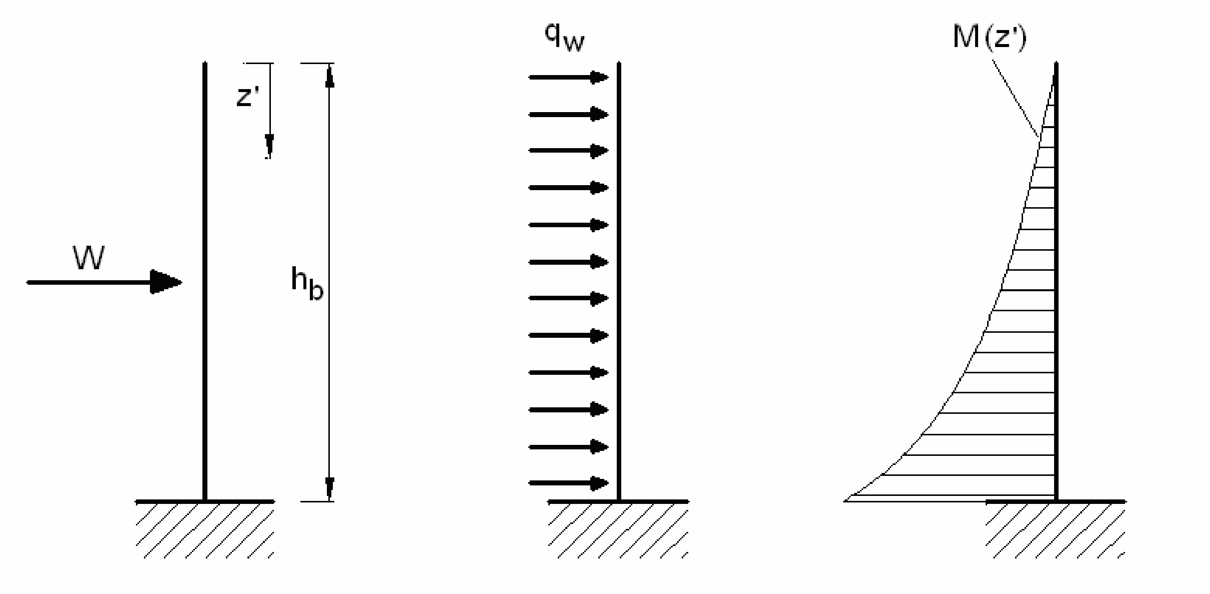


**Slika 30: Uklonski faktor zunanjega pritiska $C_\Theta$ pri različnih robnih pogojih za cilinder**

## 4.4 Membranske sile

### 4.4.1 Poln silos

Upogib cilindra silosa se upošteva tako, da se cilinder, na katerega deluje sila vetra, nadomesti z nadomestno konzolo, na katero deluje enakomerna linijska obtežba, ki izhaja iz sile vetra.



**Slika 31: Nadomestna konzola za račun upogiba cilindra**

Enakomerna linijska obtežba $q_w$ se naj izračuna po izrazu:

$$q_w = \frac{W}{d_c} \tag{I.179}$$

Koordinata $z'$ teče od vrha do dna cilindra. Upogibni moment na poljubni globini $z'$ je enak:

$$M(z') = q_w \cdot \frac{z'^2}{2} \tag{I.180}$$

Maksimalni sili $n_{z,wf}$, in $n_{\phi,wf}$, ki nastaneta zaradi upogiba cilindra polnega silosa, znašata:

$$\boxed{n_{z,wf}(z') = \frac{M(z')}{I} \cdot r \cdot t} \tag{I.181}$$

$$\boxed{n_{\phi,wf} = 0} \tag{I.182}$$

Parameter $I$ predstavlja vztrajnostni moment prereza cilindra in je podan z enačbo *(I.14)*.

## 4.4.2 Prazen silos

### 4.4.2.1 Mejno stanje plastičnosti

Kot je bilo že rečeno v poglavju *4.3.2.1*, je za račun notranjih sil pri kontroli mejnega stanja plastičnosti potrebno upoštevati delovanje zunanjega tlaka vetra $w_e(\phi)$, ki je podan v enačbi (I.162). Funkcija $c_{p0}(\phi)$ (poglavje *4.2.1*) je bila izračunana za 20 členov Fouriereve vrste. Izrazi za notranje sile pa so predstavljeni v splošni obliki, za poljubno število členov te vrste.

Enačbe za sili $n_{z,we}$, $n_{\phi,we}$ in za strižno silo $n_{z\phi,we}$, zaradi delovanja vetra na prazen cilinder, se glasijo:

$$\boxed{n_{z,we}(z,\phi) = \frac{q_p}{2r} \cdot \left( \sum_{m=1}^{\infty} m^2 c_m \cos m\phi \right) \cdot z^2} \tag{I.183}$$

$$\boxed{n_{\phi,we}(z,\phi) = -\frac{q_p r}{2} \cdot \left( c_0 + 2 \sum_{m=1}^{\infty} c_m \cos m\phi \right)} \tag{I.184}$$

$$\boxed{n_{z\phi,we}(z,\phi) = -q_p \cdot \left( \sum_{m=1}^{\infty} m c_m \sin m\phi \right) \cdot z} \tag{I.185}$$

### 4.4.2.2 Mejno stanje uklona

Ekvivalentna osno simetrična obtežba vetra $w_{eq}$ povzroča v steni cilindra naslednje notranje sile:

$$\boxed{n_{z,we}(z)=0} \tag{I.186}$$

$$\boxed{n_{\phi,we}(z,\phi)=-w_{eq}\cdot r} \tag{I.187}$$

## 4.5 Računalniški program

Izdelan je bil računalniški program, ki izračuna razporeditev pritiskov in celotno silo vetra, ki deluje na cilinder silosa. V program so vgrajene vse tri krivulje, ki jih prikazuje slika v poglavju *4.2.1*.

Program se nahaja na zgoščenki, ki je priložena tej nalogi in se nahaja na zadnji platnici. Za zagon programa je potrebno odpreti datoteko »*Wind Load on Cylindrical Silos.xls*«.

Za pravilno delovanje mora biti na računalniku nameščen *Microsoft Excel 2003* (ali novejši). Makroji morajo biti omogočeni, njihova varnost pa nastavljena na *Srednjo* oz. *Nizko* stopnjo.

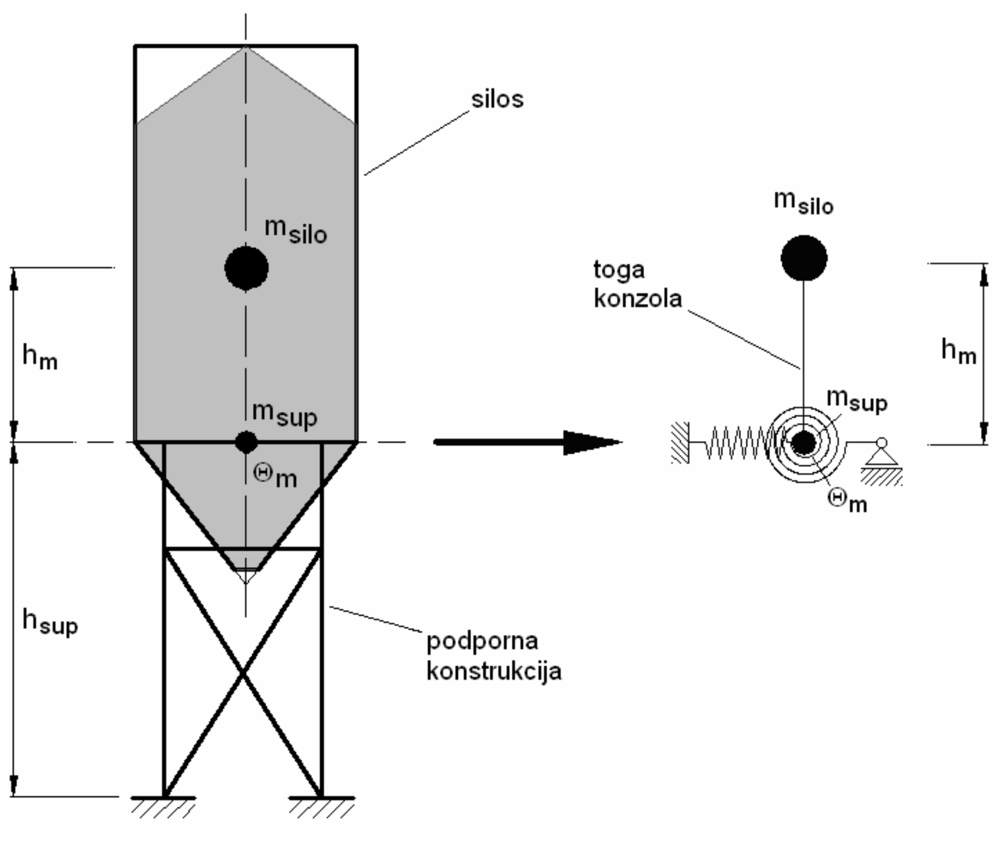
Podrobnejša navodila se nahajajo v *III. delu*, v poglavju *13*.

# 5 Potresna obremenitev silosov

## 5.1 Modeliranje silosov pri potresni analizi

### 5.1.1 Poln silos

Glavna naloga silosa je, da je v času obratovanja sposoben prenašati težo shranjenega materiala, ki je navadno mnogokrat večja od lastne teže silosne konstrukcije. Zato lahko pri obravnavi seizmične obremenitve predpostavimo, da je poln silos tog v primerjavi z njegovo podporno konstrukcijo. To pomeni, da imamo opravka z veliko maso, podprto na določeni višini, z relativno podajnimi podporami.



**Slika 32: Modeliranje silosa s togo konzolo**

Pri računu potresne obremenitve tako lahko uporabimo model toge brezmasne konzole, pri kateri je na zgornjem koncu skoncentrirana velika masa, spodnji konec pa je vpet v podlago preko dveh vzmeti, s katerimi ponazorimo togost podporne konstrukcije (*Slika 32*). Masa $m_{silo}$ predstavlja maso silosa brez podporne konstrukcije (masa lupine silosa in shranjenega materiala). Učinke nihanja podporne konstrukcije lahko zajamemo tako, da na spodnji konec konzole, na mestu vzmeti, dodamo translacijsko maso $m_{sup}$ in rotacijsko maso $\Theta_m$, pri čemer obe masi izhajata iz podporne konstrukcije.

Če zanemarimo masi podporne konstrukcije, ima model samo eno maso, podprto na višini $h_m$, in s tem tudi samo eno možno nihajno obliko v horizontalni smeri. V tem primeru imamo torej sistem z eno prostostno stopnjo in lahko pri analizi uporabimo metodo z vodoravnimi silami (ang. »lateral force method«). Tak model je v EN standardih znan tudi pod imenom »model obrnjenega nihala«. Mi ga v tej nalogi imenujmo *1. seizmični model.*

Če upoštevamo tudi masi podporne konstrukcije, ima model dve možni prostostni stopnji za gibanje v horizontalni smeri. V tem primeru obravnavamo torej model z dvema prostostnima stopnjama, zato moramo tukaj uporabiti modalno analizo s spektrom odziva. Ta model imenujmo *2. seizmični model.*

Tako metoda z vodoravnimi silami, kot tudi modalna analiza s spektrom odziva, sta statični metodi. To pomeni, da lahko dinamični problem prevedemo na ekvivalentni statični problem, z uporabo d'Alembertovega principa, ki pogojuje naslednji ravnotežni pogoj:

$$\Sigma F + F_I = 0 \tag{I.188}$$

Oznaki v zgornji enačbi pomenita:

$\Sigma F$ vsoto statičnih sil na dani sistem,

$F_I$ inercijska sila, ki zajema dinamični vpliv (2. Newtonov zakon):

$$F_I = -m \cdot a \tag{I.189}$$

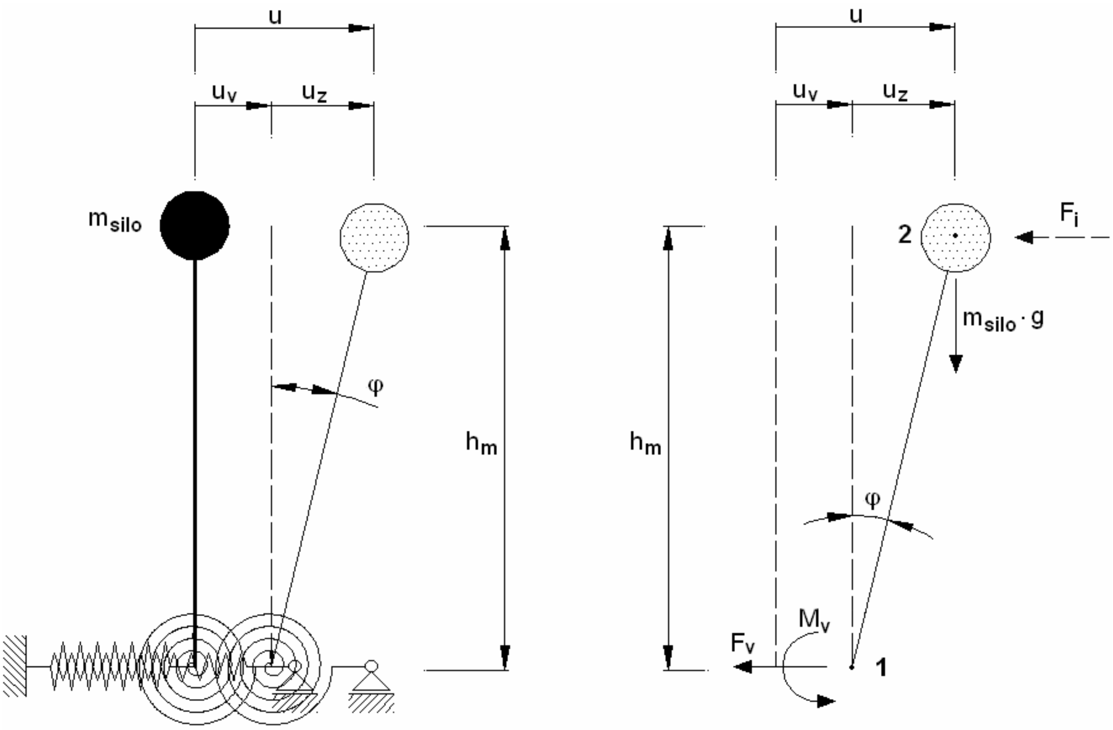
V nadaljevanju sta oba modela tudi podrobneje analizirana.

### 5.1.2 Prazen silos

Kadar je silos prazen, je podporna konstrukcija veliko bolj toga od lupine silosa. V tem primeru je potrebno notranje sile v stenah lupine izračunati z uporabo dinamične analize celotne silosne konstrukcije. Tukaj imajo prispevki višjih nihajnih oblik lahko večji vpliv na odziv konstrukcije, tako da je potrebno uporabiti modalno analizo s spektrom odziva. Kot veleva *EN 1998-1*, je potrebno upoštevati toliko nihajnih oblik, da bo vsota modalnih mas znaša vsaj *90%* celotne mase.

## 5.2 Nihajni časi obravnavanih modelov

### 5.2.1 1. seizmični model



**Slika 33: Pomiki in sile 1. seizmičnega modela**

Naj bo $u_v$ vodoravni pomik zaradi deformacije translacijske vzmeti in $u_z$ pomik zaradi zasuka rotacijske vzmeti.

Kot v večini primerov iz inženirske prakse, se tudi tukaj predpostavi, da je zasuk $\phi$ zaradi delovanja potresne obremenitve zelo majhen ( $\varphi << 0.2\ rad$ ). Velja naslednja poenostavitev:

$$sin\,\varphi \cong \varphi \tag{I.190}$$

Če upoštevamo to poenostavitev, potem lahko zapišemo:

$$u_z = h_m \cdot \varphi \tag{I.191}$$

Celotni pomik $u$ je enak:

$$u = u_v + u_z = u_v + h_m \cdot \varphi \tag{I.192}$$

Označimo s $k_v$ togost translacijske (vodoravne) vzmeti in s $k_m$ pa togost rotacijske (sučnostne) vzmeti. Sila v vodoravni vzmeti $F_v$ in moment v sučnostni vzmeti $M_v$ sta enaka:

$$F_v = k_v \cdot u_v \tag{I.193}$$

$$M_v = k_m \cdot \varphi \tag{I.194}$$

Sila $F_i$ označuje inercijsko silo, ki zajema dinamične učinke v skladu z d'Alembertovim principom.

$$F_i = m_{silo} \cdot \ddot{u} \tag{I.195}$$

Zapišimo sedaj ravnotežni pogoj za vsoto vseh sil v vodoravni smeri (koordinata $x$) in za vsoto vseh momentov na točko *1* (*Slika 33*), ob upoštevanju enačb (I.191) do (I.195):

$$\sum F_x = 0: \qquad F_i + F_v = 0$$

$$m_{silo} \cdot \ddot{u} + k_v \cdot u_v = 0$$

$$m_{silo} \cdot (\ddot{u}_v + \ddot{u}_z) + k_v \cdot u_v = 0$$

$$m_{silo} \cdot \ddot{u}_v + m_{silo} \cdot \ddot{u}_z + k_v \cdot u_v = 0 \tag{I.196}$$

$$\sum M^1 = 0: \quad M_v + F_i \cdot h_m - m_{silo} \cdot g \cdot u_z = 0$$

$$M_v + m_{silo} \cdot \ddot{u} \cdot h_m - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0$$

$$k_m \cdot \varphi + m_{silo} \cdot \ddot{u} \cdot h_m = m_{silo} \cdot g \cdot h_m \cdot \varphi$$

$$k_m \cdot \varphi + m_{silo} \cdot (\ddot{u}_v + \ddot{u}_z) \cdot h_m = m_{silo} \cdot g \cdot h_m \cdot \varphi$$

$$k_m \cdot \varphi + m_{silo} \cdot \ddot{u}_v \cdot h_m + m_{silo} \cdot \ddot{u}_z \cdot h_m - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0$$

$$k_m \cdot \varphi + m_{silo} \cdot \ddot{u}_v \cdot h_m + m_{silo} \cdot h_m^2 \cdot \ddot{\varphi} - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0 \tag{I.197}$$

Sedaj lahko tvorimo sistem diferencialnih enačb, ki predstavljata lastno nihanje sistema (nihanje brez delovanja zunanje obtežbe):

$$[M] \cdot \{\ddot{u}\} + [K] \cdot \{u\} = \{0\} \tag{I.198}$$

Simboli v zgornji enačbi označujejo:

| | |
|---|---|
| $[M]$ | masno matriko, katere koeficienti $m_{ij}$ predstavljajo vztrajnostno silo v točki *i*, zaradi enotskega pospeška v točki *j*, |
| $[K]$ | togostno matriko, katere koeficienti $k_{ij}$ predstavljajo silo v točki *i*, zaradi enotskega pomika v točki *j*, |
| $\{\ddot{u}\}$ | vektor pospeškov, |
| $\{u\}$ | vektor pomikov. |

Če upoštevamo enačbi (I.196) in (I.197), dobimo sledeči sistem navadnih diferencialnih enačb:

$$\begin{bmatrix} m_{silo} & m_{silo} \cdot h_m \\ m_{silo} \cdot h_m & m_{silo} \cdot h_m^2 \end{bmatrix} \cdot \begin{Bmatrix} \ddot{u}_v \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} k_v & 0 \\ 0 & k_m - m_{silo} \cdot g \cdot h_m \end{bmatrix} \cdot \begin{Bmatrix} u_v \\ \varphi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{I.199}$$

Uporabimo standardni nastavek za pomik *{u}*:

$$\{u\} = \{\phi\} \cdot sin(\omega t - \Theta) \tag{I.200}$$

Simboli v zgornji enačbi označujejo:

$\{\phi\}$ nihajno obliko sistema,

$\omega$ kotno frekvenco in

$\Theta$ fazo nihanja.

Drugi odvod $\{\ddot{u}\}$ je enak:

$$\{\ddot{u}\} = -\omega^2 \cdot \{\phi\} \cdot sin(\omega t - \Theta) \tag{I.201}$$

Če sedaj upoštevamo enačbi (I.200) in (I.201) v enačbi (I.198), dobimo:

$$-\omega^2 \cdot sin(\omega t - \Theta)[M]\{\phi\} + sin(\omega t - \Theta)[K]\{\phi\} = \{0\} \tag{I.202}$$

Ker je v splošnem $sin(\omega t - \Theta) \neq 0$, velja naslednje:

$$\left([K] - \omega^2 \cdot [M]\right) \cdot \{\phi\} = \{0\} \tag{I.203}$$

Vidimo, da se začetni sistem enačb, ki opisujejo lastno nihanje izbranega modela, prevede na problem lastnih vrednosti. Lastne vrednosti tega problema predstavljajo osnovno kotno frekvenco $\omega$. Določimo jih z izračunom determinante $D$:

$$D = det\left([K] - \omega^2 \cdot [M]\right) = 0 \tag{I.204}$$

Ob upoštevanju matrik $[M]$ in $[K]$ za izbran model, je determinanta $D$ enaka:

$$D = det\left(\begin{bmatrix} k_v & 0 \\ 0 & k_m - m_{silo} \cdot g \cdot h_m \end{bmatrix} - \omega^2 \cdot \begin{bmatrix} m_{silo} & m_{silo} \cdot h_m \\ m_{silo} \cdot h_m & m_{silo} \cdot h_m^2 \end{bmatrix}\right) = 0$$

$$D = \begin{vmatrix} k_v - \omega^2 \cdot m_{silo} & -\omega^2 \cdot m_{silo} \cdot h_m \\ -\omega^2 \cdot m_{silo} \cdot h_m & k_m - m_{silo} \cdot g \cdot h_m - \omega^2 \cdot m_{silo} \cdot h_m^2 \end{vmatrix} = 0$$

$$\left(k_v - \omega^2 \cdot m_{silo}\right) \cdot \left(k_m - m_{silo} \cdot g \cdot h_m - \omega^2 \cdot m_{silo} \cdot h_m^2\right) - \left(-\omega^2 \cdot m_{silo} \cdot h_m\right)^2 = 0 \tag{I.205}$$

Ko rešimo zgornjo enačbo, dobimo:

$$\boxed{\omega = \sqrt{\frac{k_v}{m_{silo}} \cdot \frac{k_m - m_{silo} \cdot g \cdot h_m}{k_m + h_m \left(h_m \cdot k_v - m_{silo} \cdot g\right)}}} \tag{I.206}$$

Mejna primera:

$$k_m \to \infty: \quad \boxed{\omega = \sqrt{\frac{k_v}{m_{silo}}}} \tag{I.207}$$

$$k_v \to \infty: \quad \boxed{\omega = \sqrt{\frac{k_m - m_{silo} \cdot g \cdot h_m}{m_{silo} \cdot h_m{}^2}}} \tag{I.208}$$

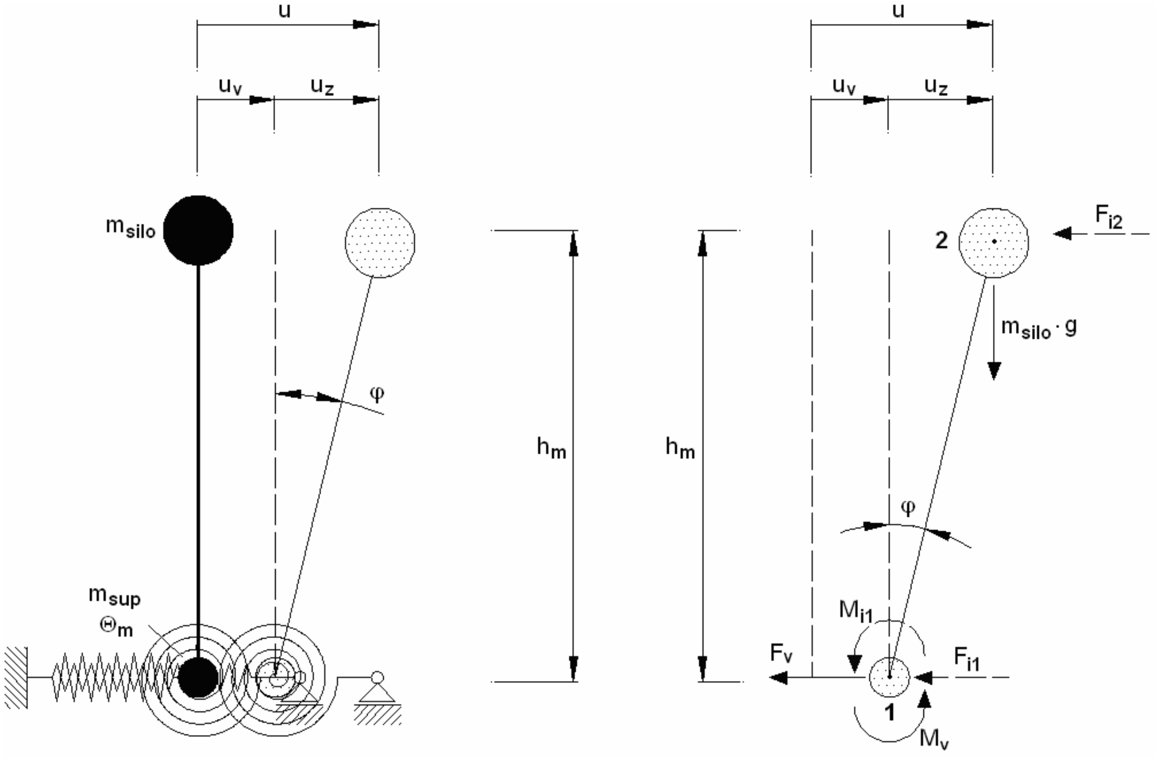
Nihajni čas je definiran kot:

$$T = \frac{2\pi}{\omega} \tag{I.209}$$

Nihajni čas *1. seizmičnega modela* je enak:

$$\boxed{T = \frac{2\pi}{\sqrt{\frac{k_v}{m_{silo}} \cdot \frac{k_m - m_{silo} \cdot g \cdot h_m}{k_m + h_m \left(h_m \cdot k_v - m_{silo} \cdot g\right)}}}} \tag{I.210}$$

### 5.2.2 2. seizmični model



**Slika 34: Pomiki in sile 2. seizmičnega modela**

V tem modelu dodamo translacijsko maso $m_{sup}$ in rotacijsko maso $\Theta_m$ na spodnji konec (točka *1*) toge konzole. S tem so zajeti tudi učinki nihanja podporne konstrukcije. Obe te masi je potrebno določiti glede na tip podporne konstrukcije.

Te dve masi lahko ocenimo, če modeliramo podporno konstrukcijo z enim podajnim, nadomestnim linijskim elementom. Diagonalna masna matrika takšnega elementa znaša:

$$[m_d] = \frac{m_{sup,total}}{420} \begin{bmatrix} 210 & & & \\ & {h_{sup}}^2 & & \\ & & 210 & \\ & & & {h_{sup}}^2 \end{bmatrix} \tag{I.211}$$

Oznaka $m_{sup,total}$ predstavlja celotno maso podporne konstrukcije.

To pomeni, da sta masi na krajnih točkah tega elementa $m_{sup}$ in $\Theta_m$ enaki:

$$\boxed{m_{sup} = 0.5\, m_{sup,total}} \tag{I.212}$$

$$\boxed{\Theta_m = \frac{m_{sup,total} \cdot h_{sup}^{\ 2}}{420}} \tag{I.213}$$

V tem modelu nastopata dve inercijski sili: $F_{i1}$, ki predstavlja inercijsko silo v točki *1* in $F_{i2}$, ki predstavlja inercijsko silo v točki *2* (*Slika 34*).

$$F_{i1} = m_{sup} \cdot \ddot{u}_v \tag{I.214}$$

$$F_{i2} = m_{silo} \cdot \ddot{u} \tag{I.215}$$

Rotacijska masa $\Theta_m$ povzroča v točki *1* inercijski moment $M_{i1}$:

$$M_{i1} = \Theta_m \cdot \ddot{\varphi} \tag{I.216}$$

Enačbe (I.190) do (I.195) veljajo tudi za ta model. Če zapišemo sedaj ravnotežni pogoj za vsoto vseh sil v vodoravni smeri (koordinata *x*) in za vsoto vseh momentov na točko *1*, dobimo:

$$\begin{aligned}
\sum F_x = 0: \quad & F_{i1} + F_{i2} + F_v = 0 \\
& m_{sup} \cdot \ddot{u}_v + m_{silo} \cdot \ddot{u} + k_v \cdot u_v = 0 \\
& m_{sup} \cdot \ddot{u}_v + m_{silo} \cdot (\ddot{u}_v + \ddot{u}_z) + k_v \cdot u_v = 0 \\
& (m_{sup} + m_{silo}) \cdot \ddot{u}_v + m_{silo} \cdot \ddot{u}_z + k_v \cdot u_v = 0 \\
& (m_{sup} + m_{silo}) \cdot \ddot{u}_v + m_{silo} \cdot h_m \cdot \ddot{\varphi} + k_v \cdot u_v = 0
\end{aligned} \tag{I.217}$$

$$\begin{aligned}
\sum M^1 = 0: \quad & M_v + M_{i1} + F_{i2} \cdot h_m - m_{silo} \cdot g \cdot u_z = 0 \\
& k_m \cdot \varphi + \Theta_m \cdot \ddot{\varphi} + m_{silo} \cdot \ddot{u} \cdot h_m - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0 \\
& k_m \cdot \varphi + \Theta_m \cdot \ddot{\varphi} + m_{silo} \cdot h_m \cdot (\ddot{u}_v + h_m \cdot \ddot{\varphi}) - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0 \\
& m_{silo} \cdot h_m \cdot \ddot{u}_v + (\Theta_m + m_{silo} \cdot h_m^{\ 2}) \cdot \ddot{\varphi} + k_m \cdot \varphi - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0
\end{aligned} \tag{I.218}$$

Sistem navadnih diferencialnih enačb za lastno nihanje izbranega sistema se glasi:

$$\begin{bmatrix} m_{silo}+m_{sup} & m_{silo}\cdot h_m \\ m_{silo}\cdot h_m & m_{silo}\cdot h_m^{\,2}+\Theta_m \end{bmatrix}\cdot\begin{Bmatrix}\ddot{u}_v\\ \ddot{\varphi}\end{Bmatrix}+\begin{bmatrix} k_v & 0 \\ 0 & k_m-m_{silo}\cdot g\cdot h_m \end{bmatrix}\cdot\begin{Bmatrix}u_v\\ \varphi\end{Bmatrix}=\begin{Bmatrix}0\\0\end{Bmatrix} \tag{I.219}$$

Rešitev spodnje determinante nam da vrednosti za lastne kotne frekvence $\omega$:

$$D=det\left(\begin{bmatrix} k_v & 0 \\ 0 & k_m-m_{silo}\cdot g\cdot h_m \end{bmatrix}-\omega^2\cdot\begin{bmatrix} m_{silo}+m_{sup} & m_{silo}\cdot h_m \\ m_{silo}\cdot h_m & m_{silo}\cdot h_m^{\,2}+\Theta_m \end{bmatrix}\right)=0$$

$$D=\begin{vmatrix} k_v-\omega^2\cdot\left(m_{silo}+m_{sup}\right) & -\omega^2\cdot m_{silo}\cdot h_m \\ -\omega^2\cdot m_{silo}\cdot h_m & k_m-m_{silo}\cdot g\cdot h_m-\omega^2\cdot\left(m_{silo}\cdot h_m^{\,2}+\Theta_m\right) \end{vmatrix}=0 \tag{I.220}$$

Vrednosti za $\omega$ znašajo:

$$\omega_1=\sqrt{\frac{A-\sqrt{B-C}}{D}} \tag{I.221}$$

$$\omega_2=\sqrt{\frac{A+\sqrt{B-C}}{D}} \tag{I.222}$$

Simboli $A$, $B$, $C$ in $D$ v zgornji enačbi označujejo:

$$A=k_m\left(m_{sup}+m_{silo}\right)+k_v\left(h_m^{\,2}\,m_{silo}+\Theta_m\right)-m_{silo}\cdot g\cdot h_m\left(m_{sup}+m_{silo}\right) \tag{I.223}$$

$$B=\left[m_{silo}\cdot g\cdot h_m\left(m_{sup}+m_{silo}\right)-k_m\left(m_{sup}+m_{silo}\right)-k_v\left(h_m^{\,2}\,m_{silo}+\Theta_m\right)\right]^2 \tag{I.224}$$

$$C=4k_v\left(k_m-m_{silo}\cdot g\cdot h_m\right)\cdot\left[h_m^{\,2}\,m_{sup}\,m_{silo}+\Theta_m\left(m_{sup}+m_{silo}\right)\right] \tag{I.225}$$

$$D=2\left[h_m^{\,2}\,m_{sup}\,m_{silo}+\Theta_m\left(m_{sup}+m_{silo}\right)\right] \tag{I.226}$$

Nihajna časa za *2. seizmični model* sta enaka:

$$\boxed{T_1 = \frac{2\pi}{\omega_1}} \qquad (I.227)$$

$$\boxed{T_2 = \frac{2\pi}{\omega_2}} \qquad (I.228)$$

## 5.3 Postopek določitve potresne obremenitve polnega silosa

Spodnji postopek prikazuje določitev potresne sile in pritiskov na steno cilindra zaradi delovanja potresne obremenitve. Predstavljena je samo metoda z vodoravnimi silami, ki se uporablja pri analizi *1. seizmičnega modela*. Postopek temelji na standardih *EN 1998-1:2004* in *EN 1998-4:2006*.

**Preglednica 19: Pregled postopka**

| Korak | Opis koraka | Lokacija koraka v standardu |
|---|---|---|
| 1 | določitev razreda in faktorja pomembnosti | EN 1998-4, poglavje 2.1.3(4) |
| 2 | izračun mase silosa pri potresni obremenitvi | EN 1998-1, poglavje 3.2.4(2)P |
| 3 | določitev togosti podporne konstrukcije | - |
| 4 | izračun nihajnega časa | - |
| 5 | določitev tipa tal | EN 1998-1, preglednica 3.1 |
| 6 | določitev projektnega pospeška tal | EN 1998-1, poglavje 3.2.1(3) |
| 7 | določitev faktorja obnašanja | EN 1998-1, poglavje 6.3 |
| 8 | določitev spektra pospeškov | EN 1998-1, poglavje 3.2.2.2 |
| 9 | izračun celotne prečne sile | EN 1998-1, poglavje 4.3.3.2.2 |
| 10 | izračun dodatnih pritiskov na steno cilindra | EN 1998-4, poglavje 3.3(5) |

Modalna analiza s spektrom odziva, ki jo je potrebno uporabiti pri *2. seizmičnem modelu*, je predstavljena na primeru v III. delu naloge. Tam je podana tudi primerjava obeh modelov za izbran računski primer.

### 5.3.1 Korak 1: Razred pomembnosti in faktor pomembnosti

Razred pomembnosti določa, kakšne bodo posledice ob porušitvi dela oz. celotnega silosa zaradi potresne obremenitve.

Obstajajo trije razredi pomembnosti. Potrebno je izbrani en razred, ki je značilen za obravnavan silos.

**Preglednica 20: Razred pomembnosti**

| Razred pomembnosti | Posledice porušitve |
|---|---|
| I. razred | majhne |
| II. razred | srednje |
| III. razred | velike |

Če so posledice zaradi porušitve **majhne**, to pomeni, da ob porušitvi silosa obstaja majhno tveganje za izgubo človeških življenj in da so vplivi na okolje zanemarljivi.

Če so posledice zaradi porušitve **srednje**, to pomeni, da ob porušitvi silosa obstaja določeno tveganje za izgubo človeških življenj in da bo prišlo do vidnih vplivov na okolje.

Če so posledice zaradi porušitve **velike**, to pomeni, da ob porušitvi silosa obstaja veliko tveganje za izgubo človeških življenj in da bo prišlo do velikih vplivov na okolje.

Potem, ko je bil izbran razred pomembnosti za obravnavani silos, je potrebno določiti še ustrezni faktor pomembnosti $\gamma_I$, ki je podan v spodnji tabeli.

**Preglednica 21: Faktor pomembnosti $\gamma_I$**

| Narava shranjenega materiala | Razred pomembnosti | | |
|---|---|---|---|
| | I | II | III |
| zdravju neškodljiv in nevnetljiv material | 0.8 | 1 | 1.2 |
| zdravju škodljiv material | 1 | 1.2 | 1.4 |
| eksploziven ali vnetljiv material | 1.2 | 1.4 | 1.6 |

## 5.3.2 Korak 2: Masa silosa pri potresni obremenitvi

Pri računi mase silosa za potresno obremenitev je potrebno upoštevati mase, ki izhajajo iz lastne teže materialov, ki tvorijo silos *G*, in maso shranjenega materiala *m*. Pri tem je potrebno upoštevati tudi ustrezen faktor pomembnosti.

$$m_{silo} = \gamma_I \left(G + 0.8\,m\right) \qquad \text{(I.229)}$$

## 5.3.3 Korak 3: Togost podporne konstrukcije

Izračunati je potrebno ustrezno horizontalno togost ($k_v$) in rotacijsko togost podporne konstrukcije ($k_m$).

## 5.3.4 Korak 4: Nihajni čas

Ob upoštevanju togosti $k_v$ in $k_m$, je potrebno določiti nihajni čas silosa, ki je podan v enačbi (I.210). Simbol *g* v tej enačbi predstavlja težnostni pospešek Zemlje:

$$g = 9.81\; m/s^2 \qquad \text{(I.230)}$$

### 5.3.5 Korak 5: Tip tal

Glede na tip tal, na katerih je temeljen silos, je potrebno določiti vrednosti parametrov $S$, $T_B(s)$, $T_C(s)$ in $T_D(s)$, ki opisujejo elastični spekter pospeškov.

**Preglednica 22: Tipi tal in parametri, ki opisujejo elastični spekter pospeškov**[6]

| Tip tal | Opis stratigrafskega profila | S | $T_B$(s) | $T_C$(s) | $T_D$(s) |
|---|---|---|---|---|---|
| A | Skala ali druga skali podobna geološka formacija, na kateri je največ 5 m slabšega površinskega materiala. | 1 | 0.15 | 0.4 | 2 |
| B | Zelo gost pesek, prod ali zelo toga glina, debeline vsaj nekaj deset metrov, pri katerih mehanske značilnosti postopoma naraščajo z globino. | 1.2 | 0.15 | 0.5 | 2 |
| C | Globoki sedimenti gostega ali srednje gostega peska, proda ali toge gline globine nekaj deset do več sto metrov. | 1.15 | 0.2 | 0.6 | 2 |
| D | Sedimenti rahlih do srednje gostih nevezljivih zemljin (z nekaj mehkimi vezljivimi plastmi ali brez njih) ali pretežno mehkih do trdnih vezljivih zemljin. | 1.35 | 0.2 | 0.8 | 2 |
| E | Profil tal, kjer površinska aluvialna plast debeline med okrog 5 in 20 metri. (*podrobnosti v EN 1998-1, 3.1.1(2)*) | 1.4 | 0.15 | 0.5 | 2 |
| S1 | Sedimenti, ki so sestavljeni iz (ali vsebujejo) najmanj 10 m debele plasti mehke gline/melja. Z visokim indeksom plastičnosti (PI > 40) in visoko vsebnostjo | potrebne so posebne študije za določitev ustreznih vrednosti (*EN 1998-1, 3.1.1(4)P*) | | | |
| S2 | Tla podvržena likvefakciji, občutljive gline ali drugi profili tal, ki niso vključeni v tipe A-E ali S1. | | | | |

---

[6] Povzeto po *EN 1998-1, Preglednica 3.1* in *3.2*.

### 5.3.6 Korak 6: Projektni pospešek tal

Projektna vrednost pospeška tal je podana z naslednjo enačbo:

$$a_g = \gamma_I \cdot a_{gR} \tag{I.231}$$

Referenčni maksimalni pospešek tal $a_{gR}$, se odčita iz karte potresne nevarnosti, ki se nahaja v *Nacionalnem dodatku* standarda *EN 1998*.

### 5.3.7 Korak 7: Faktor obnašanja

Faktor obnašanja je potrebno določiti v skladu z razredom duktilnosti:

**Preglednica 23: Faktor obnašanja**

| Razred duktilnosti | Faktor obnašanja (q) |
|---|---|
| Nizek (DCL) | 1.0 |
| Srednji (DCM) | 1.0 ali 1.25<br>*(odvisno od podpor)*[7] |
| Visok (DCH) | |

### 5.3.8 Korak 8: Projektni spekter pospeškov

Za izračunan nihajni čas *T* je potrebno določiti pripadajočo vrednost v projektnem spektru pospeškov $S_d(T)$. Projektni spekter pospeškov je določen za dve mejni stanji:

- mejno stanje nosilnosti (MSN),
- mejno stanje poškodb (MSP).

Pri analizi je potrebno upoštevati obe mejni stanji.

---

[7] Silosi, ki so podprti s plaščem, imajo $q = 1.0$.

Silosi, ki so podprti s točkovnimi podporami, imajo $q = 1.25$.

Enačbe, ki določajo projektni spekter so podane za obe mejni stanji v spodnjih dveh preglednicah. Določene so bile ob upoštevanju ustreznega vpliva dušenja za posamezno mejno stanje. Koeficiente *S*, $T_B(s)$, $T_C(s)$ in $T_D(s)$ določa *Preglednica 22*.

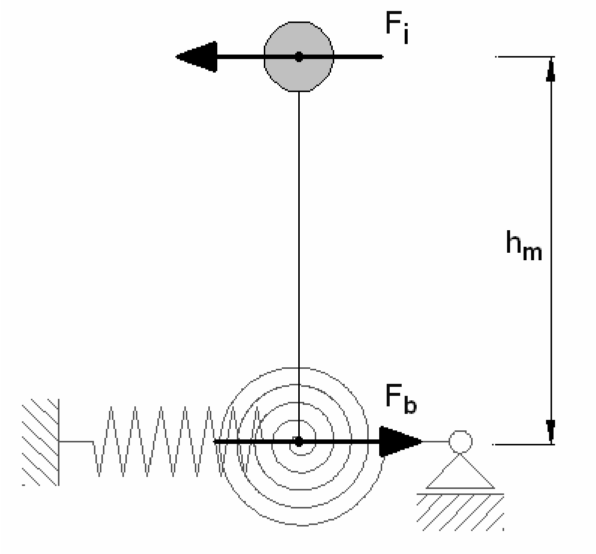
**Preglednica 24: Projektni spekter pospeškov za MSN**

| Nihajni čas | Mejno stanje nosilnosti (MSN) |
|---|---|
| $0 \leq T \leq T_B$ | $S_d(T) = a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$ |
| $T_B < T \leq T_C$ | $S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}$ |
| $T_C < T \leq T_D$ | $S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C}{T}\right] \geq 0.2a_g$ |
| $T_D < T$ | $S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C \cdot T_D}{T^2}\right] \geq 0.2a_g$ |

**Preglednica 25: Projektni spekter pospeškov za MSP**

| Nihajni čas | Mejno stanje poškodb (MSP) |
|---|---|
| $0 \leq T \leq T_B$ | $S_d(T) = a_g \cdot S \cdot \left[\frac{1}{2} + \frac{T}{T_B} \cdot \left(\frac{1.5}{q} - \frac{1}{2}\right)\right]$ |
| $T_B < T \leq T_C$ | $S_d(T) = a_g \cdot S \cdot \frac{1.5}{q}$ |
| $T_C < T \leq T_D$ | $S_d(T) = a_g \cdot S \cdot \frac{1.5}{q}\left[\frac{T_C}{T}\right] \geq 0.2a_g$ |
| $T_D < T$ | $S_d(T) = a_g \cdot S \cdot \frac{1.5}{q}\left[\frac{T_C \cdot T_D}{T^2}\right] \geq 0.2a_g$ |

### 5.3.9 Korak 9: Celotna prečna sila



**Slika 35: Celotna prečna sila**

Celotna prečna sila $F_b$ deluje na mestu vpetja konstrukcije. Predstavlja potresno silo, ki deluje na silos. Ker uporabljamo model s togo konzolo in vzmetmi (*Slika 32*), deluje ta sila na mestu podpor konzole. Določimo jo z enačbo:

$$F_b = S_d(T) \cdot m_{silo} \tag{I.232}$$

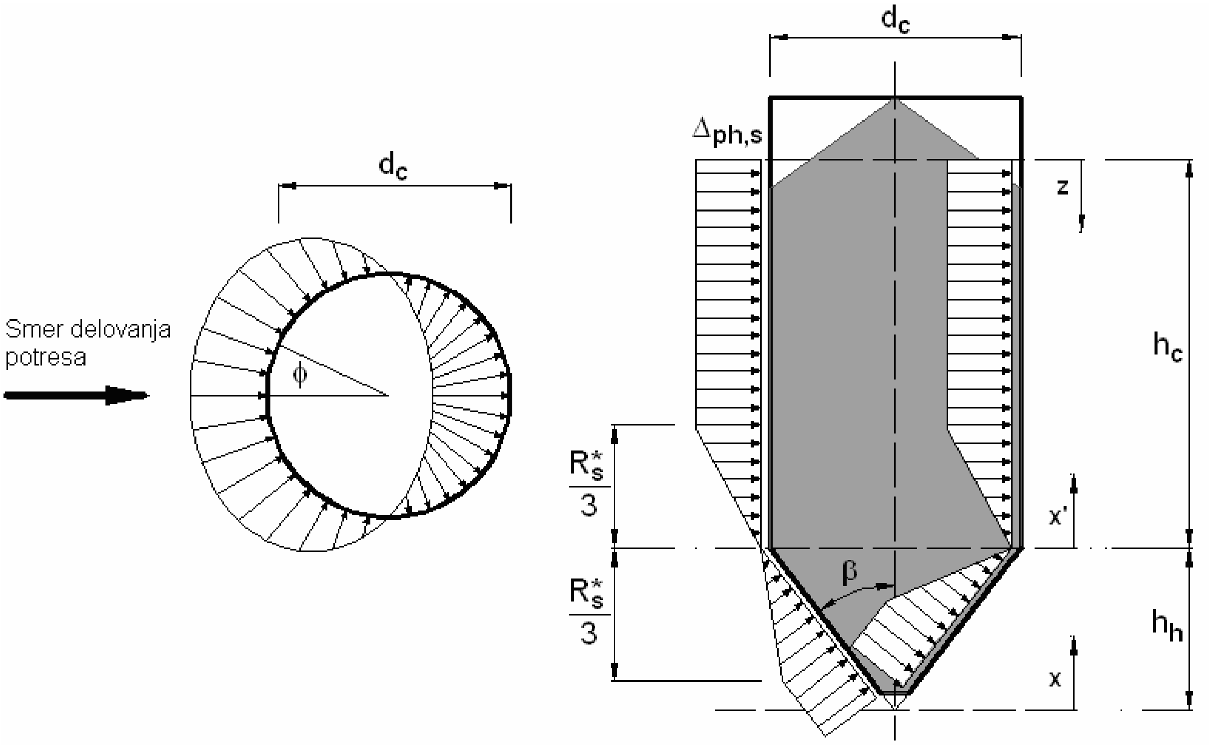
Celotno prečno silo je potrebno izračunati za obe mejni stanji (MSN in MSP), in sicer ob upoštevanju ustreznega projektnega spektra pospeškov $S_d(T)$.

Sila $F_i$ predstavlja inercijsko silo, ki je v tem primeru nasprotno enaka celotni prečni sili:

$$F_i = F_b \tag{I.233}$$

Deluje v težišču silosa, na višini $h_m$ (enačba (I.13)). Potrebujemo jo pri kontroli prevrnitve.

### 5.3.10 Korak 10: Dodatni pritisk na steno cilindra



**Slika 36: Dodatni normalni pritisk na stene silosa zaradi potresne obremenitve**

Dodatni normalni pritisk $\Delta_{ph,s}$, ki deluje na stene cilindra zaradi potresne obremenitve, se določi v skladu s postopkom, prikazanim na *Diagramu 4*.

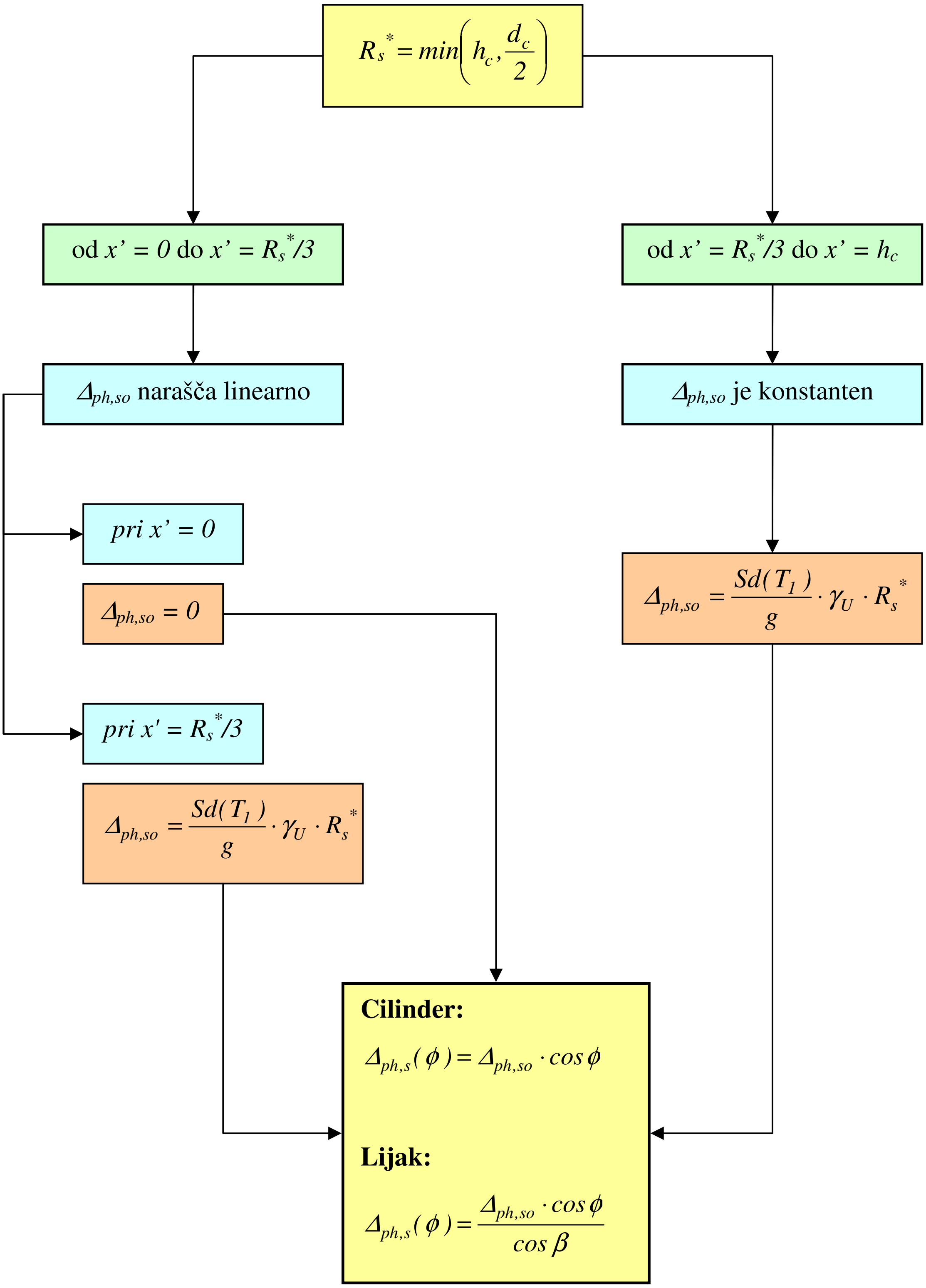
Ta pritisk je v vsaki točki omejen s pogojem, da mora biti vsota statičnih pritiskov, ki jih povzroča shranjen material, in dodatnih pritiskov, ki nastanejo kot posledica potresne obremenitve, večja ali enaka nič.

Velja torej:

**Cilinder:** $$p_h + \Delta_{ph,s} \geq 0 \qquad \text{(I.234)}$$

**Lijak:** $$p_n + \Delta_{ph,s} \geq 0 \qquad \text{(I.235)}$$

**Diagram 4: Določitev dodatnega pritiska na stene silosa pri potresni obremenitvi**



Standard *EN 1998-4* določa samo porazdelitev pritiskov, ki jih prikazuje *Slika 36*, kjer dodatni pritisk $\Delta_{ph,s}$ na dnu cilindra najprej narašča linearno, potem pa ima konstanto vrednost po celotni višini cilindra. Če je višina $R_s^* / 3$ veliko manjša od $h_c$, lahko namesto takšne razporeditve vzamemo razporeditev, kjer je $\Delta_{ph,s}$ konstanten po celotni višini cilindra in lijaka. Ta poenostavitev je na varni strani, saj z njo dobimo večje pritiske in s tem tudi večje notranje sile v stenah silosa.

Predlagam sledeč kriterij za določitev tipa porazdelitve dodatnega pritiska $\Delta_{ph,s}$:

$R_s^* / 3 < 0.1 h_c$ → konstantna porazdelitev po celotni višini cilindra in lijaka, ki je enaka:

$$\Delta_{ph,so} = \frac{Sd(T_1)}{g} \cdot \gamma_U \cdot R_s^* \qquad \text{(I.236)}$$

$R_s^* / 3 \geq 0.1 h_c$ → porazdelitev $\Delta_{ph,s}$, kot jo podaja *Diagram 4.*

## *5.4 Membranske sile*

### 5.4.1 Poln silos

Podani so izrazi za membranske sile, ki jih povzroča dodatni normalni pritisk $\Delta_{ph,s}$.

### 5.4.1.1 Cilinder

Vertikalna sila $n_{z,efs}$, obročna sila $n_{\phi,\ efs}$ in strižna sila $n_{z\phi,\ efs}$ v cilindru polnega silosa, zaradi delovanja pritiska $\Delta_{ph,s}(\phi)$:

$$\boxed{n_{z.efs}(\phi) = \frac{\Delta_{ph,s}(\phi)}{d_c} \cdot z^2} \qquad \text{(I.237)}$$

$$\boxed{n_{\phi.efs} = -\Delta_{ph,s}(\phi) \cdot r} \qquad \text{(I.238)}$$

$$\boxed{n_{z\phi,efs}(z) = -\Delta_{ph,so} \cdot \sin\phi \cdot z} \tag{I.239}$$

## 5.4.1.2 Lijak

Sila $n_{s,efs}$, obročna sila $n_{\phi,\ efs}$ in strižna sila $n_{s\phi,\ efs}$ v lijaku polnega silosa, zaradi delovanja pritiska $\Delta_{ph,s}(\phi)$:

$$\boxed{n_{s,efs}(\phi) = \frac{\Delta_{ph,so}}{6} \cdot \left[\frac{1}{\sin\beta} - 3\sin\beta\right] \cdot \cos\phi \cdot x} \tag{I.240}$$

$$\boxed{n_{\phi,efs}(\phi) = -\frac{\Delta_{ph,so}}{\cos\beta} \cdot \tan\beta \cdot \cos\phi \cdot x} \tag{I.241}$$

$$\boxed{n_{s\phi}(\phi) = -\frac{\Delta_{ph,so}}{3} \cdot \sin\phi \cdot x} \tag{I.242}$$

## 5.4.2 Prazen silos

Kot je bilo že omenjeno v poglavju *5.1*, je v primeru praznega silosa potrebno opraviti modalno analizo s spektrom odziva za celotno silosno konstrukcijo. Notranje sile je potrebno določiti ob upoštevanju vseh nihajnih oblik, katerih vsota modalnih mas znaša vsaj *90%* celotne mase konstrukcije. Priporočena je uporaba ustreznega računalniškega programa.

# 6 Druge vrste obtežb

## *6.1 Temperaturna obtežba*

Če je razlika med temperaturo shranjenega materiala in silosno konstrukcijo velika, je potrebno upoštevati temperaturno obremenitev. Določiti se jo mora v skladu s standardom *EN 1991-1-5*.

## *6.2 Obtežba zaradi eksplozije prahu*

Materiali kot so razna gnojila, krmila za živali, zdrobljena žita, itd. lahko proizvajajo prah, ki je eksploziven. Do eksplozije lahko pride, ko so delci prahu dovolj drobni in homogeno razporejeni po zraku. Ob stiku s kisikom lahko reagirajo in povzročijo kontinuirano eksotermno reakcijo. Če ustrezno zračenje ni prisotno, se lahko pojavijo dodatni pritiski velikosti od *0.8 MPa* do *1 MPa*.

Popoln seznam potencialno eksplozivnih materialov je podan v *EN 1991-4, Preglednica E.1.* V primeru, da se uporablja takšen material, je potrebno upoštevati določila standarda *EN 26184-1*.

# 7 Obtežne kombinacije in kontrole mejnih stanj

Obtežne kombinacije, ki so obravnavane v tem poglavju, se nanašajo na obtežbe opisane v poglavju *2*. Simboli, ki so navedeni v spodnji preglednici, predstavljajo karakteristično vrednost notranje sile za posamezno vrsto obtežbe.

**Preglednica 26: Simboli, ki označujejo posamezno vrsto obtežbe**

| Simbol | Vrsta obtežbe |
|---|---|
| $G$ | lastna teža |
| $Q_f$ | polnjenje shranjenega materiala |
| $Q_d$ | praznjenje shranjenega materiala |
| $Q_s$ | obtežba snega |
| $Q_w$ | obtežba vetra |
| $Q_t$ | temperaturna obtežba |
| $Q_{ef,MSN}$ | potresna obremenitev na poln silos pri mejnem stanju nosilnosti |
| $Q_{ef,MSP}$ | potresna obremenitev na poln silos pri mejnem stanju poškodb |
| $Q_{ee,MSN}$ | potresna obremenitev na prazen silos pri mejnem stanju nosilnosti |
| $Q_{ee,MSP}$ | potresna obremenitev na prazen silos pri mejnem stanju poškodb |

## 7.1 Mejno stanje nosilnosti (MSN)

### 7.1.1 Obtežne kombinacije

**Preglednica 27: Obtežne kombinacije za mejno stanje nosilnosti**

| Kombinacija | Kratek opis | Formula |
|---|---|---|
| CO1 | Praznjenje materiala | $1.35 \cdot G + 1.5 \cdot Q_d + 0.9 \cdot ( Q_s + Q_w )$ |
| CO2 | Sneg | $1.35 \cdot G + 1.5 \cdot Q_f + 0.9 \cdot Q_s$ |
| CO3 | Veter – poln silos | $1.35 \cdot G + 1.5 \cdot Q_f + 0.9 \cdot Q_w$ |
| CO4 | Veter – prazen silos | $1.35 \cdot G + 0.9 \cdot Q_{w,fs}$ |
| CO5 | Temperatura | $1.35 \cdot G + 1.0 \cdot Q_f + 0.6 \cdot Q_t$ |
| CO6 | Potres – poln silos | $1.0 \cdot G + 0.8 \cdot Q_f + Q_{ef,MSN}$ |
| CO7 | Potres – prazen silos | $1.0 \cdot G + Q_{ee,MSN}$ |

### 7.1.2 Kontrole

Znotraj mejnega stanja nosilnosti je potrebni izvesti sledeče kontrole, in sicer v skladu s standardom *prEN-1993-1-6*. Te kontrole so:

- **Meja plastičnosti (LS1)**

  Nanaša se na mejno stanje pri katerem je konstrukcija še sposobna prenesti dano obremenitev, ne da bi bila dosežena (prevelika) plastifikacija materiala.

- **Ciklična plastičnost (LS2)**

  Nanaša se na mejno stanje, kjer ponavljajoči cikli obremenjevanja in razbremenjevanja povzročajo plastično tečenje v nategu in tlaku, kar posledično lahko vodi do pojava razpok v materialu.

- **Uklon (LS3)**

  Nanaša se na izgubo stabilnosti zaradi delovanja tlačnih napetosti. Ko je kritično uklonsko napetostno stanje preseženo, se pojavijo veliki hipni premiki konstrukcije, ki lahko vodijo v porušitev.

- **Utrujanje (LS4)**

  Nanaša se na mejno stanje, kjer se zaradi ponavljajočih se ciklov obremenjevanja pojavijo razpoke v materialu (Podobno kot LS2, le da tukaj meja plastičnosti ni presežena). To kontrolo je potrebno izvesti, kadar je število ciklov večje od 10 000 v življenjski dobi konstrukcije.

## 7.2 Mejno stanje uporabnosti (MSU)

Pri silosih se mejno stanje poškodb (MSP) obravnava kot mejno stanje uporabnosti (MSU).

### 7.2.1 Obtežne kombinacije

Obtežne kombinacije za mejno stanje uporabnosti so podane v spodnji preglednici.

**Preglednica 28: Obtežne kombinacije za mejno stanje uporabnosti**

| Kombinacija | Kratek opis | Formula |
|---|---|---|
| CO8 | Veter – poln silos | $1.0 \cdot G + 0.9 \cdot Q_f + 0.6 \cdot Q_w$ |
| CO9 | Potres – poln silos | $1.0 \cdot G + 0.8 \cdot Q_f + Q_{ef,MSP}$ |

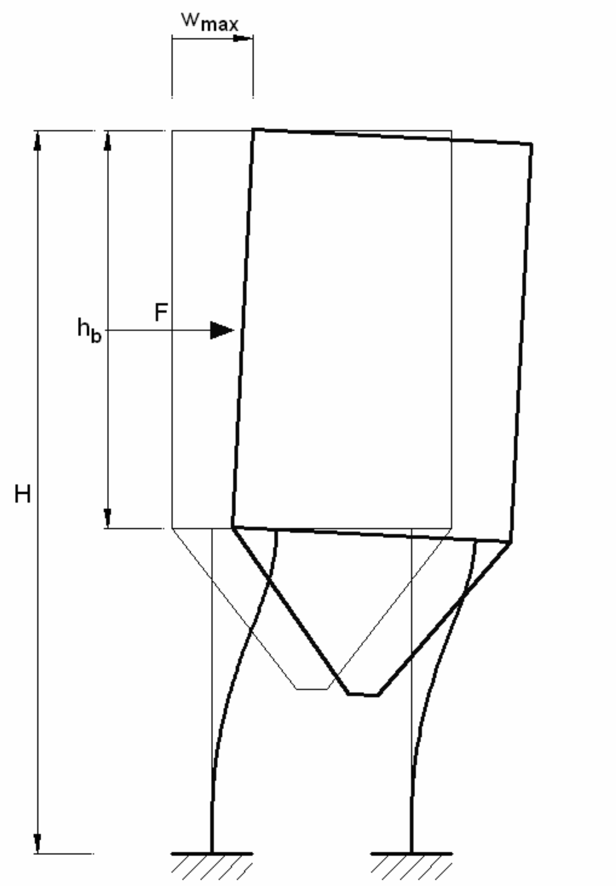
### 7.2.2 Kontrola pomikov

Maksimalna dovoljena vrednost vodoravnega pomika znaša:

$$w_{max} = 0.02 \cdot H \tag{I.243}$$

Simbol $H$ označuje celotno višino silosa, merjeno od temeljev, do vrha.

Preverjamo pomike, ki jih povzročata projektni vrednosti horizontalnih sil vetra oz. potresa na poln silos (označeno z $F$ na spodnji sliki).



**Slika 37: Maksimalni horizontalni pomik silosa zaradi delovanja projektne sile vetra oz. potresa**

### 7.2.2.1 Maksimalni pomik zaradi obtežbe vetra

Sila *F* na zgornji sliki v tem primeru označuje projektno vrednost sile vetra $W_d$, ki jo je potrebno izračunati v skladu s kombinacijo *CO8* (*Preglednica 28*).

Ob upoštevanju vodoravne togosti $k_v$ in upogibne togosti $k_m$ podporne konstrukcije, lahko določimo maksimalni pomik na vrhu silosa $w_w$ zaradi delovanja vetra.

$$w_W = W_d \left( \frac{1}{k_v} + \frac{h_b^{\,2}}{2k_m} \right) \tag{I.244}$$

Maksimalni pomik na vrhu silosa zaradi delovanja vetra mora biti manjši od maksimalnega dovoljenega pomika:

$$\boxed{w_W \leq w_{max}} \tag{I.245}$$

### 7.2.2.2 Maksimalni pomik zaradi potresne obremenitve

Projektno vrednost vodoravne sile zaradi delovanja potresa $F_{ed}$ je potrebno izračunati v skladu s kombinacijo *CO9* (*Preglednica 28*).

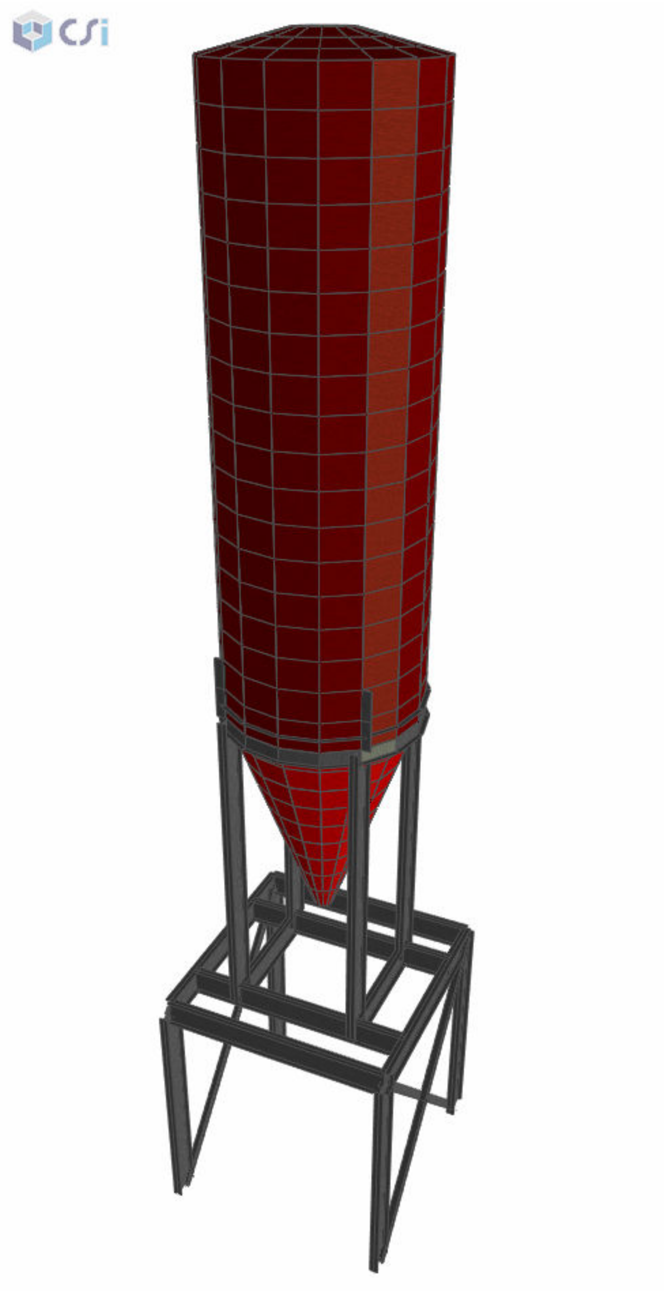
Ob upoštevanju vodoravne togosti $k_v$ in upogibne togosti $k_m$ podporne konstrukcije ter faktorja obnašanja $q$, lahko določimo maksimalni pomik na vrhu silosa $w_{Fe}$ pri potresni obremenitvi:

$$w_{Fe} = F_{ed} \cdot q \cdot \left( \frac{1}{k_v} + \frac{{h_b}^2}{2\,k_m} \right) \tag{I.246}$$

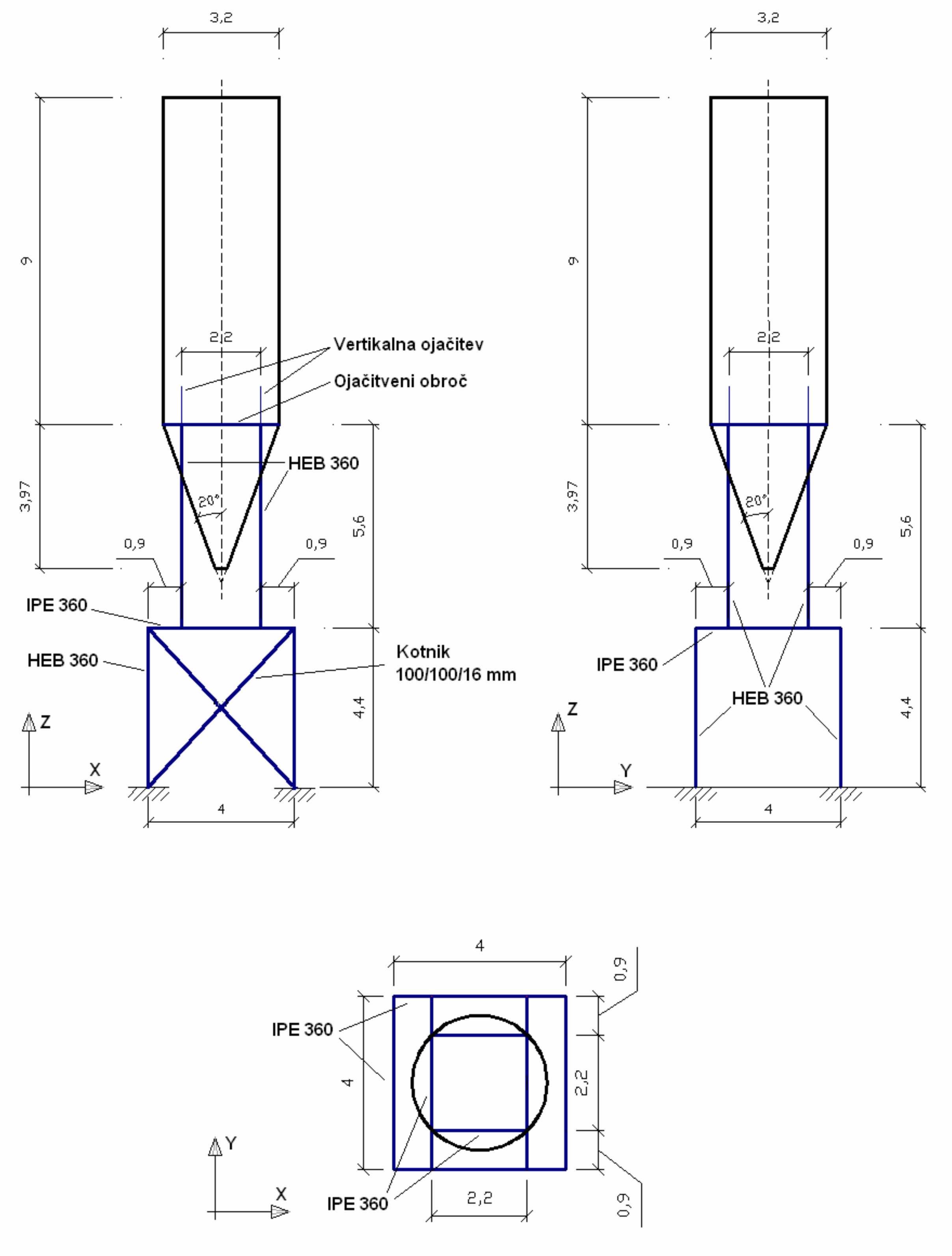
Maksimalni pomik na vrhu silosa zaradi delovanja potresa mora biti manjši od maksimalnega dovoljenega pomika:

$$\boxed{w_{Fe} \leq w_{max}} \tag{I.247}$$

# II. DEL – RAČUNSKI PRIMER



Slika 38: 3D predstavitev računskega modela



***Vse dimenzije so v metrih.***

**Slika 39: Dimenzije silosa in podporne konstrukcije**

# 8 Sestava silosa

## 8.1 Lastnosti shranjenega materiala

**Tip shranjenega materiala:** elektrofiltrski pepel (ang. »fly ash«)

Lastnosti so vzete iz preglednice v *Prilogi C*, oz. *Preglednice E.1* v *EN 1991-4*. Izbrana kategorija stene je *D2*.

| | | |
|---|---|---|
| $\gamma_l$ | 8.0 | [kN/m$^3$] |
| $\gamma_u$ | 15.0 | [kN/m$^3$] |
| $\phi_r$ | 41.0 | [º] |
| $\phi_{im}$ | 35.0 | [º] |
| $a_\phi$ | 1.16 | [ - ] |

| | | |
|---|---|---|
| $K_m$ | 0.46 | [ - ] |
| $a_K$ | 1.20 | [ - ] |
| $\mu$ | 0.62 | [ - ] |
| $a_\mu$ | 1.07 | [ - ] |
| $C_{op}$ | 0.50 | [ - ] |

## 8.2 Geometrijski parametri

**Neodvisni parametri**

Višina cilindra: $h_b = 9\ m$ (II.1)

Radij cilindra: $r = 1.6\ m$ (II.2)

Kot lijaka glede na simetrijsko os: $\beta = 20\ °$ (II.3)

Debelina stene: $t = 6.8\ mm \cong 7\ mm$ (povprečna debelina stene) (II.4)

**Odvisni parametri**

Višina lijaka:

$$h_h = \frac{r}{\tan\beta} = \frac{1.6}{\tan 20°} = 4.4\,m \qquad \text{(II.5)}$$

Višina zgornjega kupa:

$$h_{tp} = r \cdot \tan\phi_r = 1.6 \cdot \tan 41° = 1.39\,m \qquad \text{(II.6)}$$

Globina pod ekvivalentno površino: $h_0 = \frac{r}{3} \cdot \tan \phi_r = \frac{1.6}{3} \cdot \tan 41° = 0.46\ m$ (II.7)

Ekvivalentna višina cilindra:

$$h_c = h_b - h_{tp} + h_0 = 9 - 1.39 + 0.46$$

$$h_c = 8.07\ m \qquad \text{(II.8)}$$

Ekvivalentna višina shranjenega metriala: $h_s = h_h + h_c = 4.4 + 8.07 = 12.47\ m$ (II.9)

Premer cilindra: $d_c = 2 \cdot r = 2 \cdot 1.6 = 3.2\ m$ (II.10)

Dodatna geometrijska omejitev:

$$h_s / d_c < 10$$

$$12.47 / 3.2 = 3.9 < 10 \qquad \text{(II.11)}$$

Obseg silosa: $U = \pi \cdot d_c = \pi \cdot 3.2 = 10.05\ m$ (II.12)

Prečni prerez silosa: $A = \pi \cdot \frac{d_c^{\,2}}{4} = \pi \cdot \frac{3.2^2}{4} = 8.04\ m^2$ (II.13)

Volumen shranjenega materiala:

$$V_m = 8.04 \cdot \left( 8.07 - 0.46 + \frac{4.4 + 1.39}{3} \right)$$

$$V_m = 76.7\ m^3 \qquad \text{(II.14)}$$

Teža shranjenega materiala: $G_m = 15 \cdot 76.7 = 115\ kN$ (II.15)

Teža lupine silosa:

$$G_s = \pi \cdot \left(1.607^2 - 1.6^2\right) \cdot \left(9 + \frac{4.4}{3}\right) \cdot 78.5$$

$$G_s = 57.9 kN \qquad \text{(II.16)}$$

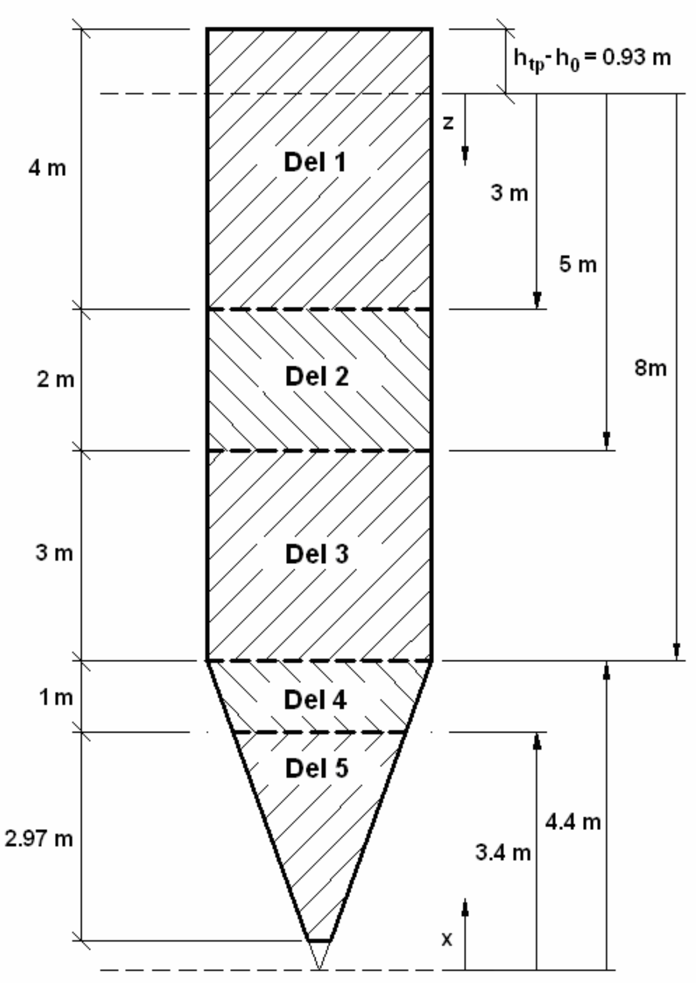
Težišče silosa:

$$h_m = \frac{1 + \frac{1}{18} \cdot \left(\frac{1.39}{8.07}\right)^2 - \frac{1}{6} \cdot \left(\frac{4.4}{8.07}\right)^2}{1 + \frac{1}{3} \cdot \frac{4.4}{8.07}} \cdot \frac{8.07}{2} = 3.19\ m \qquad \text{(II.17)}$$

Vztrajnostni moment cilindra (okoli vodoravne osi):

$$I = \frac{\pi \cdot 0.007 \cdot (3.207) \cdot \left( 3.2^2 + 2 \cdot 3.2 \cdot 0.007 + 2 \cdot 0.007^2 \right)}{8} = 0.09067\ m^4 \qquad \text{(II.18)}$$

## Sestava in debelina stene:



**Slika 40: Sestavni deli silosa**

Cilinder je sestavljen iz treh delov:

- *Del 1* - od vrha silosa do $z = 3\ m$; debelina stene $t_1 = 3\ mm$,
- *Del 2* - od $z = 3\ m$ do $z = 5\ m$; debelina stene $t_2 = 5\ mm$,
- *Del 3* - od $z = 5\ m$ do $z = 8.1\ m$; debelina stene $t_3 = 8\ mm$.

Lijak je sestavljen iz dveh delov:

- *Del 4* - od $x = 4.4\ m$ do $x = 3.4\ m$; debelina stene $t_4 = 6\ mm$,
- *Del 5* - od $x = 3.4\ m$ do $x = 0\ m$; debelina stene $t_5 = 5\ mm$.

## 8.3 Tip cilindra in tip lijaka

Tip cilindra: $$\frac{h_c}{d_c}=\frac{8.07}{3.2}=2.52>2.0 \quad \rightarrow \quad \textit{Vitki silos} \qquad \text{(II.19)}$$

Tip lijaka: $$\mu_{min}=\frac{\mu}{a_\mu}=\frac{0.62}{1.07}=0.58 \qquad \tan\beta=\tan 20°=0.36 \quad \rightarrow \quad \textit{Strm lijak} \qquad \text{(II.20)}$$

# 9 Obtežbe silosa in membranske sile

Pri določitvi obtežbe in membranski sil, obravnavamo maksimalne obremenitve, ki se pojavijo znotraj določenega dela stene silosa. Navadno se te vrednosti pojavljajo na stikih dveh delov.

Silos je povezan s podporno konstrukcijo preko štirih točkovnih podpor. V bližini teh podpor lahko pričakujemo veliko koncentracije napetosti. Nad vsako podporo je zato dodana vertikalna ojačitev višine *1 m*. V tem primeru je zato obravnavana tudi obremenitev, ki se pojavi na vrhnjem robu ojačitev (na globini $z$ = *7.5 m*).

## 9.1 Stalna obtežba

**Lastna teža strehe silosa:**

Teža na $m^2$ strehe (ocena): $$g_{p,r}=2.5\,kN/m^2 \qquad \text{(II.21)}$$

Celotna teža strehe: $$W_r=g_{p,r}\cdot A=2.5\cdot 8.04=20\,kN \qquad \text{(II.22)}$$

**Lastna teža cilindra:**

Lastna teža v odvisnosti od koordinate $z$:

$$W_{sc}(z)=\pi\cdot\left((r+t)^2-r^2\right)\cdot z\cdot 78.5 \qquad \text{(II.23)}$$

$$W_{sc}(z)=\pi\cdot\left((1.6+0.007)^2-1.6^2\right)\cdot z\cdot 78.5=5.33\cdot z\ \ [kN] \qquad \text{(II.24)}$$

**Membranske sile zaradi stalnih obtežb:**

$$\boxed{n_{z,p} = \frac{F_{p,r} + W_{sc}}{2\pi r} = \frac{20 + 5.536 \cdot z}{2 \cdot \pi \cdot 1.6} = 2 + 0.53\, z \;\; [kN/m]} \qquad \text{(II.25)}$$

$$\boxed{n_{\phi,p} = 0 \;\; kN/m} \qquad \text{(II.26)}$$

## *9.2 Obtežba zaradi shranjenega materiala*

### 9.2.1 Osno simetrična obtežba

### 9.2.1.1 Razred obremenitve

Masa shranjenega materiala je enaka:

$$m_{solid} = 0.1 \cdot W_m = 0.1 \cdot 1151 = 115\, t \rightarrow AAC = 2 \qquad \text{(II.27)}$$

### 9.2.1.2 Kombinacije materialnih parametrov

**Cilinder:**

**Preglednica 29: Kombinacije materialnih parametrov za določitev maksimalnih obtežb na stene cilindra računskega primera**

| **Kombinacija** | **Namen** | **μ** | **K** | **$\phi_i$** |
|---|---|---|---|---|
| **2** | maksimalni normalni pritisk ($p_h$) | **0.58** | **0.55** | **30.17** |
| **3** | maksimalno trenje ($p_w$) | **0.66** | **0.55** | **30.17** |

**Lijak:**

**Preglednica 30: Kombinacije materialnih parametrov za določitev maksimalnih obtežb na stene lijaka računskega primera**

| Kombinacija | Velja za | Namen | $\mu$ | K | $\phi_i$ |
|---|---|---|---|---|---|
| **4** | steno cilindra | maksimalni vertikalni pritisk ($p_v$) | **0.58** | **0.38** | **40.60** |
| | steno lijaka | maksimalni pritisk ob polnjenju | **0.58** | **0.38** | **30.17** |
| **5** | steno cilindra | maksimalni vertikalni pritisk ($p_v$) | **0.58** | **0.38** | **40.60** |
| | steno lijaka | maksimalni pritisk ob praznjenju | **0.58** | **0.55** | **40.60** |

### 9.2.1.3 Obtežba na steno cilindra

Pritiski na steno cilindra so bili izračunani z uporabo programa »*Stored Solid Load on Cylindrical Silos*«.

**Polnjenje:**

**Preglednica 31: Pritiski in notranje sile v cilindru, pri polnjenju**

**Kombinacija 2**

| $z/h_c$ | z | $p_{hf}$ | $p_{wf}$ | $p_{vf}$ | $n_{zf}$ | $n_{\phi f}$ |
|---|---|---|---|---|---|---|
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| *0.07* | *0.5* | 4.03 | 2.33 | 7.30 | -0.65 | 6.44 |
| *0.13* | *1.1* | 7.23 | 4.19 | 13.09 | -2.41 | 11.56 |
| *0.20* | *1.6* | 9.84 | 5.70 | 17.82 | -5.08 | 15.74 |
| *0.27* | *2.1* | 11.94 | 6.92 | 21.63 | -8.48 | 19.10 |
| *0.33* | *2.7* | 13.63 | 7.90 | 24.70 | -12.47 | 21.81 |
| *0.40* | *3.2* | 15.00 | 8.69 | 27.17 | -16.93 | 24.00 |
| *0.47* | *3.8* | 16.10 | 9.33 | 29.17 | -21.77 | 25.76 |

**se nadaljuje …**

**… nadaljevanje**

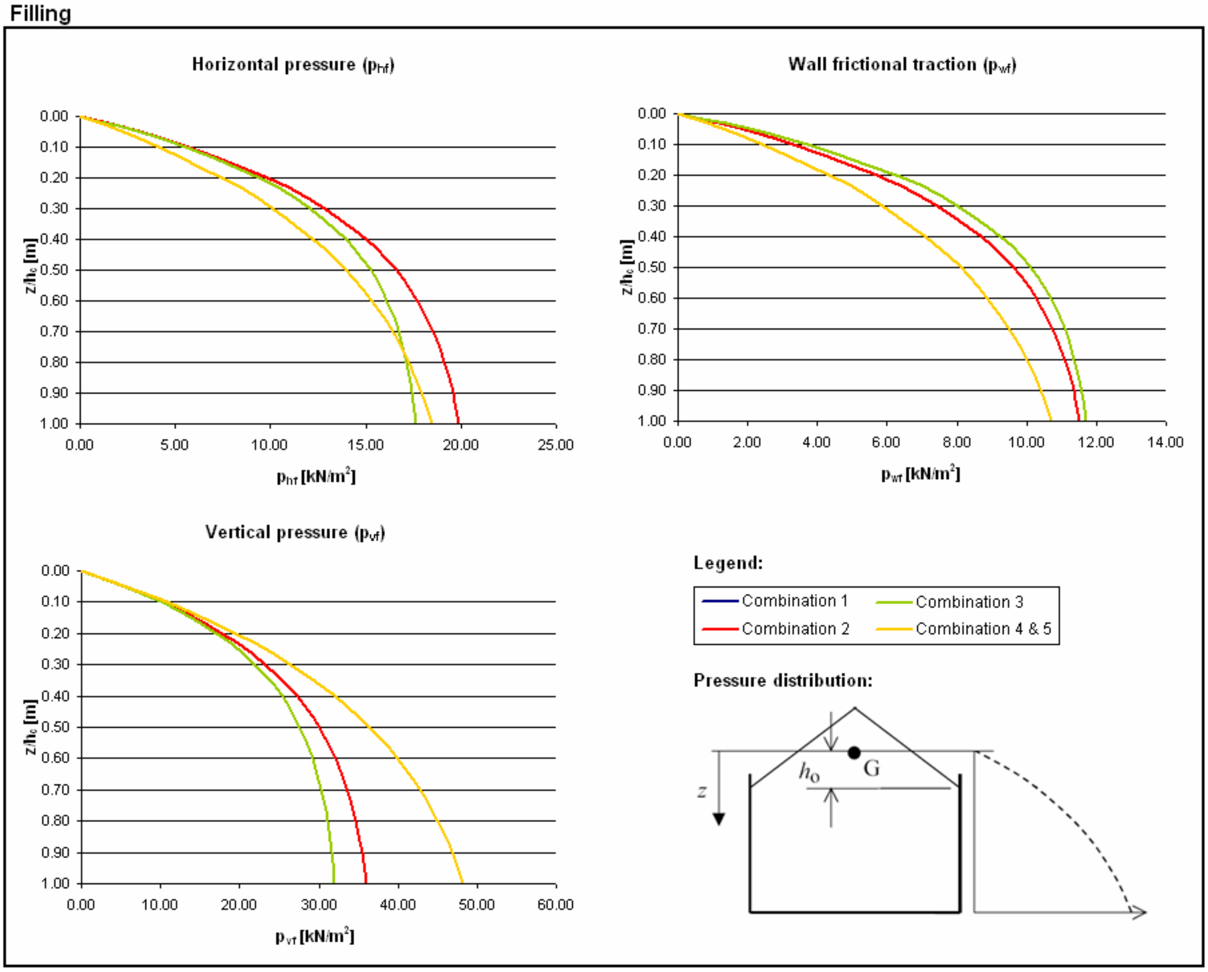
| | | | | | | |
|---|---|---|---|---|---|---|
| *0.53* | *4.3* | 16.99 | 9.85 | 30.78 | -26.93 | 27.19 |
| *0.60* | *4.8* | 17.71 | 10.26 | 32.08 | -32.33 | 28.34 |
| *0.67* | *5.4* | 18.29 | 10.60 | 33.13 | -37.93 | 29.26 |
| *0.73* | *5.9* | 18.76 | 10.87 | 33.98 | -43.69 | 30.01 |
| *0.80* | *6.4* | 19.13 | 11.09 | 34.66 | -49.59 | 30.62 |
| *0.86* | *7.0* | 19.44 | 11.26 | 35.22 | -55.59 | 31.10 |
| *0.93* | *7.5* | 19.68 | 11.41 | 35.66 | -61.68 | 31.49 |
| *1.00* | *8.1* | 19.88 | 11.52 | 36.02 | -67.83 | 31.81 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |

### Kombinacija 3

| $z/h_c$ | $z$ | $p_{hf}$ | $p_{wf}$ | $p_{vf}$ | $n_{zf}$ | $n_{\phi f}$ |
|---|---|---|---|---|---|---|
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| *0.07* | *0.5* | 3.97 | 2.63 | 7.19 | -0.74 | 6.35 |
| *0.13* | *1.1* | 7.02 | 4.66 | 12.72 | -2.71 | 11.24 |
| *0.20* | *1.6* | 9.44 | 6.26 | 17.10 | -5.66 | 15.10 |
| *0.27* | *2.1* | 11.32 | 7.51 | 20.51 | -9.37 | 18.12 |
| *0.33* | *2.7* | 12.80 | 8.49 | 23.18 | -13.68 | 20.48 |
| *0.40* | *3.2* | 13.95 | 9.25 | 25.27 | -18.45 | 22.32 |
| *0.47* | *3.8* | 14.85 | 9.85 | 26.91 | -23.59 | 23.76 |
| *0.53* | *4.3* | 15.56 | 10.32 | 28.18 | -29.01 | 24.89 |
| *0.60* | *4.8* | 16.11 | 10.69 | 29.18 | -34.65 | 25.77 |
| *0.67* | *5.4* | 16.54 | 10.97 | 29.96 | -40.47 | 26.46 |
| *0.73* | *5.9* | 16.88 | 11.20 | 30.58 | -46.42 | 27.00 |
| *0.80* | *6.4* | 17.14 | 11.37 | 31.05 | -52.48 | 27.43 |
| *0.86* | *7.0* | 17.35 | 11.51 | 31.43 | -58.62 | 27.76 |
| *0.93* | *7.5* | 17.51 | 11.62 | 31.72 | -64.83 | 28.01 |
| *1.00* | *8.1* | 17.64 | 11.70 | 31.95 | -71.09 | 28.22 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |

**Kombinaciji 4 & 5**

| $z/h_c$ | $z$ | $p_{hf}$ | $p_{wf}$ | $p_{vf}$ | $n_{zf}$ | $n_{\phi f}$ |
|---|---|---|---|---|---|---|
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| *0.07* | *0.5* | 2.89 | 1.67 | 7.53 | -0.46 | 4.62 |
| *0.13* | *1.1* | 5.34 | 3.09 | 13.93 | -1.74 | 8.54 |
| *0.20* | *1.6* | 7.47 | 4.33 | 19.49 | -3.75 | 11.96 |
| *0.27* | *2.1* | 9.31 | 5.39 | 24.27 | -6.37 | 14.89 |
| *0.33* | *2.7* | 10.88 | 6.31 | 28.39 | -9.51 | 17.41 |
| *0.40* | *3.2* | 12.24 | 7.10 | 31.94 | -13.11 | 19.59 |
| *0.47* | *3.8* | 13.42 | 7.77 | 35.00 | -17.11 | 21.47 |
| *0.53* | *4.3* | 14.43 | 8.36 | 37.64 | -21.44 | 23.08 |
| *0.60* | *4.8* | 15.30 | 8.86 | 39.91 | -26.07 | 24.48 |
| *0.67* | *5.4* | 16.05 | 9.30 | 41.86 | -30.95 | 25.67 |
| *0.73* | *5.9* | 16.69 | 9.67 | 43.54 | -36.04 | 26.71 |
| *0.80* | *6.4* | 17.25 | 9.99 | 45.00 | -41.32 | 27.60 |
| *0.86* | *7.0* | 17.73 | 10.27 | 46.25 | -46.77 | 28.36 |
| *0.93* | *7.5* | 18.14 | 10.51 | 47.32 | -52.35 | 29.03 |
| *1.00* | *8.1* | 18.50 | 10.72 | 48.25 | -58.05 | 29.59 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |



Slika 41: Pritiski na steno cilindra po polnjenju (slika vzeta iz programa)

## Praznjenje:

**Preglednica 32: Pritiski in notranje sile v cilindru, pri praznjenju**

**Kombinacija 2**

| $z/h_c$ | z | $p_{he}$ | $p_{we}$ | $n_{ze}$ | $n_{\phi e}$ |
|---|---|---|---|---|---|
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 |
| *0.07* | *0.5* | 4.63 | 2.57 | -0.72 | 7.41 |
| *0.13* | *1.1* | 8.31 | 4.61 | -2.65 | 13.30 |
| *0.20* | *1.6* | 11.31 | 6.27 | -5.59 | 18.10 |
| *0.27* | *2.1* | 13.73 | 7.61 | -9.33 | 21.97 |
| *0.33* | *2.7* | 15.68 | 8.69 | -13.72 | 25.08 |

**se nadaljuje …**

**… nadaljevanje**

| | | | | | |
|---|---|---|---|---|---|
| *0.40* | *3.2* | 17.25 | 9.56 | -18.62 | 27.60 |
| *0.47* | *3.8* | 18.52 | 10.26 | -23.95 | 29.63 |
| *0.53* | *4.3* | 19.54 | 10.83 | -29.62 | 31.27 |
| *0.60* | *4.8* | 20.37 | 11.29 | -35.56 | 32.59 |
| *0.67* | *5.4* | 21.03 | 11.66 | -41.72 | 33.65 |
| *0.73* | *5.9* | 21.57 | 11.96 | -48.06 | 34.51 |
| *0.80* | *6.4* | 22.00 | 12.20 | -54.55 | 35.21 |
| *0.86* | *7.0* | 22.35 | 12.39 | -61.15 | 35.77 |
| *0.93* | *7.5* | 22.64 | 12.55 | -67.84 | 36.22 |
| *1.00* | *8.1* | 22.86 | 12.67 | -74.62 | 36.58 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |

**Kombinacija 3**

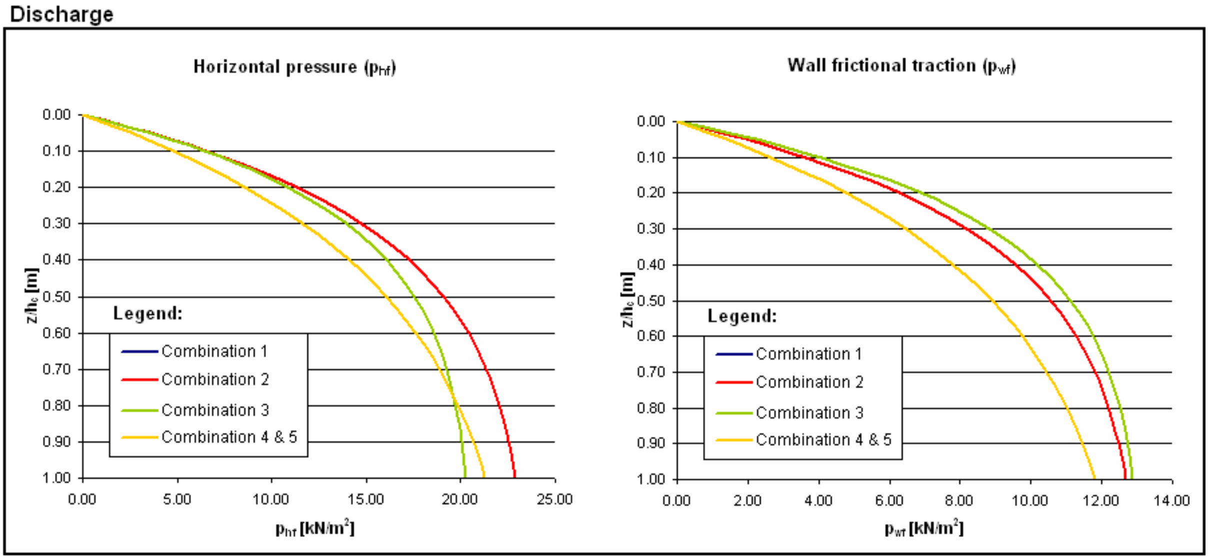
| $z/h_c$ | $z$ | $p_{he}$ | $p_{we}$ | $n_{ze}$ | $n_{\phi e}$ |
|---|---|---|---|---|---|
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 |
| *0.07* | *0.5* | 4.56 | 2.89 | -0.82 | 7.30 |
| *0.13* | *1.1* | 8.08 | 5.13 | -2.98 | 12.92 |
| *0.20* | *1.6* | 10.86 | 6.89 | -6.23 | 17.37 |
| *0.27* | *2.1* | 13.02 | 8.26 | -10.31 | 20.84 |
| *0.33* | *2.7* | 14.72 | 9.34 | -15.05 | 23.55 |
| *0.40* | *3.2* | 16.04 | 10.18 | -20.30 | 25.67 |
| *0.47* | *3.8* | 17.08 | 10.84 | -25.94 | 27.33 |
| *0.53* | *4.3* | 17.89 | 11.35 | -31.91 | 28.63 |
| *0.60* | *4.8* | 18.53 | 11.76 | -38.11 | 29.64 |
| *0.67* | *5.4* | 19.02 | 12.07 | -44.51 | 30.43 |
| *0.73* | *5.9* | 19.41 | 12.32 | -51.06 | 31.05 |
| *0.80* | *6.4* | 19.71 | 12.51 | -57.73 | 31.54 |
| *0.86* | *7.0* | 19.95 | 12.66 | -64.48 | 31.92 |
| *0.93* | *7.5* | 20.14 | 12.78 | -71.31 | 32.22 |
| *1.00* | *8.1* | 20.28 | 12.87 | -78.20 | 32.45 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |

**se nadaljuje …**

**… nadaljevanje**

**Kombinacija 4 & 5**

| $z/h_c$ | z | $p_{he}$ | $p_{we}$ | $n_{ze}$ | $n_{\phi e}$ |
|---|---|---|---|---|---|
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 |
| *0.07* | *0.5* | 3.32 | 1.84 | -0.51 | 0.87 |
| *0.13* | *1.1* | 6.14 | 3.40 | -1.92 | 1.72 |
| *0.20* | *1.6* | 8.59 | 4.76 | -4.12 | 2.58 |
| *0.27* | *2.1* | 10.70 | 5.93 | -7.00 | 3.44 |
| *0.33* | *2.7* | 12.52 | 6.94 | -10.46 | 4.30 |
| *0.40* | *3.2* | 14.08 | 7.80 | -14.43 | 5.16 |
| *0.47* | *3.8* | 15.43 | 8.55 | -18.82 | 6.01 |
| *0.53* | *4.3* | 16.59 | 9.20 | -23.59 | 6.87 |
| *0.60* | *4.8* | 17.59 | 9.75 | -28.68 | 7.73 |
| *0.67* | *5.4* | 18.45 | 10.23 | -34.04 | 8.59 |
| *0.73* | *5.9* | 19.20 | 10.64 | -39.65 | 9.45 |
| *0.80* | *6.4* | 19.84 | 10.99 | -45.46 | 10.31 |
| *0.86* | *7.0* | 20.39 | 11.30 | -51.44 | 11.17 |
| *0.93* | *7.5* | 20.86 | 11.56 | -57.58 | 12.03 |
| *1.00* | *8.1* | 21.27 | 11.79 | -63.85 | 12.89 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |



**Slika 42: Pritiski na steno cilindra pri praznjenju (slika vzeta iz programa)**

### 9.2.1.4 Obtežba na steno lijaka

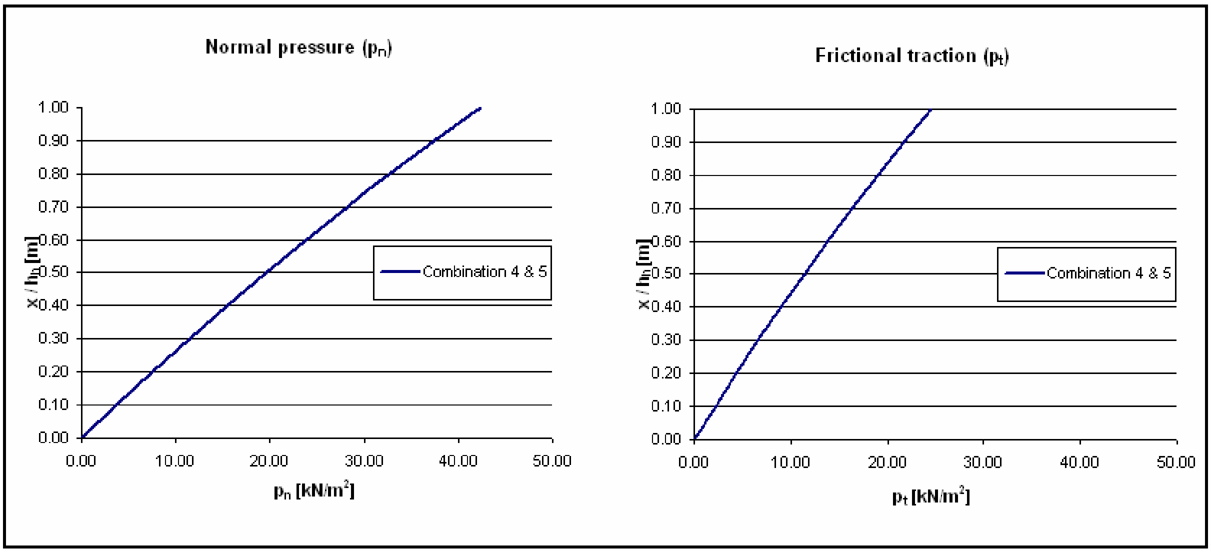
Pritiski na steno lijaka so bili izračunani z uporabo programa »*Stored Solid Load on Cylindrical Silos*«.

**Polnjenje:**

**Preglednica 33: Pritiski in notranje sile v lijaku, pri polnjenju**

**Kombinacija 4 & 5**

| $x/h_h$ | x | $p_{nf}$ | $p_{tf}$ | $n_{sf}$ | $n_{\phi f}$ |
|---|---|---|---|---|---|
| *1.00* | *4.4* | 42.35 | 24.54 | 59.82 | 72.11 |
| *0.90* | *4.0* | 37.44 | 21.70 | 47.86 | 57.38 |
| *0.80* | *3.5* | 32.72 | 18.96 | 37.38 | 44.57 |
| *0.70* | *3.1* | 28.17 | 16.32 | 28.31 | 33.58 |
| *0.60* | *2.6* | 23.78 | 13.78 | 20.59 | 24.30 |
| *0.50* | *2.2* | 19.54 | 11.32 | 14.16 | 16.64 |
| *0.40* | *1.8* | 15.43 | 8.94 | 8.99 | 10.51 |
| *0.30* | *1.3* | 11.45 | 6.63 | 5.02 | 5.85 |
| *0.20* | *0.9* | 7.56 | 4.38 | 2.22 | 2.57 |
| *0.10* | *0.4* | 3.75 | 2.17 | 0.55 | 0.64 |
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |



**Slika 43: Pritiski na steno lijaka po polnjenju (slika vzeta iz programa)**

## Praznjenje:

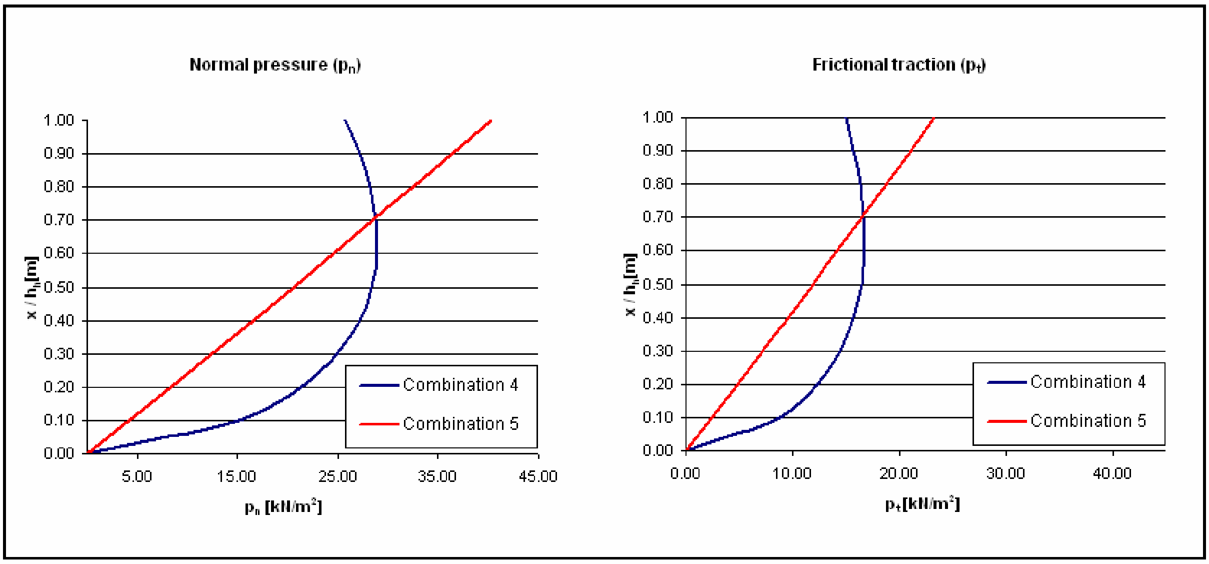
**Preglednica 34: Pritiski in notranje sile v lijaku, pri praznjenju**

**Kombinacija 4**

| $x/h_h$ | x | $p_{ne}$ | $p_{te}$ | $n_{se}$ | $n_{\phi e}$ |
|---|---|---|---|---|---|
| *1.00* | *4.4* | 25.78 | 14.94 | 59.82 | 43.90 |
| *0.90* | *4.0* | 27.16 | 15.74 | 54.12 | 41.62 |
| *0.80* | *3.5* | 28.17 | 16.32 | 47.91 | 38.37 |
| *0.70* | *3.1* | 28.77 | 16.67 | 41.27 | 34.29 |
| *0.60* | *2.6* | 28.87 | 16.73 | 34.35 | 29.49 |
| *0.50* | *2.2* | 28.38 | 16.44 | 27.30 | 24.16 |
| *0.40* | *1.8* | 27.13 | 15.72 | 20.30 | 18.48 |
| *0.30* | *1.3* | 24.89 | 14.42 | 13.59 | 12.72 |
| *0.20* | *0.9* | 21.22 | 12.30 | 7.52 | 7.23 |
| *0.10* | *0.4* | 15.11 | 8.76 | 2.60 | 2.57 |
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |

**Kombinacija 5**

| $x/h_h$ | x | $p_{ne}$ | $p_{te}$ | $n_{se}$ | $n_{\phi e}$ |
|---|---|---|---|---|---|
| *1.00* | *4.4* | 40.21 | 23.30 | 58.97 | 68.46 |
| *0.90* | *4.0* | 36.36 | 21.07 | 47.71 | 55.72 |
| *0.80* | *3.5* | 32.47 | 18.81 | 37.66 | 44.23 |
| *0.70* | *3.1* | 28.54 | 16.53 | 28.79 | 34.01 |
| *0.60* | *2.6* | 24.56 | 14.23 | 21.12 | 25.09 |
| *0.50* | *2.2* | 20.55 | 11.91 | 14.64 | 17.49 |
| *0.40* | *1.8* | 16.50 | 9.56 | 9.34 | 11.24 |
| *0.30* | *1.3* | 12.42 | 7.19 | 5.24 | 6.34 |
| *0.20* | *0.9* | 8.30 | 4.81 | 2.32 | 2.83 |
| *0.10* | *0.4* | 4.16 | 2.41 | 0.57 | 0.71 |
| *0.00* | *0.0* | 0.00 | 0.00 | 0.00 | 0.00 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |



**Slika 44: Pritiski na steno lijaka pri praznjenju (slika vzeta iz programa)**

### 9.2.2 Nesimetrična obtežba

Ker se obravnavani silos nahaja v razredu obremenitve 2 ($AAC = 2$), se nesimetrična obtežba ponazori z enakomernim povečanjem pritiskov, ki nastanejo zaradi osno simetrične obtežbe.

Faktorja nesimetrične obtežbe za polnjenje $C_{pf}$ in praznjenje $C_{pe}$ znašata:

$$C_{pf} = 0.21 \cdot C_{op} \cdot \left(1 - e^{-1.5(h_c / d_c - 1)}\right) = 0.21 \cdot 0.5 \cdot \left(1 - e^{-1.5(8.07 / 3.2 - 1)}\right) = 0.094 \qquad \text{(II.28)}$$

$$C_{pe} = 0.42 \cdot C_{op} \cdot \left(1 - e^{-1.5(h_c / d_c - 1)}\right) = 0.21 \cdot 0.5 \cdot \left(1 - e^{-1.5(8.07 / 3.2 - 1)}\right) = 0.189 \qquad \text{(II.29)}$$

Povečanje pritiska je enako:

- Polnjenje: $p_{hf,u} = p_{hf}{}^{\cdot}\ (1 + 0.5{}^{\cdot}C_{pf}) = 1.047\ p_{hf}$ (II.30)

  $p_{wf,u} = p_{wf}{}^{\cdot}\ (1 +{}^{\cdot}C_{pf}) = 1.094\ p_{wf}$ (II.31)

- Praznjenje: $p_{he,u} = p_{he}{}^{\cdot}\ (1 + 0.5{}^{\cdot}C_{pe}) = 1.094\ p_{he}$ (II.32)

  $p_{we,u} = p_{we}{}^{\cdot}\ (1 +{}^{\cdot}C_{pe}) = 1.189\ p_{we}$ (II.33)

Notranje sile, ki nastanejo zaradi osno simetrične obtežbe, se nato pomnožijo z ustreznimi faktorji na sledeč način:

— *polnjenje:* sila $n_{\phi f}$ se pomnoži z *1.047*, sila $n_{zf}$ pa z *1.094*.

— *praznjenje:* sila $n_{\phi e}$ se pomnoži z *1.094*, sila $n_{ze}$ pa z *1.189*.

## 9.2.3 Membranske sile, pomembne za dimenzioniranje

Membranske sile v cilindru in lijaku, ki so uporabljene pri dimenzioniranju, so podane v spodnjih preglednicah. Koordinata $z$ predstavlja mesto največje sile, pri določenem sestavnem delu cilindra.

V prvi vrstici vsake izmed globin $z$ je podana maksimalna sila $n_z$, skupaj s pripadajočo vrednostjo sile $n_\phi$. V drugi vrstici vsake izmed globin $z$ je podana maksimalna sila $n_\phi$, skupaj s pripadajočo vrednostjo sile $n_z$.

### 9.2.3.1 Cilinder

**Polnjenje:**

**Preglednica 35: Maksimalne membranske sile v cilindru po polnjenju, za vsak sestavni del cilindra**

| | | | | **Maksimalna in pripadajoča sila** | | | |
|---|---|---|---|---|---|---|---|
| | | | | **brez nesimetrične obtežbe** | | **z nesimetrično obtežbo** | |
| **Sestavni del** | **z [m]** | **Kombinacija** | **Maksimalna sila** | **$n_{zf}$ [kN/m]** | **$n_{\phi f}$ [kN/m]** | **$n_{zf}$ [kN/m]** | **$n_{\phi f}$ [kN/m]** |
| 1 | 3 | 3 | $n_{zf}$ | -16.42 | *21.61* | **-17.97** | ***20.60*** |
| | | 2 | $n_{\phi f}$ | *-15.03* | 23.15 | ***-16.44*** | **22.06** |
| 2 | 5 | 3 | $n_{zf}$ | -36.44 | *26.01* | **-39.87** | ***24.78*** |
| | | 2 | $n_{\phi f}$ | *-34.05* | 28.65 | ***-37.25*** | **27.30** |
| 3 | 7.5 | 3 | $n_{zf}$ | -64.83 | *28.01* | **-70.92** | ***29.32*** |
| | | 2 | $n_{\phi f}$ | *-61.68* | 31.49 | ***-67.50*** | **33.00** |

**Praznjenje:**

**Preglednica 36: Maksimalne membranske sile v cilindru pri praznjenju, za vsak sestavni del cilindra**

|  |  |  |  | Maksimalna in pripadajoča sila |  |  |  |
|---|---|---|---|---|---|---|---|
|  |  |  |  | brez nesimetrične obtežbe |  | brez nesimetrične obtežbe |  |
| **Sestavni del** | **z [m]** | **Kombinacija** | **Maksimalna sila** | **$n_{ze}$ [kN/m]** | **$n_{\phi e}$ [kN/m]** | **$n_{ze}$ [kN/m]** | **$n_{\phi e}$ [kN/m]** |
| 1 | 3 | 3 | $n_{ze}$ | -18.07 | *24.85* | **-21.50** | ***22.49*** |
|  |  | 2 | $n_{\phi e}$ | *-16.53* | *26.62* | ***-19.68*** | **24.09** |
| 2 | 5 | 3 | $n_{ze}$ | -40.09 | *29.91* | **-47.70** | ***27.07*** |
|  |  | 2 | $n_{\phi e}$ | *-37.46* | *32.94* | ***-44.57*** | **29.81** |
| 3 | 7.5 | 3 | $n_{ze}$ | -61.68 | *31.49* | **-73.33** | ***34.45*** |
|  |  | 2 | $n_{\phi e}$ | *-67.84* | 36.22 | ***-80.66*** | **39.62** |

## 9.2.3.2 Lijak

**Polnjenje:**

**Preglednica 37: Maksimalne membranske sile v lijaku po polnjenju, za vsak sestavni del lijaka**

|  |  |  |  | Maksimalna in pripadajoča sila |  |
|---|---|---|---|---|---|
| **Sestavni del** | **x [m]** | **Kombinacija** | **Maksimalna sila** | **$n_{sf}$ [kN/m]** | **$n_{\phi f}$ [kN/m]** |
| 4 | 4.4 | 4 | $n_{sf}$ | **59.82** | ***72.11*** |
|  |  |  | $n_{\phi f}$ | ***59.82*** | **72.11** |
| 5 | 3.4 | 4 | $n_{sf}$ | **37.38** | ***44.57*** |
|  |  |  | $n_{\phi f}$ | ***37.38*** | **44.57** |

## Praznjenje:

**Preglednica 38: Maksimalne membranske sile v lijaku pri praznjenju, za vsak sestavni del lijaka**

| | | | | Maksimalna in pripadajoča sila | |
|---|---|---|---|---|---|
| **Sestavni del** | **x [m]** | **Kombinacija** | **Maksimalna sila** | **$n_{se}$ [kN/m]** | **$n_{\phi e}$ [kN/m]** |
| 4 | 4.4 | 5 | $n_{se}$ | **59.82** | ***43.90*** |
| | | | $n_{\phi e}$ | ***59.82*** | **43.90** |
| 5 | 3.4 | 5 | $n_{se}$ | **47.91** | ***38.37*** |
| | | | $n_{\phi e}$ | ***47.91*** | **38.37** |

## 9.3 Obtežba snega

Obravnavana je obtežba snega na strehi silosa, s karakteristično vrednostjo pritiska $q_s = 2\ kN/m^2$.

Celotna vertikalna sila, ki jo povzroča sneg je enaka:

$$F_s = q_s \cdot A = 2 \cdot 8.04 = 16\ kN \tag{II.34}$$

Osni sili na poljubni koordinati $z$ sta enaki:

$$\boxed{n_{z,sneg} = -\frac{F_s}{2\pi r} = -\frac{16}{2 \cdot \pi \cdot 1.6} = -1.6\ kN/m} \tag{II.35}$$

$$\boxed{n_{\phi,sneg} = 0\ kN/m} \tag{II.36}$$

## 9.4 Obtežba vetra

### 9.4.1 Vhodni podatki

Vetrna cona: *Cona A*

Kategorija terena: *IV*

Referenčna hitrost vetra: $v_{b,0} = 25\ m/s$.

Višina hrapavosti: $z_0 = 1.0\ m$.

Minimalna višina, kjer je hitrost vetra še konstantna: $z_{min} = 10\ m$.

Gostota zraka: $\rho = 1.25\ kg/m^3$.

### 9.4.2 Konični tlak vetra

Osnovni tlak vetra znaša:

$$q_b = 0.5 \rho {v_b}^2 = 0.5 \times 1.25 \times 25^2 = 391 N/m^2 = 0.39 kN/m^2 \qquad \text{(II.37)}$$

$v_b$ je osnovna hitrost vetra:

$$v_b = c_{dir}\, c_{season}\, v_{b,0} \qquad \text{(II.38)}$$

Simbola $c_{dir}$ in $c_{season}$ označujeta:

- $c_{dir}$ označuje koeficient smeri vetra (= *1.0*),
- $c_{season}$ označuje sezonski koeficient (= *1.0*).

Določimo sedaj še faktor izpostavljenosti. Za tubrulenčni faktor $k_I$ vzemimo:

$$k_I = 1.0 \qquad \text{(II.39)}$$

Faktor terena $k_r$ znaša:

$$k_r = 0.19 \cdot \left( \frac{z_0}{z_{0,II}} \right)^{0.07} = 0.19 \cdot \left( \frac{1.0}{0.05} \right)^{0.07} = 0.234 \qquad \text{(II.40)}$$

Za faktor topologije terena $c_0$ vzemimo:

$$c_0 = 1.0 \tag{II.41}$$

Faktor hrapavosti $c_r(z)$ znaša:

$$c_r( z = h_s ) = k_r \cdot ln\left( \frac{z}{z_0} \right) = 0.234 \cdot ln\left( \frac{19}{1.0} \right) = 0.69 \tag{II.42}$$

Simbol *hs* označuje celotno višino silosa (*Slika 39*).

$$h_s = 4.4 + 5.6 + 9 = 19\ m \tag{II.43}$$

Faktor izpostavljenosti znaša:

$$c_e = \left[ 1 + 7\frac{k_I k_r}{c_0 c_r} \right] \cdot c_0{}^2 c_r{}^2 = \left[ 1 + 7 \cdot \frac{1.0 * 0.234}{1.0 * 0.668} \right] \cdot 1.0^2 \cdot 0.69^2 = 1.61 \tag{II.44}$$

Sedaj lahko določimo konično tlak vetra:

$$q_p = c_e\, q_b = 1.54 * 0.39\ kN/m^2 = 0.63\ kN/m^2 \tag{II.45}$$

## 9.4.3 Poln silos

Celotna sila vetra na poln silos znaša

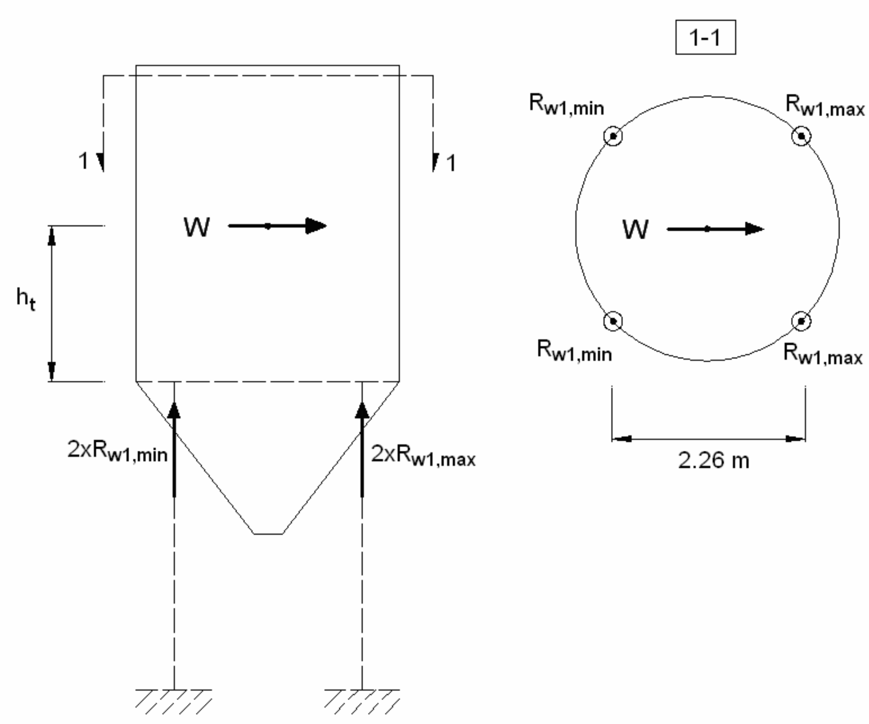
$$W = 0.372 \cdot \pi \cdot r \cdot h_b \cdot q_p$$

$$W = 0.372 \cdot \pi \cdot 1.6 \cdot 9 \cdot 0.63 = 10.6\ kN \tag{II.46}$$

### 9.4.3.1 Kontrola prevrnitve

Tukaj izračunamo povečanje reakcij v podporah na stiku med silosom in podporno konstrukcijo zaradi delovanja sile vetra na poln silos. Prevrnitev je prepečena, kadar so podpore sposobne prevzeti povečane reakcije. Delovanje vetra obravnavamo v dveh smereh.

## Smer 1:



**Slika 45: Smer vetra 1**

Višina $h_t$ znaša:

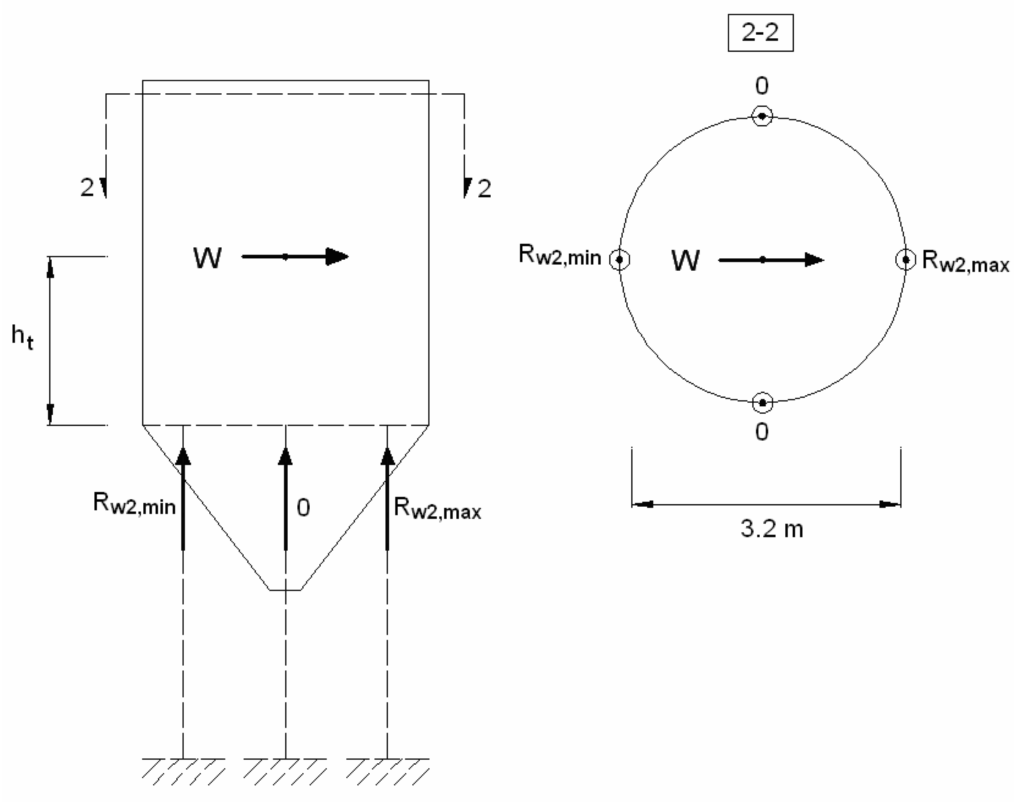
$$h_t = h_b / 2 = 9 / 2 = 4.5\,m \qquad \text{(II.47)}$$

Dodatne reakcije v podporah so enake:

$$R_{w1,max} = \frac{1}{2} \cdot \frac{|W| \cdot h_t}{2.26} = \frac{1}{2} \cdot \frac{10.6 \cdot 4.5}{2.26} = 10.55kN \qquad \text{(II.48)}$$

$$R_{w1,min} = -\frac{1}{2} \cdot \frac{|W| \cdot h_t}{2.26} = -\frac{1}{2} \cdot \frac{27 \cdot 4.5}{2.26} = -10.55kN \qquad \text{(II.49)}$$

## Smer 2:



Slika 46: Smer vetra 2

Dodatne reakcije v podporah so enake:

$$R_{w2,max} = \frac{|W| \cdot h_t}{3.2} = \frac{7.3 \cdot 4.5}{3.2} = 14.91\, kN \tag{II.50}$$

$$R_{w2,min} = -\frac{|W| \cdot h_t}{3.2} = -\frac{7.3 \cdot 4.5}{3.2} = -14.91 kN \tag{II.51}$$

Vidimo, da je merodajna smer vetra *Smer 2*, ker nam da večje dodatne reakcije v podporah.

### 9.4.3.2 Membranske sile zaradi upogiba cilindra

Cilinder nadomestimo s konzolo, na katero deluje enakomerna linijska obtežba, ki izhaja iz sile vetra.

Enakomerna linijska obtežba na konzolo znaša:

$$q_w = \frac{W}{d_c} = \frac{10.6}{3.2} = 3.31\ kN/m \tag{II.52}$$

Upogibni moment na globini $z$ je enak:

$$M(z) = q_w \cdot \frac{z^2}{2} = 1.66 \cdot z^2\ [kNm] \tag{II.53}$$

Notranji sili na poljubni globini $z$ znašata:

$$n_{z,wf}(z) = \frac{M(z)}{I} \cdot r \cdot t = \frac{1.66 \cdot z^2}{0.09067} \cdot 1.6 \cdot 0.007$$

$$\boxed{n_{z,wf}(z) = 0.205 \cdot z^2\ [kN/m]} \tag{II.54}$$

$$\boxed{n_{\phi,wf} = 0\ kN/m} \tag{II.55}$$

## 9.4.4 Prazen silos

### 9.4.4.1 Membranske sile za kontrolo mejnega stanja plastičnosti

Pri določitvi zunanjega pritiska vetra $w_e(\phi)$, upoštevamo funkcijo $c_{p0}(\phi)$, izračunano za 20 členov Fouriereve vrste (*Preglednica 17*). Maksimalna vrednost zunanjega pritiska se pojavi pri kotu $\phi = 90^\circ$ (*Slika 25*).

Enačbe za sili $n_{z,we}$, $n_{\phi,we}$ in za strižno silo $n_{z\phi,we}$, zaradi delovanja vetra na prazen cilinder, se za izbran primer glasijo:

$$n_{z,we}(z,\pi/2) = \frac{q_p}{2r} \cdot \left( \sum_{m=1}^{19} m^2\, c_m \cos m \cdot \frac{\pi}{2} \right) \cdot z^2$$

$$n_{z,we}(z,\pi/2) = \frac{0.63}{2 \cdot 1.6} \cdot \left( \sum_{m=1}^{19} m^2\, c_m \cos m \cdot \frac{\pi}{2} \right) \cdot z^2$$

$$\boxed{n_{z,we} = -0.89 \cdot z^2 \ [kN/m]} \tag{II.56}$$

$$n_{\phi,we}(z,\pi/2) = -\frac{q_p\, r}{2} \cdot \left( c_0 + 2 \sum_{m=1}^{19} c_m \cos m \cdot \frac{\pi}{2} \right)$$

$$n_{\phi,we}(z,\pi/2) = -\frac{0.63 \cdot 1.6}{2} \cdot \left( c_0 + 2 \sum_{m=1}^{19} c_m \cos m \cdot \frac{\pi}{2} \right)$$

$$\boxed{n_{\phi,we} = 2.24\ kN/m} \tag{II.57}$$

$$n_{z\phi,we}(z,\pi/2) = -q_p \cdot \left( \sum_{m=1}^{19} m\, c_m \sin m \cdot \frac{\pi}{2} \right) \cdot z$$

$$n_{z\phi,we}(z,\pi/2) = -0.63 \cdot \left( \sum_{m=1}^{19} m\, c_m \sin m \cdot \frac{\pi}{2} \right) \cdot z$$

$$\boxed{n_{z\phi,we} = 0.01 \cdot z \ [kN/m]} \tag{II.58}$$

### 9.4.4.2 Membranske sile za kontrolo mejnega stanja uklona

Brezdimenzijski parameter $\omega$ je enak:

$$\omega = \frac{h_c}{\sqrt{r \cdot t}} = \frac{9}{\sqrt{1.6 \cdot 0.007}} = 86.3 \tag{II.59}$$

Upoštevana je bila povprečna debelina stene ($t = 7\ mm$).

Robni pogoji: spodnji rob je zaradi ojačitvenega obroča togo vpet, zgornji rob je zaradi strehe členkasto podprt. Uklonski faktor zunanjega pritiska ja tako enak (*Slika 30*):

$$C_{\Theta} = 1.25 \tag{II.60}$$

Parameter $k_w$ je enak:

$$k_w = 0.46 \cdot \left( 1 + 0.1\sqrt{\frac{C_{\Theta}}{\omega} \cdot \frac{r}{t}} \right) = 0.46 \cdot \left( 1 + 0.1\sqrt{\frac{1.25}{86.3} \cdot \frac{1.6}{0.007}} \right) = 0.65 \tag{II.61}$$

Ekvivalentni osno simetrični pritisk je enak:

$$w_{eq} = k_w \cdot q_p = 0.65 \cdot 0.63 = 0.39\ kN/m^2 \tag{II.62}$$

Membranski sili zaradi obtežbe $w_{eq}$ znašata:

$$\boxed{n_{z,we}(z) = 0\ \ kN/m} \tag{I.63}$$

$$n_{\phi,we}(z) = -w_{eq} \cdot r$$

$$\boxed{n_{\phi,we}(z) = -0.39 \cdot 1.6 = -0.62\ kN/m} \tag{I.64}$$

## *9.5 Potresna obremenitev*

Za primerjavo bosta obravnavana oba seizmična modela, ki sta bila prestavljena v poglavju *5.1.1*. Pri prvem modelu je uporabljena metoda z vodoravnimi silami, pri drugem modelu pa je uporabljena modalna analiza s spektrom odziva.

### 9.5.1 Poln silos

### 9.5.1.1 1. seizmični model

#### *9.5.1.1.1 Razred pomembnosti*

Shranjen material je elektrofiltrski pepel, ki je ne strupen in nevnetljiv material. Porušitev silosa v primeru potresne obremenitve ne bi predstavljala velikega tveganja za življenja ljudi, kot tudi ne bi imela velikih negativnih učinkov na okolje.

To pomeni, da lahko za razred pomembnosti izberemo *Razred 1*. Kot faktor pomembnosti izberemo :

$$\gamma_I = 0.8 \qquad \text{(II.65)}$$

#### *9.5.1.1.2 Masa silosa pri potresni obremenitvi*

Masa stalnih obtežb $G$ je enaka masi lupine silosa in strehe silosa:

$$G = \frac{W_s + F_{p,r}}{g} = \frac{56 + 20}{9.81} = 7.6\,t \qquad \text{(II.66)}$$

Masa shranjenega materiala $m$ je enaka:

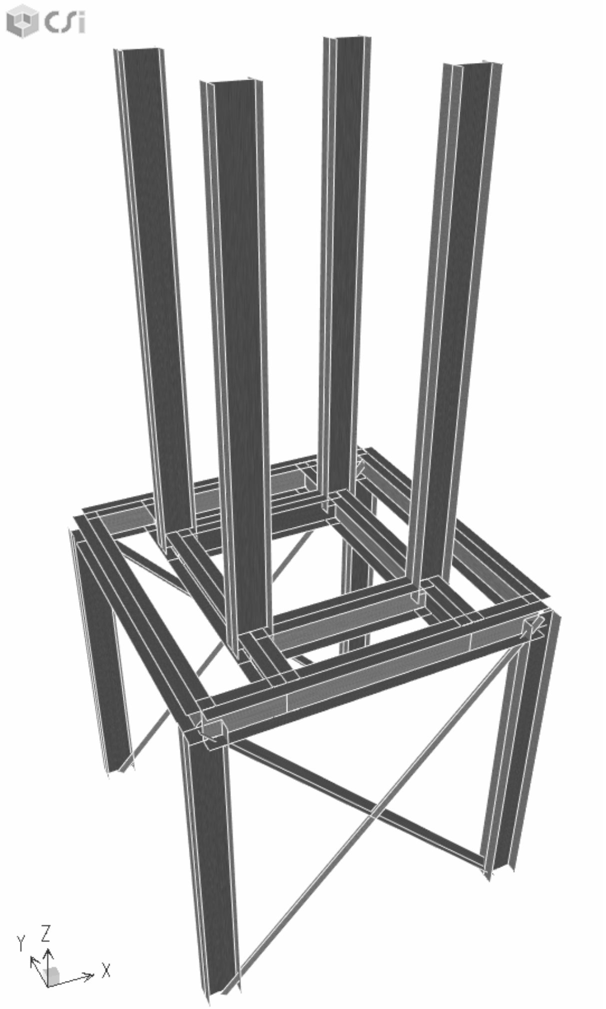
$$m = W_m / g = 1151 / 9.81 = 117.3\,t \qquad \text{(II.67)}$$

Celotna masa silosa pri potresni obremenitvi znaša:

$$m_{silo} = \gamma_I \left(G + 0.8\, m\right) = 0.8 \cdot \left(7.6 + 0.8 \cdot 117.3\right) = 81.15\, t \tag{II.68}$$

### *9.5.1.1.3 Togost podporne konstrukcije*

Togost podporne konstrukcije (*Slika 39*) je bila določena z računalniškim programom *CSi SAP2000*.



**Slika 47: Računalniški model podporne konstrukcije**

Rezultati so podani za vsako smer posebej:

smer x: $k_{vx} = 6666.7\, kN / m$, (II.69)

$k_{my} = 157133.9\, kNm / rad$. (II.70) smer y: $k_{vy} = 1760.6\,kN/m,$ (II.71)

$k_{mx} = 365363.5\,kNm/rad\,.$ (II.72)

### 9.5.1.1.4 Nihajni čas

Smer *x*:

$$\omega_x = \sqrt{\frac{6666.7}{81.15} \cdot \frac{157133.9 - 81.15 \cdot 9.81 \cdot 3.19}{157133.9 + 3.19 \cdot (3.19 \cdot 6666.7 - 81.15 \cdot 9.81)}} = 7.56/s \qquad \text{(II.73)}$$

$$\boxed{T_x = \frac{2\pi}{\omega_x} = \frac{2\pi}{4.55} = 0.83\,s} \qquad \text{(II.74)}$$

Smer *y*:

$$\omega_y = \sqrt{\frac{1760.6}{81.15} \cdot \frac{365363.5 - 81.15 \cdot 9.81 \cdot 3.19}{365363.5 + 3.19 \cdot (3.19 \cdot 1760.6 - 81.15 \cdot 9.81)}} = 4.55/s \qquad \text{(II.75)}$$

$$\boxed{T_y = \frac{2\pi}{\omega_y} = \frac{2\pi}{4.55} = 1.38\,s} \qquad \text{(II.76)}$$

### 9.5.1.1.5 Tip tal

Silos je temeljen na sloju savskega prodca, debeline *15 – 20 m* → tip tal *B*.

### 9.5.1.1.6 Projektni pospešek tal

Lokacija: *Ljubljana, Slovenija* → $a_{gR} = 0.2$.

Projektni pospešek tal za izbrano lokacijo znaša:

$$a_g = \gamma_I \cdot a_{gR} = \gamma_I \cdot 0.2 \cdot g = 0.8 \cdot 0.2 \cdot 9.81 = 1.57\,m/s^2 \qquad \text{(II.77)}$$

### *9.5.1.1.7 Razred duktilnosti*

Izberimo srednji razred duktilnosti (*DCM*). Ker je silos podpor na točkovnih podporah, znaša faktor obnašanja:

$$q = 1.25 \tag{II.78}$$

### *9.5.1.1.8 Projektni spekter pospeškov*

Glede na tip tal najprej določimo parametre spektra (*Preglednica 22*):

**Preglednica 39: Parametri, ki definirajo spekter pospeškov**

| S | $T_B$(s) | $T_C$(s) | $T_D$(s) |
|---|---|---|---|
| 1.2 | 0.15 | 0.5 | 2.0 |

**Mejno stanje nosilnosti:**

Smer *x:*

$T_C = 0.5\ s < T_x = 0.83\ s \leq T_D = 2\ s$

$$\rightarrow S_d(T_x) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C}{T_x}\right] = 1.57 \cdot 1.2 \cdot \frac{2.5}{1.25}\left[\frac{0.5}{0.83}\right] = 2.27\, m/s^2 \tag{II.79}$$

Smer y*:*

$T_C = 0.5\ s < T_y = 1.38\ s \leq T_D = 2\ s$

$$\rightarrow S_d(T_y) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C}{T_y}\right] = 1.57 \cdot 1.2 \cdot \frac{2.5}{1.25}\left[\frac{0.5}{1.38}\right] = 1.36\, m/s^2 \tag{II.80}$$

**Mejno stanje poškodb:**

Smer $x$:

$T_C = 0.5\ s < T_x = 0.83\ s \leq T_D = 2\ s$

$$\rightarrow S_d(T_x) = a_g \cdot S \cdot \frac{1.5}{q}\left[\frac{T_C}{T_x}\right] = 1.57 \cdot 1.2 \cdot \frac{1.5}{1.25}\left[\frac{0.5}{0.83}\right] = 1.36\ m/s^2 \qquad \text{(II.81)}$$

Smer $y$:

$T_C = 0.5\ s < T_y = 1.38\ s \leq T_D = 2\ s$

$$\rightarrow \quad S_d(T_y) = a_g \cdot S \cdot \frac{1.5}{q}\left[\frac{T_C}{T_y}\right] = 1.57 \cdot 1.2 \cdot \frac{1.5}{1.25}\left[\frac{0.5}{1.38}\right] = 0.60 m/s^2 \qquad \text{(II.82)}$$

Vidimo, da je smer $x$ vzbujanja bolj neugodna kot $y$, ker da večje vrednosti za $S_d$.

### 9.5.1.1.9 Celotna prečna sila

**Mejno stanje nosilnosti:**

Smer $x$: $F_{bx} = S_d(T_x) \cdot m_{silo} = 2.27 \cdot 81.15 = 184.2\,kN$ (II.83)

Smer $y$: $F_{by} = S_d(T_y) \cdot m_{silo} = 1.36 \cdot 81.15 = 110.4\,kN$ (II.84)

**Mejno stanje poškodb:**

Smer $x$: $F_{bx,DLS} = S_d(T_x) \cdot m_{silo} = 1.36 \cdot 81.15 = 110.36\ kN$ (II.85)

Smer $y$: $F_{by,DLS} = S_d(T_y) \cdot m_{silo} = 0.60 \cdot 81.15 = 48.7\,kN$ (II.86)

### *9.5.1.1.10 Dodatni normalni pritisk na steno cilindra*

Določimo ga na podlagi procedure, ki je podana v *Diagramu 4*:

$$R_s^* = min\left( h_c, \frac{d_c}{2} \right) = min(8.1, 1.6) = 1.6\,m \qquad \text{(II.87)}$$

$$\frac{R_s^*}{3} = \frac{1.6}{3} = 0.53\,m \qquad \text{(II.88)}$$

Ker predstavlja $\frac{R_s^*}{3} = 0.53\,m$ zgolj $6.5\%$ višine $h_c$, predpostavimo, da je pritisk $\Delta_{ph,s}(\phi)$ konstanten po celotni višini *x'* (od *0* do $h_c$). Enako predpostavimo tudi za lijak.

Maksimalna vrednost dodatnega pritiska $\Delta_{ph,so}$ je enaka:

$$\Delta_{ph,so} = \frac{Sd(T_1)}{g} \cdot \gamma_U \cdot R_s^* = \frac{2.27}{9.81} \cdot 15 \cdot 1.6 = 5.55\,kN/m^2 \qquad \text{(II.89)}$$

Dodatni normalni pritisk $\Delta_{ph,s}(\phi)$ je tako enak:

<u>Cilinder</u>: $\Delta_{ph,s}(\phi) = \Delta_{ph,so} \cdot cos\,\phi = 5.55 \cdot cos\,\phi\ kN/m^2$ (II.90)

Pri $\phi = 0°$: $\Delta_{ph,s} = 5.55\ kN/m^2$,

pri $\phi = 180°$: $\Delta_{ph,s} = -5.55\ kN/m^2$.

<u>Lijak</u>: $\Delta_{ph,s}(\phi) = \frac{\Delta_{ph,so} \cdot cos\,\phi}{cos\,\beta} = \frac{5.55 \cdot cos\,\phi}{cos\,20} = 5.9 \cdot cos\,\phi\ kN/m^2$ (II.91)

Pri $\phi = 0°$: $\Delta_{ph,s} = 5.9\ kN/m^2$,

pri $\phi = 180°$: $\Delta_{ph,s} = -5.9\ kN/m^2$.

### 9.5.1.2 2. seizmični model

Pri analizi 1. seizmičnega modela smo videli, da je bolj neugodna *x* smer vzbujanja, zato pri analizi tega modela upoštevamo samo *x* smer vzbujanja. Poglavja *9.5.1.1.1* do *9.5.1.1.3* in *9.5.1.1.5* do *9.5.1.1.8* veljajo tudi za ta model.

Celotna masa podporne konstrukcije znaša (izračunano s pomočjo računalniškega modela):

$$m_{sup,total} = 10\,t \tag{II.92}$$

Translacijska in rotacijska masa podporne konstrukcije za potresno analizo modela znašata:

$$m_{sup} = 0.5\, m_{sup,total} = 5\,t \tag{II.93}$$

$$\Theta_m = \frac{m_{sup,total} \cdot h_{sup}^{\;2}}{420} = \frac{10 \cdot (\,4.4 + 5.6\,)^2}{420} = 2.38\,t\,m^2 \tag{II.94}$$

#### *9.5.1.2.1 Nihajna časa*

Parametri *A*, *B*, *C* in *D*, ki so potrebni za izračun nihajnih časov, znašajo (enačba (I.223) do (I.226)):

$$A = 1.88395 \cdot 10^7 \tag{II.95}$$

$$B = 3.54926 \cdot 10^{14} \tag{II.96}$$

$$C = 1.78674 \cdot 10^{12} \tag{II.97}$$

$$D = 8668.14 \tag{II.98}$$

Vrednosti za $\omega$ so enake:

$$\omega_1 = \sqrt{\frac{1.88395 \cdot 10^7 - \sqrt{3.54926 \cdot 10^{14} - 1.78674 \cdot 10^{12}}}{8668.14}} = 7.44\,/\,s \tag{II.99}$$

$$\omega_2 = \sqrt{\frac{1.88395 \cdot 10^7 + \sqrt{3.54926 \cdot 10^{14} - 1.78674 \cdot 10^{12}}}{8668.14}} = 65.50\,/\,s \tag{II.100}$$

Nihajna časa *2. seizmičnega modela* sta enaka:

$$\boxed{T_1 = \frac{2\pi}{\omega_1} = \frac{2\pi}{5.55} = 0.84\ s} \tag{II.101}$$

$$\boxed{T_2 = \frac{2\pi}{\omega_2} = \frac{2\pi}{54.45} = 0.10\ s} \tag{II.102}$$

### 9.5.1.2.2 Projektni spekter pospeškov

Ker analiziramo *2. seizmični model* samo za namen primerjave s *1. seizmičnim modelom*, obravnavamo samo mejno stanje nosilnosti in *x* smer vzbujanja.

**Mejno stanje nosilnosti:**

$T_C = 0.5\ s < T_1 = 0.84\ s \leq T_D = 2\ s$

$$\rightarrow\ S_d(T_1) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C}{T_1}\right] = 1.57 \cdot 1.2 \cdot \frac{2.5}{1.25}\left[\frac{0.5}{0.84}\right] = 2.24\ m/s^2 \tag{II.103}$$

$T_2 = 0.10\ s < T_B = 0.15\ s$

$$\rightarrow\ S_d(T_2) = a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T_2}{T_B} \cdot \left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$$

$$S_d(T_2) = 1.57 \cdot 1.2 \cdot \left[\frac{2}{3} + \frac{0.10}{0.15} \cdot \left(\frac{2.5}{1.25} - \frac{2}{3}\right)\right] = 2.93\ m/s^2 \tag{II.104}$$

### *9.5.1.2.3 Nihajne oblike*

Vsaka nihajna oblika predstavlja enega od možnih načinov nihanja izbranega sistema. V vektorju $\{\phi_i\}$ predstavlja komponenta $\phi_{i1}$ pomik $u_v$, komponenta $\phi_{i2}$ pa predstavlja zasuk $\varphi$ *i*-te nihajne oblike (*Slika 34*).

**Prva nihajna oblika**

$$\phi_{11} = 1$$

$$\phi_{12} = \frac{k_v - (m_{silo} + m_{sup}) \cdot \omega_1^2}{m_{silo} \cdot h_m \cdot \omega_1^2} = \frac{3333.3 - (81.15 + 5) \cdot 7.44^2}{81.15 \cdot 3.19 \cdot 7.44^2} = 0.0855$$

$$\{\phi_1\} = \begin{Bmatrix} 1 \\ 0.1320 \end{Bmatrix} \tag{II.105}$$

**Druga nihajna oblika**

$$\phi_{21} = 1$$

$$\phi_{22} = \frac{k_v - (m_{silo} + m_{sup}) \cdot \omega_1^2}{m_{silo} \cdot h_m \cdot \omega_1^2} = \frac{3333.3 - (81.15 + 5) \cdot 65.50^2}{81.15 \cdot 3.19 \cdot 65.50^2} = -0.3268$$

$$\{\phi_2\} = \begin{Bmatrix} 1 \\ -0.3268 \end{Bmatrix} \tag{II.106}$$

### *9.5.1.2.4 Masna participacijska faktorja*

Te dva faktorja nam podajata delež določene nihajne oblike na nihanje sistema.

Ker imamo vzbujanje samo v $x$ smeri, je smerni vektor $\{s\}$ enak:

$$\{s\}=\begin{Bmatrix}1\\0\end{Bmatrix} \tag{II.107}$$

Masna matrika izbranega modela je enaka:

$$[M]=\begin{bmatrix}m_{silo}+m_{sup} & m_{silo}\cdot h_m\\ m_{silo}\cdot h_m & m_{silo}\cdot h_m^{\ 2}+\Theta_m\end{bmatrix}=\begin{bmatrix}81.15+5 & 81.15\cdot 3.19\\ 81.15\cdot 3.19 & 81.15\cdot 3.19^2+2.38\end{bmatrix} \tag{II.108}$$

$$[M]=\begin{bmatrix}81.15+5 & 81.15\cdot 3.19\\ 81.15\cdot 3.19 & 81.15\cdot 3.19^2+2.38\end{bmatrix}=\begin{bmatrix}86.15 & 258.87\\ 258.87 & 828.17\end{bmatrix} \tag{II.109}$$

Masna participacijska faktorja sta tako enaka:

**Prva nihajna oblika:** $$\Gamma_1=\frac{\{\phi_1\}^T\cdot[M]\cdot\{s\}}{\{\phi_1\}^T\cdot[M]\cdot\{\phi_1\}}=0.712 \tag{II.110}$$

**Druga nihajna oblika:** $$\Gamma_2=\frac{\{\phi_2\}^T\cdot[M]\cdot\{s\}}{\{\phi_2\}^T\cdot[M]\cdot\{\phi_2\}}=0.288 \tag{II.111}$$

### *9.5.1.2.5 Vektor potresnih sil*

Ker predstavljajo komponente vektorja posamezne nihajne oblike pomik $u_v$ in zasuk $\varphi$, predstavljajo komponente vektroja posplošenih potresnih sil $\{F_e\}$ silo $F_v$ v vodoravni vzmeti in moment $M_v$ v rotacijski vzmeti (*Slika 34*).

$$\{F_e\}=\begin{Bmatrix}F_v\\M_v\end{Bmatrix} \tag{II.112}$$

Posplošene potresne sile[8] znašajo:

**Prva nihajna oblika**

$$F_{e1,max} = [M]\cdot\{\phi_1\}\cdot\Gamma_1\cdot S_d(T_1) \qquad \text{(II.113)}$$

$$F_{e1,max} = \begin{bmatrix} 86.15 & 258.87 \\ 258.87 & 828.17 \end{bmatrix}\cdot\begin{Bmatrix} 1 \\ 0.1320 \end{Bmatrix}\cdot 0.793\cdot 1.67 = \begin{Bmatrix} 191.29\,kN \\ 585.37\,kNm \end{Bmatrix} \qquad \text{(II.114)}$$

**Druga nihajna oblika**

$$F_{e2,max} = [M]\cdot\{\phi_2\}\cdot\Gamma_2\cdot S_d(T_2) \qquad \text{(II.115)}$$

$$F_{e2,max} = \begin{bmatrix} 86.15 & 258.87 \\ 258.87 & 828.17 \end{bmatrix}\cdot\begin{Bmatrix} 1 \\ -0.3285 \end{Bmatrix}\cdot 0.207\cdot 3.27 = \begin{Bmatrix} 1.31\,kN \\ -9.92\,kNm \end{Bmatrix} \qquad \text{(II.116)}$$

Vektor potresnih sil se nato določi s kombiniranjem potresnih sil obeh nihajnih oblik. Uporabimo SRSS kombinacijo:

$$\{F_e\} = \sqrt{\{F_{e1,max}\}^2 + \{F_{e2,max}\}^2}$$

$$\{F_e\} = \sqrt{\begin{Bmatrix} 191.29 \\ 585.37 \end{Bmatrix}^2 + \begin{Bmatrix} 1.31 \\ -9.92 \end{Bmatrix}^2} = \begin{Bmatrix} 191.29\,kN \\ 585.45\,kNm \end{Bmatrix} \qquad \text{(II.117)}$$

### *9.5.1.2.6 Celotna prečna sila in celotna inercijska sila*

Celotna prečna sila je enaka sili v vodoravni vzmeti $F_v$, ki predstavlja prvo komponento vektorja potresnih sil $\{F_e\}$.

$$F_b = F_v = 191.29\,kN \qquad \text{(II.118)}$$

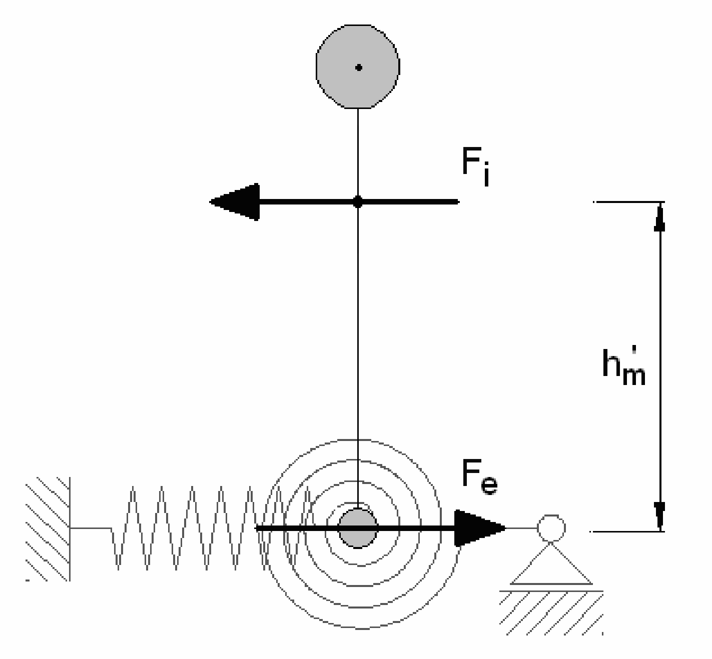
---

[8] Posplošena sila predstavlja silo ali moment.

Izbrani model ima dve inercijski sili: $F_{i1}$ in $F_{i2}$ (*Slika 34*). Skupna inercijska sila $F_i$ je enaka vsoti obeh posameznih inercijskih sil.

$$F_i = F_{i1} + F_{i2} \tag{II.119}$$

Njeno prijemališče se nahaja na višini $h_m^{'}$, v masnem središču obeh mas.



**Slika 48: Celotna prečna sila in celotna inercijska sila 2. seizmičnega modela**

$$h_m^{'} = \frac{m_{silo} \cdot h_m}{m_{silo} + m_{sup}} = \frac{85.15 \cdot 3.19}{85.15 + 5} = 3.05\,m \tag{II.120}$$

Po drugi strani pa mora biti celotna inercijska enaka celotni prečni sili:

$$F_i = F_b = 191.29\,kN\,, \tag{II.121}$$

oziroma mora biti enaka kovcientu momenta v rotacijski vzmeti $M_v$ (2. komponenta v vektorju potresnih sil) in ročice $h_m^{'}$:

$$F_b = \frac{M_v}{h_m^{'}} = \frac{585.45\,kNm}{3.05\,m} = 191.95\,kN \cong 191.29\,kN \tag{II.122}$$

### 9.5.1.3 Primerjava rezultatov obeh modelov

V prvem modelu smo imeli eno prostostno stopnjo in zato samo en (osnovni) nihajni čas $T_1$. V drugem modelu smo imeli dve prostostni stopnji in zato tudi dva nihajna časa. Primerjajmo samo osnovna nihajna časa obeh modelov.

Spodnja tabela povzema rezultate za oba modela:

**Preglednica 40: Primerjava obeh modelov**

| | **Model 1** | **Model 2** | **Razlika [%]** |
|---|---|---|---|
| **$T_1$ [s]** | 0.83 | 0.84 | **1.2** |
| **$F_b$ [kN]** | 184.2 | 191.3 | **3.8** |

Vidimo, da je razlika med obema osnovnima nihajnima časoma okoli *2%*, medtem ko je razlika med celotnima prečnima silama okoli *4%*. To pomeni, da razlika *2%* pride od 2. nihajne oblike *2. seizmičnega modela*.

Čeprav je prečna sila v prvem modelu manjša, je ta model na varni strani, ker imamo v tem primeru inercijsko silo, ki deluje na večji ročici, kot pri drugem modelu. Primerjajmo prevrnitvene momente, ki jih obe inercijski sili povzročata na dnu nadomestne konzole:

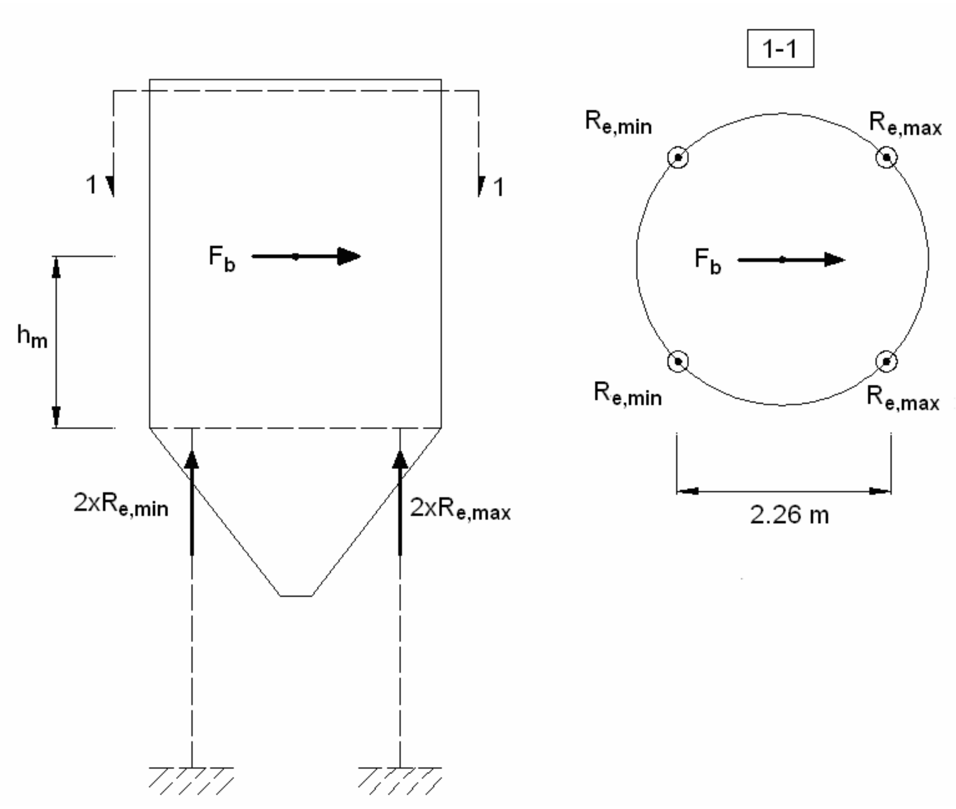
$$M_1 = F_{i1} \cdot h_m = 184.2 \cdot 3.19 = 587.6\,kNm$$

$$M_2 = F_{i2} \cdot h_m' = 191.3 \cdot 3.05 = 583.5\,kNm$$

Vidimo, da je $M_1$ večji kot $M_2$. To pomeni, da povzroča *1. seizmični model* večji prevrnitveni moment kot *2. seizmični model* in je zato na varni strani.

V nadaljnjih izračunih so uporabljeni samo rezultati, ki jih podaja *1. seizmični model*.

### 9.5.1.4 Kontrola prevrnitve



**Slika 49: Dodatne reakcije v podporah zaradi delovanje potresne obremenitve**

Povečanje reakcij v podporah silosa zaradi delovanja potresne obremenitve znaša:

$$Re,max = \frac{1}{2} \cdot \frac{F_b \cdot h_e}{2.26} = \frac{1}{2} \cdot \frac{184.2 \cdot 3.19}{2.26} = 130.0\ kN \qquad \text{(II.123)}$$

$$Re,min = -\frac{1}{2} \cdot \frac{F_b \cdot h_e}{2.26} = -\frac{1}{2} \cdot \frac{138 \cdot 3.19}{2.26} = -130.0\ kN \qquad \text{(II.124)}$$

### 9.5.1.5 Membranske sile

Membranske sile v polnem silosu pri potresni obremenitvi povzroča dodatni normalni pritisk $\Delta_{ph,s}(\phi)$, ki deluje na steno cilindra in lijaka.

**Cilinder:**

**Preglednica 41: Membranske sile zaradi potresne obremenitve na poln cilinder**

| | | Ekstremne vrednosti sil | | | |
|---|---|---|---|---|---|
| **Sestavni del** | **z [m]** | $n_{z,efs}$ **[kN/m]** | $n_{\phi,efs}$ **[kN/m]** | $n_{z,efs}$ **[kN/m]** | $n_{\phi,efs}$ **[kN/m]** |
| 1 | 3 | 15.61 | -8.88 | -15.61 | 8.88 |
| 2 | 5 | 43.36 | -8.88 | -43.36 | 8.88 |
| 3 | 7.5 | 84.98 | -8.88 | -84.98 | 8.88 |

**Lijak:**

**Preglednica 42: Membranske sile zaradi potresne obremenitve na poln lijak**

| | | Ekstremne vrednosti sil | | | |
|---|---|---|---|---|---|
| **Sestavni del** | **x [m]** | $n_{s,efs}$ **[kN/m]** | $n_{\phi,efs}$ **[kN/m]** | $n_{s,efs}$ **[kN/m]** | $n_{\phi,efs}$ **[kN/m]** |
| 4 | 3.4 | 5.97 | -7.31 | -5.97 | 7.31 |
| 5 | 4.4 | 7.73 | -9.46 | -7.73 | 9.46 |

## 9.5.2 Prazen silos

### 9.5.2.1 Vhodni podatki za računalniško analizo

Analiza potresne obremenitve praznega silosa je bila izvedena z uporabo računalniškega programa *CSi SAP2000*. Uporabljena je bila modalna analiza s spektrom odziva.

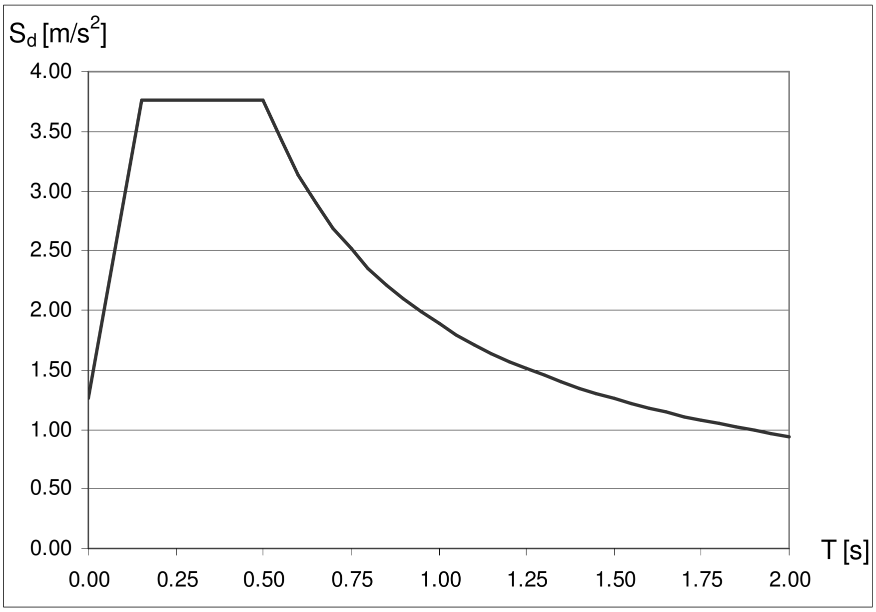
Spekter je bil določen na podlagi parametrov, ki so že bili izračunani v poglavju *9.5.1.1*. Vhodni podatki za spekter so zbrani v spodnji preglednici.

**Preglednica 43: Vhodni podatki za določitev spektra pospeškov**

| | |
|---|---|
| **q** | 1.25 |
| **S** | 1.20 |
| **$a_g$ [m/s$^2$]** | 1.57 |

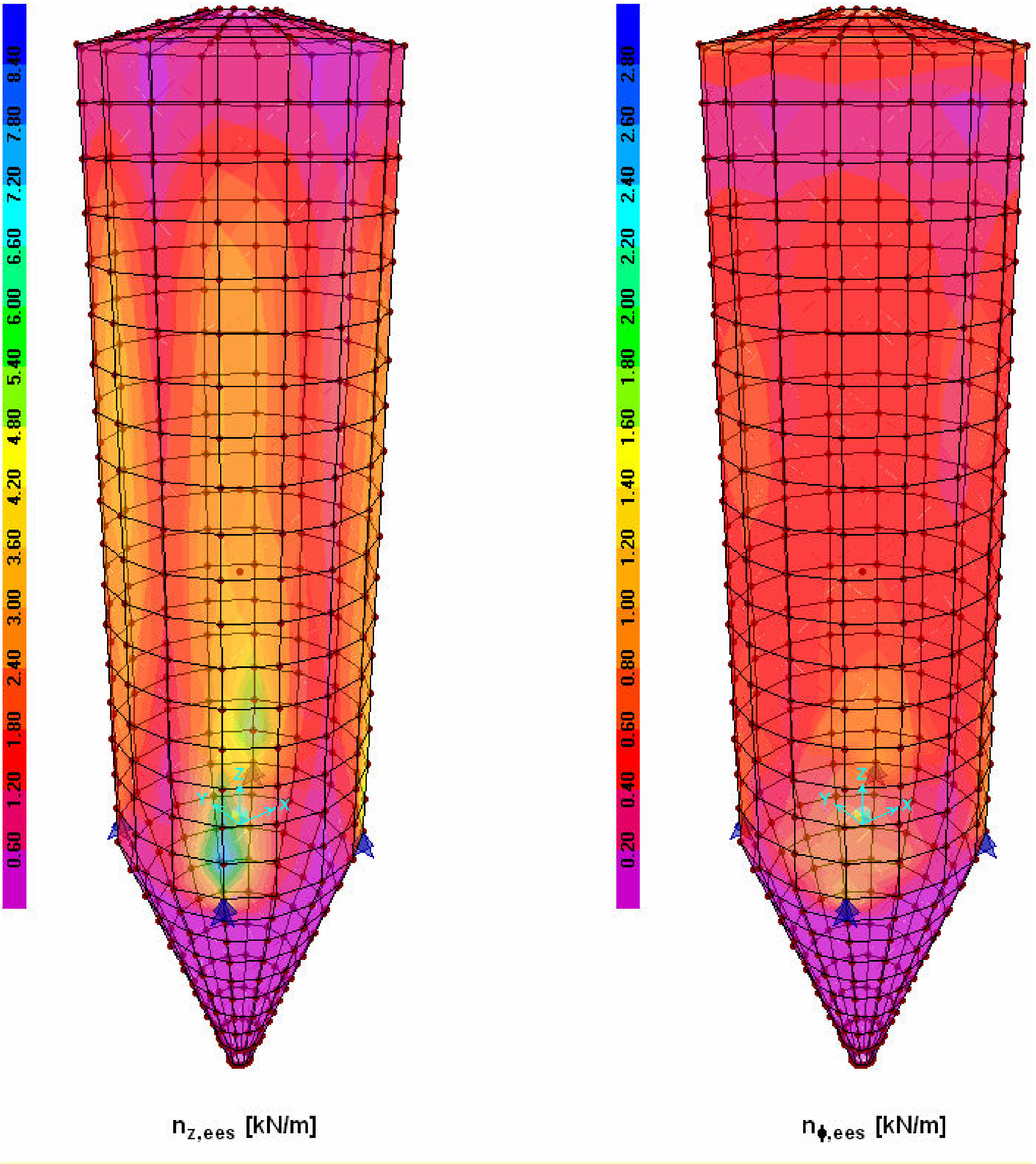
| | |
|---|---|
| **$T_B$ [s]** | 0.15 |
| **$T_C$ [s]** | 0.50 |
| **$T_D$ [s]** | 2.00 |

Spekter pospeškov je bil nato izračunan z enačbami, ki jih podaja *Preglednica 24*.



**Slika 50: Projektni spekter za modalno analizo praznega silosa**

### 9.5.2.2 Membranske sile dobljene s pomočjo računalniške analize



**Slika 51: Membranske sile na prazen silos pri delovanju potresne obremenitve**

## Cilinder:

**Preglednica 44: Membranske sile zaradi potresne obremenitve na prazen cilinder**

| | | Ekstremne vrednosti sil | | | |
|---|---|---|---|---|---|
| **Sestavni del** | **z [m]** | **$n_{z,ees}$ [kN/m]** | **$n_{\phi,ees}$ [kN/m]** | **$n_{z,ees}$ [kN/m]** | **$n_{\phi,ees}$ [kN/m]** |
| 1 | 3 | 3.04 | *0.54* | - 3.04 | *-0.54* |
| 2 | 5 | 3.06 | *0.55* | - 3.06 | *-0.55* |
| 3 | 7.5 | 6.35 | *0.90* | - 6.35 | *-0.90* |

## Lijak:

**Preglednica 45: Membranske sile zaradi potresne obremenitve na prazen lijak**

| | | Ekstremne vrednosti sil | | | |
|---|---|---|---|---|---|
| **Sestavni del** | **x [m]** | **$n_{s,ees}$ [kN/m]** | **$n_{\phi,ees}$ [kN/m]** | **$n_{s,ees}$ [kN/m]** | **$n_{\phi,ees}$ [kN/m]** |
| 4 | 3.4 | 0.51 | -0.08 | 0.51 | 0.08 |
| 5 | 4.4 | 4.30 | -1.35 | -4.30 | 1.35 |

# 10 Projektne vrednosti notranjih sil

V spodnjih preglednicah so podane projektne vrednosti membranskih notranjih sil, izračunane po ustreznih kombinacijah (poglavji *7.1.1* in *7.2.1*).

V prvi vrstici, vsake izmed kombinacij, je podana maksimalna projektna vrednost sile $n_{zd}$, skupaj s pripadajočo projektno vrednostjo sile $n_{\phi d}$. V drugi vrstici, vsake izmed kombinacij, je podana maksimalna projektna vrednost sile $n_{\phi d}$, skupaj s pripadajočo projektno vrednostjo sile $n_{zd}$. Pripadajoče vrednosti so v obeh premerih označene s poševno ležečim tekstom.

Sestavne dele stene silosa ter potek koordinat $z$ in $x$ prikazuje *Slika 40*.

Kombinacija *CO4*, ki zajema vpliv vetra na prazen silos, je razdeljena na dve podkombinaciji:

- *CO4a*, ki zajema membranske sile zaradi vetra, za kontrolo nosilnosti v mejnem stanju plastičnosti (poglavje *9.4.4.1*),
- *CO4b*, ki zajema membranske sile zaradi vetra, za kontrolo mejnega stanja uklona (poglavje *9.4.4.2*).

# *10.1 Mejno stanje nosilnosti*

## 10.1.1 Cilinder

**Preglednica 46: Projektne vrednosti membranskih sil v cilindru za del 1**

| **Sestavni del 1, z = 3 m** | | **Projektna vrednost membranske sile** | | | |
|---|---|---|---|---|---|
| | | **Maksimum** | | **Minimum** | |
| **Obtežna kombinacija** | | $n_{zd}$ | $n_{\phi d}$ | $n_{zd}$ | $n_{\phi d}$ |
| CO1 | Praznjenje materiala | -38.90 | *33.74* | Simetrično | |
| | | *-39.90* | 33.74 | | |
| CO2 | Sneg | -34.65 | *30.89* | | |
| | | *-32.37* | *33.09* | | |
| CO3 | Veter – poln silos | -31.56 | *30.89* | -34.88 | *30.89* |
| | | *-29.27* | 33.09 | *-32.59* | 33.09 |
| CO4a | Veter – prazen silos (nosilnost) | -13.48 | *2.02* | Simetrično | |
| | | *-13.48* | 2.02 | | |
| CO4b | Veter – prazen silos (uklon) | -6.27 | *-0.62* | | |
| | | *-6.27* | -0.62 | | |
| CO6 | Potres - poln silos | -7.32 | *9.82* | -30.72 | *23.14* |
| | | *-6.10* | 10.99 | *-29.50* | 24.31 |
| CO7 | Potres - prazen silos | -4.5 | *0.6* | -4.8 | *-0.6* |
| | | *-4.5* | 0.6 | *-4.8* | -0.6 |
| | | [kN/m] | [kN/m] | [kN/m] | [kN/m] |

| **Ekstremna vrednost** | **Kombinacija** | $n_{zd}$ | $n_{\phi d}$ | |
|---|---|---|---|---|
| **min** $n_{zd}$ | **CO1** | **-39.9** | ***33.7*** | [kN/m] |
| **min** $n_{\phi d}$ | **CO4** | ***-6.3*** | **-0.6** | [kN/m] |

| | | | | |
|---|---|---|---|---|
| **max** $n_{zd}$ | **CO6** | **-6.1** | ***11.0*** | [kN/m] |
| **max** $n_{\phi d}$ | **CO1** | ***-32.6*** | **33.7** | [kN/m] |

**Preglednica 47: Projektne vrednosti membranskih sil v cilindru za del 2**

| Sestavni del 2, z = 5 m | | Projektna vrednost membranske sile | | | |
|---|---|---|---|---|---|
| | | Maksimum | | Minimum | |
| Obtežna kombinacija | | $n_{zd}$ | $n_{\phi d}$ | $n_{zd}$ | $n_{\phi d}$ |
| CO1 | Praznjenje materiala | -76.07 | *40.60* | Simetrično | |
| | | *-76.07* | 40.60 | | |
| CO2 | Sneg | -68.93 | *37.18* | | |
| | | *-65.01* | 40.95 | | |
| CO3 | Veter – poln silos | -62.89 | *37.18* | -72.11 | *37.18* |
| | | *-58.97* | 40.95 | *-68.19* | 40.95 |
| CO4a | Veter – prazen silos (nosilnost) | -27.72 | *2.02* | Simetrično | |
| | | *-27.72* | 2.02 | | |
| CO4b | Veter – prazen silos (uklon) | -7.70 | *-0.62* | | |
| | | *-7.70* | -0.62 | | |
| CO6 | Potres - poln silos | -5.10 | *13.17* | -70.10 | *26.49* |
| | | *-3.01* | 15.18 | *-68.01* | 28.50 |
| CO7 | Potres - prazen silos | -5.3 | *0.6* | -6.1 | *-0.6* |
| | | *-5.3* | 0.6 | *-6.1* | -0.6 |
| | | [kN/m] | [kN/m] | [kN/m] | [kN/m] |

| Ekstremna vrednost | Kombinacija | $n_{zd}$ | $n_{\phi d}$ | |
|---|---|---|---|---|
| **min $n_{zd}$** | **CO1** | **-76.1** | ***40.6*** | [kN/m] |
| **min $n_{\phi d}$** | **CO4** | ***-7.7*** | **-0.6** | [kN/m] |

| | | | | |
|---|---|---|---|---|
| **max $n_{zd}$** | **CO6** | **-6.1** | ***-0.6*** | [kN/m] |
| **max $n_{\phi d}$** | **CO1** | ***-68.2*** | **41.0** | [kN/m] |

**Preglednica 48: Projektne vrednosti membranskih sil v cilindru za del 3**

| **Sestavni del 3, z = 7 m** | | **Projektna vrednost membranske sile** | | | |
|---|---|---|---|---|---|
| | | **Maksimum** | | **Minimum** | |
| **Obtežna kombinacija** | | $n_{zd}$ | $n_{\phi d}$ | $n_{zd}$ | $n_{\phi d}$ |
| CO1 | Praznjenje materiala | -103.07 | *54.99* | Simetrično | |
| | | *-95.70* | 52.56 | | |
| CO2 | Sneg | -106.76 | *41.54* | | |
| | | *-101.79* | 46.54 | | |
| CO3 | Veter – poln silos | -96.29 | *41.54* | -114.37 | *43.59* |
| | | *-91.32* | 46.54 | *-109.40* | 48.84 |
| CO4a | Veter – prazen silos (nosilnost) | -48.38 | *2.02* | Simetrično | |
| | | *-48.38* | 2.02 | | |
| CO4b | Veter – prazen silos (uklon) | -9.13 | *-0.62* | | |
| | | *-9.13* | -0.62 | | |
| CO6 | Potres - poln silos | 5.63 | *15.50* | -143.36 | *29.91* |
| | | *8.28* | 18.16 | *-119.12* | 32.71 |
| CO7 | Potres - prazen silos | -0.41 | *1.18* | -13.11 | *-1.18* |
| | | *-0.41* | 1.18 | *-13.11* | -1.18 |
| | | [kN/m] | [kN/m] | [kN/m] | [kN/m] |

| **Ekstremna vrednost** | **Kombinacija** | $n_{zd}$ | $n_{\phi d}$ | |
|---|---|---|---|---|
| **min** $n_{zd}$ | **CO1** | **-143.4** | ***29.9*** | [kN/m] |
| **min** $n_{\phi d}$ | **CO4** | ***-13.1*** | **-1.2** | [kN/m] |

| | | | | |
|---|---|---|---|---|
| **max** $n_{zd}$ | **CO6** | **-0.41** | ***1.18*** | [kN/m] |
| **max** $n_{\phi d}$ | **CO1** | ***-109.4*** | **48.8** | [kN/m] |

### 10.1.2 Lijak

**Preglednica 49: Projektne vrednosti membranskih sil v lijaku za del 4**

| **Sestavni del 4, x = 4.4 m** | | **Projektna vrednost membranske sile** | | | |
|---|---|---|---|---|---|
| | | **Maksimum** | | **Maksimum** | |
| **Obtežna kombinacija** | | **$n_{sd}$** | **$n_{\phi d}$** | **$n_{sd}$** | **$n_{\phi d}$** |
| CO1 | Praznjenje materiala | 90.74 | *65.85* | Simetrično | |
| | | *90.73* | 65.85 | | |
| CO6 | Potres - poln silos | 54.39 | *50.60* | 42.81 | *64.78* |
| | | *54.39* | 50.60 | *42.81* | 64.78 |
| CO7 | Potres - prazen silos | 5.05 | *-1.35* | -3.55 | *1.35* |
| | | *5.05* | -1.35 | *-3.55* | 1.35 |
| | | [kN/m] | [kN/m] | [kN/m] | [kN/m] |

| **Ekstremna vrednost** | **Kombinacija** | **$n_{sd}$** | **$n_{\phi d}$** | |
|---|---|---|---|---|
| **max $n_{sd}$** | **CO6** | **90.7** | ***65.9*** | [kN/m] |
| **max $n_{\phi d}$** | **CO1** | ***90.7*** | **65.9** | [kN/m] |

**Preglednica 50: Projektne vrednosti membranskih sil v lijaku za del 5**

| **Sestavni del 5, x = 3.4 m** | | **Projektna vrednost membranske sile** | | | |
|---|---|---|---|---|---|
| | | **Maksimum** | | **Maksimum** | |
| **Obtežna kombinacija** | | **$n_{sd}$** | **$n_{\phi d}$** | **$n_{sd}$** | **$n_{\phi d}$** |
| CO1 | Praznjenje materiala | 72.65 | *57.56* | Simetrično | |
| | | *72.65* | 57.56 | | |
| CO6 | Potres - poln silos | 34.96 | *30.18* | 26.00 | *41.14* |
| | | *34.96* | 30.18 | *26.00* | 41.14 |
| CO7 | Potres - prazen silos | 4.88 | *-1.35* | -3.72 | *1.35* |
| | | *4.88* | -1.35 | *-3.72* | 1.35 |
| | | [kN/m] | [kN/m] | [kN/m] | [kN/m] |

| **Ekstremna vrednost** | **Kombinacija** | **$n_{sd}$** | **$n_{\phi d}$** | |
|---|---|---|---|---|
| **max $n_{sd}$** | **CO1** | **72.7** | ***57.6*** | [kN/m] |
| **max $n_{\phi d}$** | **CO1** | ***72.7*** | **57.6** | [kN/m] |

### 10.1.3 Podpore silosa

Kot podpore silosa smatramo točke, preko katerih je silos vpet v podporno konstrukcijo.

#### 10.1.3.1 Reakcije v podporah

Uporabimo reakcije iz kontrol prevrnitve.

Lastna teža: $$R_{sw} = \frac{W_s + W_r}{4} = \frac{56+20}{4} = 19\ kN \qquad \text{(II.125)}$$

Shranjen material: $$R_{ss} = \frac{W_m}{4} = \frac{1151}{4} = 288\ kN \qquad \text{(II.126)}$$

Teža snega: $$R_s = \frac{F_s}{4} = \frac{16}{4} = 4\ kN \qquad \text{(II.127)}$$

Sila vetra na poln silos: $$R_w = R_{w2,max} = 38\ kN \qquad \text{(II.128)}$$

Potresna sila na poln silos: $$R_e = R_{e,max} = 130\ kN \qquad \text{(II.129)}$$

#### 10.1.3.2 Obtežne kombinacije

**CO1 (Praznjenje materiala)**:

$$R_{C1} = 1.35 \cdot R_{sw} + 1.5 \cdot R_{ss} + 1.5 \cdot 0.6 \cdot (R_s + R_w)$$

$$R_{C1} = 1.35 \cdot 19 + 1.5 \cdot 288 + 1.5 \cdot 0.6 \cdot (4 + 38) = 495.5\ kN \qquad \text{(II.130)}$$

**CO6 (Potres – poln silos)**:

$$R_{C2} = 1.0 \cdot R_{sw} + 0.8 \cdot R_{ss} + R_e$$

$$R_{C2} = 1.0 \cdot 19 + 0.8 \cdot 288 + 130 = 379.4\ kN \qquad \text{(II.131)}$$

→ Vidimo, da je prevladujoča kombinacija *CO1*.

## 10.2 Mejno stanje uporabnosti

V mejnem stanju uporabnosti bosta izračunani projektna vrednost sile vetra in potresne sile, ki se bosta uporabili za kontrolo pomikov.

Projektna vrednost sile vetra, ki deluje na cilinder silosa, se izračuna po kombinaciji *CO8*, ki jo podaja *Preglednica 28.*

$$W_d = 1.0 \cdot 0 + 0.9 \cdot 0 + 0.6 \cdot W$$

$$W_d = 0.6 \cdot 7.3 = 4.38\, kN \qquad \text{(II.132)}$$

Projektna vrednost potresne sile se izračuna po kombinaciji *CO9*, ki jo podaja *Preglednica 29.*

$$F_{ed} = 1.0 \cdot G + 0.8 \cdot Q_f + Q_{ef,DLS}$$

$$F_{ed} = 1.0 \cdot 0 + 0.8 \cdot 0 + F_{b,DLS}$$

Smer $x$: $F_{ed,x} = 110.36\, kN$ (II.133)

Smer $y$: $F_{ed,y} = 48.7\, kN$ (II.134)

# 11 Dimenzioniranje

V tem poglavju je predstavljeno samo dimenzioniranje sten silosa. Dimenzioniranje podporne konstrukcije ni predstavljeno.

## 11.1 Sestava stene silosa

| | | |
|---|---|---|
| Material: | Jeklo, *S275* | |
| Napetost na meji tečenja: | $f_{yk} = 27.5\, kN / cm^2$ | (II.135) |
| Varnostni faktor za material: | $\gamma_{M0} = 1.0$ | (II.136) |

Projektna vrednost napetosti na meji tečenja: $f_{yd} = \frac{f_{yk}}{\gamma_{M0}} = \frac{27.5}{1.0} = 27.5\ kN/cm^2$ (II.137)

Modul elastičnosti: $E = 21000\ kN/cm^2$ (II.138)

## 11.2 Dimenzioniranje cilindra

Ker so debeline sten cilindra že izbrane, moramo preveriti, če so stene cilindra, pri danih debelinah, sposobne prenesti zunanjo obremenitev. Uporabljene so kontrole, ki so definirane v *prEN 1993-1-6:2004*, *Poglavje 6*. Kratek opis teh kontrol se nahaja v poglavju *7.1.2*.

Kontrole so izvedene samo za sestavni del *1*, z debelino stene *t = 3 mm* (*Slika 40*). Po enakih postopkih je potrebno preveriti tudi sestavna dela *2* in *3*.

### 11.2.1 Kontrola mejnega stanja plastičnosti (LS1)

Maksimalna vertikalna sila in pripadajoča obročna sila znašata:

$$n_{zd} = -39.9\ kN/m \tag{II.139}$$

$$n_{\phi d} = 33.7\ kN/m \tag{II.140}$$

Ekvivalentna napetost znaša:

$$\sigma_{eq,Ed} = \frac{1}{t}\sqrt{{n_{zd}}^2 + {n_{\phi d}}^2 - n_{zd} \cdot n_{\phi d}}$$

$$\sigma_{eq,Ed} = \frac{1}{0.003}\sqrt{(-39.9)^2 + 33.7^2 - (-39.9) \cdot 33.7} = 21301\ kN/m^2$$

$$\sigma_{eq,Ed} = 2.13\ kN/cm^2 \tag{II.141}$$

Kontrola napetosti:

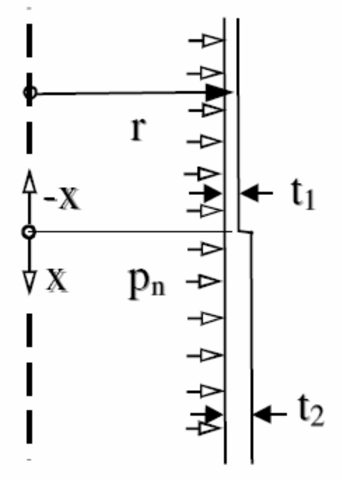
$$\sigma_{eq,Ed} = 2.13\ kN / cm^2 \leq f_{yd} = 27.5\ kN / cm^2 \qquad \text{(II.142)}$$

→ Kontrola napetosti je izpolnjena.

### 11.2.2 Kontrola mejnega stanje ciklične plastičnosti (LS2)

Kontrolo izvedemo na stiku med *1.* in *2.* sestavnim delom. Predpostavimo, da je vodoravni pritisk na steno v bližini stika obe delov konstanten.

Uporabljen je postopek, določen v *prEN 1993-1-6*, *Poglavju 7.4* in *Prilogi C*.



$t_1 = 3\ mm$

$t_2 = 5\ mm$

**Slika 52: Robni pogoji za 1. del pri kontroli ciklične plastičnosti**

Kot vrednost $p_n$ je vzeta karakteristična vrednost vodoravnega pritiska pri praznjenju materiala $p_{he}$, za kombinacijo parametrov *2*, na globini $z = 3\ m$:

$$p_n = p_{he}(\ Kombinacija\ 2; z = 3\ ) = 16.64\ kN / m^2 \qquad \text{(II.143)}$$

$$\frac{t_1}{t_2} = \frac{3}{5} = 0.6 \cong 0.667\ \rightarrow\ k_{eq,m} = 0.815 \qquad \text{(II.144)}$$

Maksimalna von Misesova ekvivalentna napetost je enaka:

$$\Delta\sigma_{eq,Ed} = k_{eq,m} \cdot p_n \cdot \frac{r}{t_1} = 0.815 \cdot 16.64 \cdot \frac{1.6}{0.003}$$

$$\Delta\sigma_{eq,Ed} = 7233\ kN/m^2 = 0.72\ kN/cm^2 \tag{II.145}$$

Kontrola napetosti:

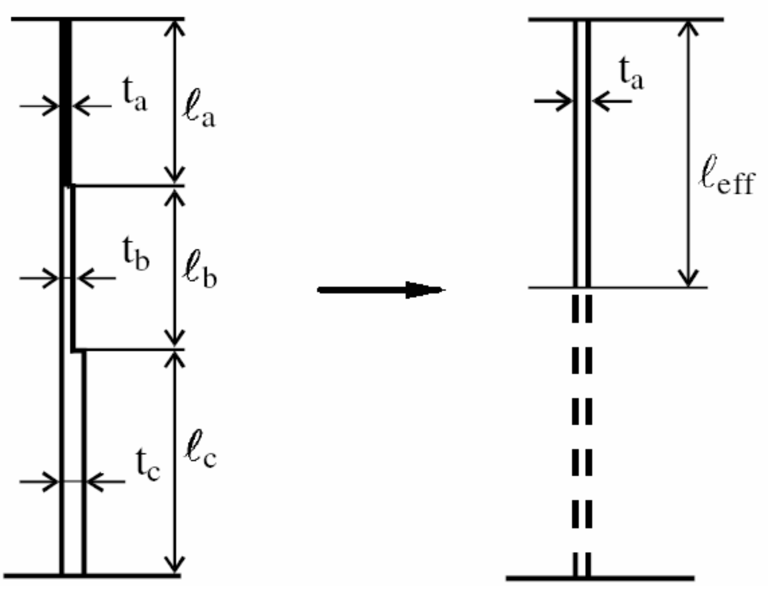
$$\Delta\sigma_{eq,Ed} = 0.72\ kN/cm^2 \leq \Delta f_{eq,Rd} = 55\ kN/cm^2 \tag{II.146}$$

→ Kontrola napetosti je izpolnjena .

### 11.2.3 Kontrola uklona (LS3)

Uporabljen je postopek, določen v *prEN 1993-1-6:2004*, *Poglavju 8.5 ; Prilogi D*.

Spremenljivo debelino stene nadomestimo z nadomestnim cilindrom, s stalno debelino stene.



$l_a = 4\ m$, $t_a = 3\ mm$

$l_b = 2\ m$, $t_b = 5\ mm$

$l_c = 2\ m$, $t_c = 8\ mm$

**Slika 53: Nadomestni cilinder s konstantno debelino stene**

Iz *prEN 1993-1-6:2004, Slika D.6*, dobimo:

$$\frac{l_a}{l} = \frac{4}{9} = 0.44 \quad ; \quad \frac{t_b}{t_a} = \frac{5}{3} = 1.67 \quad ; \quad \frac{t_c}{t_a} = \frac{8}{3} = 2.67$$

→ Faktor efektivne dolžine znaša: $\kappa = 0.95$ (II.147)

Efektivna dolžina nadomestnega cilindra je enaka:

$$l_{eff} = \frac{l_a}{\kappa} = \frac{4}{0.95} = 4.21\ m \qquad \text{(II.148)}$$

**Kritična vertikalna uklonska napetost**

Brezdimenzijski parameter $\omega$:

$$\omega = \frac{l_{eff}}{\sqrt{r \cdot t}} = \frac{4.21}{\sqrt{1.6 \cdot 0.003}} = 60.77 \qquad \text{(II.149)}$$

$$1.7 \le \omega = 37.21 \le 0.5\frac{r}{t} = 0.5\frac{1.6}{0.003} = 266.7 \rightarrow C_x = 1.0 \qquad \text{(II.150)}$$

Kritična vertikalna uklonska napetost:

$$\sigma_{z,Rcr} = 0.605 \cdot E \cdot C_x \cdot \frac{t}{r} = 0.605 \cdot 21000 \cdot 1.0 \cdot \frac{0.003}{1.6} = 23.8\ kN \qquad \text{(II.151)}$$

Relativna vitkost:

$$\bar{\lambda}_z = \sqrt{\frac{f_{yk}}{\sigma_{z,Rcr}}} = \sqrt{\frac{27.5}{23.8}} = 1.07 \qquad \text{(II.152)}$$

Kakovostni razred izdelave:

$$Razred\ B \qquad \rightarrow \qquad Q = 25 \qquad \text{(II.153)}$$

Amplituda nepopolnosti:

$$\Delta w_k = \frac{1}{Q}\sqrt{\frac{r}{t}}\, t = \frac{1}{25}\sqrt{\frac{1600}{5}} \cdot 5 = 2.77\ mm \qquad \text{(II.154)}$$

Meridianski faktor nepopolnosti:

$$\alpha_x = \frac{0.62}{1 + 1.91\left(\Delta w_k / t\right)^{1.44}} = \frac{0.62}{1 + 1.91\left(4.53 / 5\right)^{1.44}} = 0.229 \qquad \text{(II.155)}$$

Faktor plastičnosti:

$$\beta = 0.6 \tag{II.156}$$

Mejna plastična vitkost:

$$\bar{\lambda}_p = \sqrt{\frac{\alpha_x}{1-\beta}} = \sqrt{\frac{0.229}{1-0.6}} = 0.76 \tag{II.157}$$

Faktor redukcije napetosti $\chi$ znaša:

$$\bar{\lambda}_z = 1.07 \geq \bar{\lambda}_p = 0.76 \rightarrow$$

$$\chi = \frac{\alpha_x}{\bar{\lambda}_z^{\,2}} = \frac{0.23}{1.07^2} = 0.19 \tag{II.158}$$

Odpornost na uklon znaša:

$$\sigma_{z,Rd} = \frac{\chi \cdot f_{yk}}{\gamma_{M1}} = \frac{0.19 \cdot 27.5}{1.1} = 4.97\, kN / cm^2 \tag{II.159}$$

Maksimalna vertikalna membranska sila in napetost:

$$n_{zd} = -40.0\ kN / m \tag{II.160}$$

$$\sigma_{z,Ed} = \frac{|n_{zd}|}{t} = \frac{40.0}{0.003} = 13333.3\ kN / m^2 = 1.33\ kN / cm^2 \tag{II.161}$$

Kontrola napetosti:

$$\sigma_{z,Ed} = 1.33\ kN / cm^2 \leq \sigma_{z,Rd} = 4.97\, kN / cm^2 \tag{II.162}$$

→ Pogoj napetosti je izpolnjen.

## Kritična radialna uklonska napetost:

Kritična radialna uklonska napetost je enaka:

$$\sigma_{\phi,Rcr} = 0.92 \cdot E \cdot \frac{C_\Theta}{\omega} \cdot \frac{t}{r}$$

$$\sigma_{\phi,Rcr} = 0.92 \cdot 21000 \cdot \frac{1.0}{37.21} \cdot \frac{0.003}{1.6} = 0.60\ kN/cm^2 \qquad \text{(II.163)}$$

Parameter relativne vitkosti:

$$\bar{\lambda}_\phi = \sqrt{\frac{f_{yk}}{\sigma_{\phi,Rcr}}} = \sqrt{\frac{27.5}{0.60}} = 6.79 \qquad \text{(II.164)}$$

Kakovostni razred izdelave:

$$Razred\ B \quad \rightarrow \quad \alpha_\Theta = 0.65 \qquad \text{(II.165)}$$

Redukcijski faktor za uklon v radialni smeri:

$$\bar{\lambda}_\phi = 6.79 > \bar{\lambda}_p = 0.76$$

$$\rightarrow \chi_\phi = \frac{\alpha_\Theta}{\bar{\lambda}_\phi^{\,2}} = \frac{0.65}{6.79^2} = 0.014 \qquad \text{(II.166)}$$

Odpornost na uklon v radialni smeri:

$$\sigma_{\phi,Rd} = \frac{\chi_\phi \cdot f_{yk}}{\gamma_{M1}} = \frac{0.014 \cdot 27.5}{1.1} = 0.35\ kN/cm^2 \qquad \text{(II.167)}$$

Maksimalna obročna tlačna sila in napetost:

$$n_{\phi d} = -0.6\ kN/m \qquad \text{(II.168)}$$

$$\sigma_{\phi,Ed} = \frac{|n_{\phi d}|}{t} = \frac{0.6}{0.003} = 200.0\ kN/m^2 = 0.02\ kN/cm^2 \qquad \text{(II.169)}$$

Kontrola napetosti:

$$\sigma_{\phi,Ed} = 0.02\ kN/cm^2 \leq \sigma_{\phi,Rd} = 0.35\,kN/cm^2 \qquad \text{(II.170)}$$

→ Kontrola napetosti je izpolnjena.

## Kontrola interakcije obeh napetosti:

Kritična kombinacija membranskih sil:

$$n_{zd} = -6.3\ kN/m \tag{II.171}$$

$$n_{\phi d} = -0.6\ kN/m \tag{II.172}$$

Kritični napetosti:

$$\sigma_{z,Ed} = \frac{|n_{zd}|}{t} = \frac{6.3}{0.003} = 0.21\ kN/cm^2 \tag{II.173}$$

$$\sigma_{\phi,Ed} = \frac{|n_{\phi d}|}{t} = \frac{0.6}{0.003} = 0.02\ kN/cm^2 \tag{II.174}$$

Interakcijski parametri:

$$k_z = 1.25 + 0.75 \cdot \chi = 1.40 \tag{II.175}$$

$$k_\phi = 1.25 + 0.75 \cdot \chi_\phi = 1.26 \tag{II.176}$$

$$k_i = \chi \cdot \chi_\phi = 7.83 \cdot 10^{-6} \tag{II.177}$$

Kontrola interakcije:

$$\left(\frac{\sigma_{z,Ed}}{\sigma_{z,Rd}}\right)^{k_z} + \left(\frac{\sigma_{\phi,Ed}}{\sigma_{\phi,Rd}}\right)^{k_\phi} - k_i \cdot \left(\frac{\sigma_{z,Ed}}{\sigma_{z,Rd}}\right) \cdot \left(\frac{\sigma_{z,Ed}}{\sigma_{z,Rd}}\right) \leq 1$$

$$0.039 \leq 1 \tag{II.178}$$

→ Kontrola interakcije je izpolnjena.

### 11.2.4 Kontrola utrujanja (LS4)

| | |
|---|---|
| Življenjska doba konstrukcije: | *30 let.* |
| Frekvenca polnjenja in praznjenja: | *Silos se popolnoma napolni in izprazni približno 1 krat na teden (52 krat na leto).* |

Število ciklov polnjenja in praznjenja v življenjski dobi:
*N = 30 let · 52 ciklov / leto = 1560 ciklov.*

Ker je število ciklov *N* manjše od *10 000*, utrujanja ni potrebno upoštevati.

## 11.3 Dimenzioniranje lijaka

Podobno kot pri cilindru, so kontrole izvedene samo za sestavni del *4*, z debelino stene *t = 6 mm* (*Slika 40*). Po enakih postopkih je potrebno preveriti tudi sestavni del *5*.

### 11.3.1 Kontrola mejnega stanje plastičnosti (LS1)

Maksimalna vertikalna sila in pripadajoča obročna sila:

$$n_{sd} = 90.7\ kN/m \tag{II.179}$$

$$n_{\phi d} = 65.9\ kN/m \tag{II.180}$$

Ekvivalentna napetost:

$$\sigma_{eq,Ed} = \frac{1}{t}\sqrt{n_{sd}^{\ 2} + n_{\phi d}^{\ 2} - n_{sd} \cdot n_{\phi d}}$$

$$\sigma_{eq,Ed} = \frac{1}{0.006}\sqrt{90.7^2 + 65.9^2 - 90.7 \cdot 65.9} = 22697\ kN/m^2$$

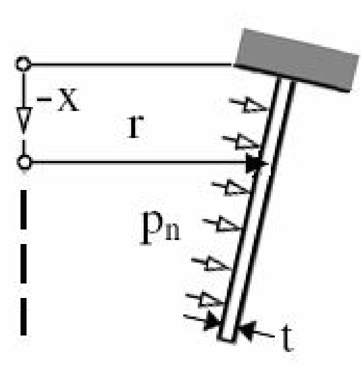
$$\sigma_{eq,Ed} = 2.27\ kN/cm^2 \tag{II.181}$$

Kontrola napetosti:

$$\sigma_{eq,Ed} = 2.27\ kN/cm^2 \leq f_{yd} = 27.5\ kN/cm^2 \qquad \text{(II.182)}$$

→ Kontrola napetosti je izpolnjena.

## 11.3.2 Kontrola mejnega stanja ciklične plastičnosti (LS2)



**Slika 54: Robni pogoj za 4. del**

Za vrednost $p_n$ je vzeta karakteristična vrednost normalnega pritiska pri praznjenju $p_{ne}$ (Kombinacija parametrov 5), na višini $x = 4.4\ m$:

$$p_n = p_{ne}(\,Case\,5;\,x = 4.4\,) = 25.87\ kN/m^2 \qquad \text{(II.183)}$$

Maksimalna von Misesova ekvivalentna napetost je enaka:

$$\Delta\sigma_{eq,Ed} = 1.614 \cdot p_n \cdot \frac{r}{t} = 1.614 \cdot 25.87 \cdot \frac{1.6}{0.006}$$

$$\Delta\sigma_{eq,Ed} = 11134\ kN/m^2 = 1.11\,kN/cm^2 \qquad \text{(II.184)}$$

Odpornost materiala je enaka:

$$\Delta f_{eq,Rd} = 2 \cdot f_{yd} = 55\ kN/cm^2 \qquad \text{(II.185)}$$

Kontrola napetosti:

$$\Delta\sigma_{eq,Ed} = 1.11\ kN/cm^2 \leq \Delta f_{eq,Rd} = 55\ kN/cm^2 \qquad \text{(II.186)}$$

→ Kontrola napetosti je izpolnjena.

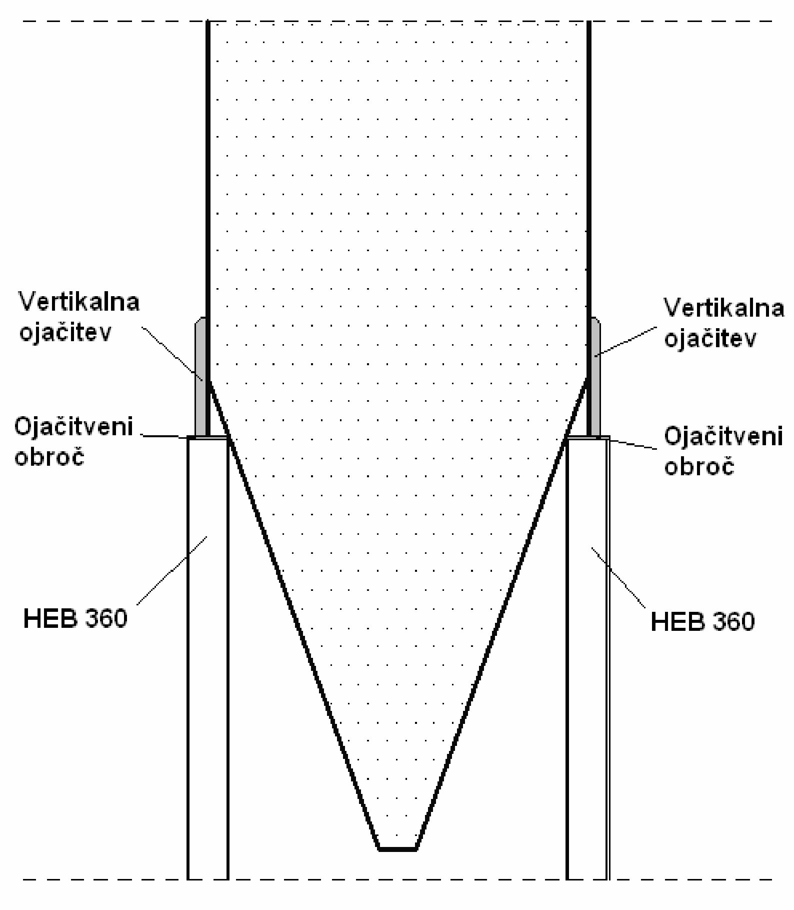
### 11.3.3 Kontrola uklona (LS3)

Ker so vse notranje sile v lijaku natezne (pozitivne), do pojava uklona ne more priti.

### 11.3.4 Kontrola utrujanja (LS4)

Utrujanja ni potrebno preverjati (poglavje *11.2.4.*)

## 11.4 Dimenzioniranje ojačitev



**Slika 55: Stik med silosom in podporno konstrukcijo**

### 11.4.1 Ojačitveni obroč

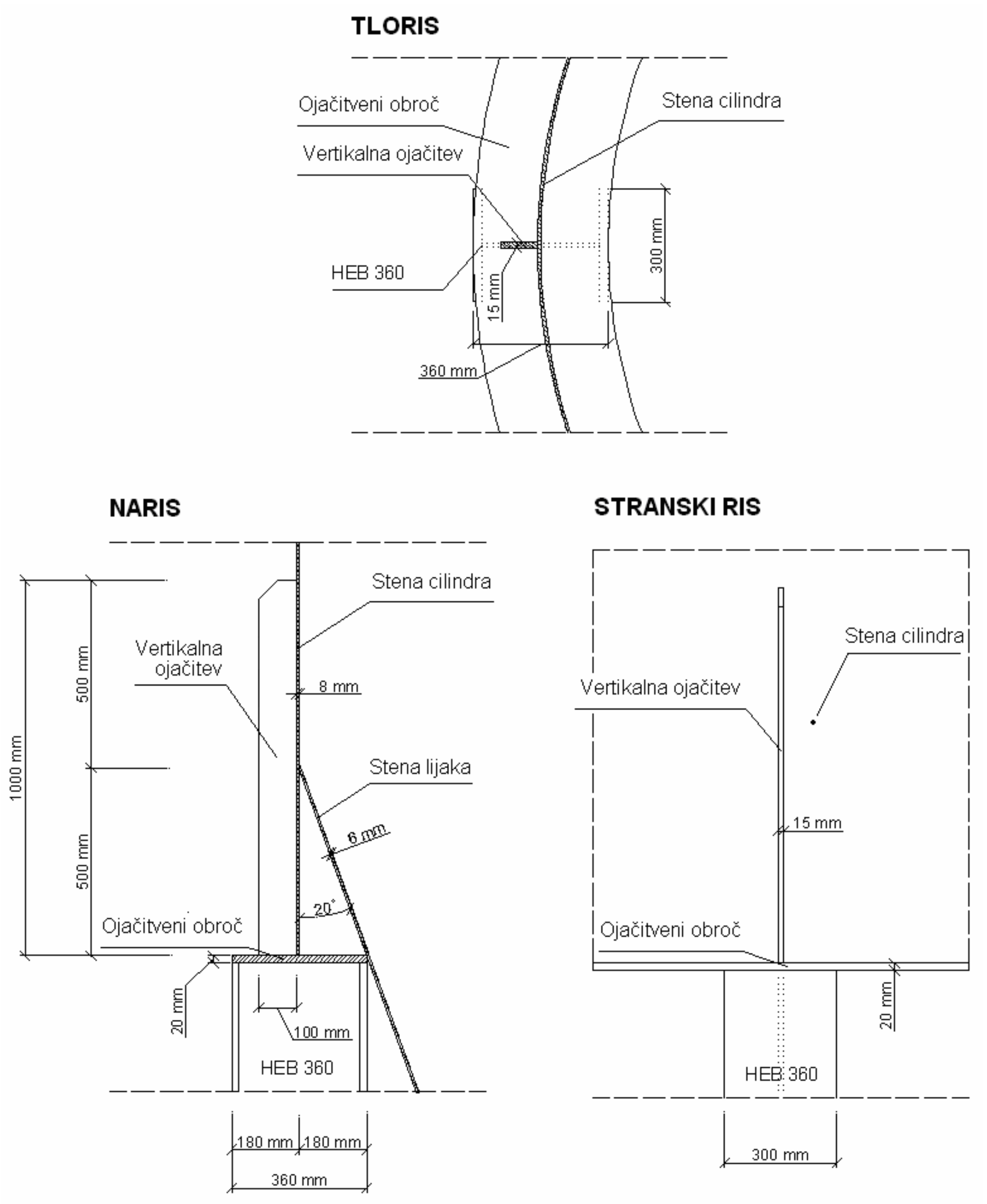
Ojačitveni obroč služi prerazporeditvi obremenitve iz silosa v podporno konstrukcijo. Ker je širina stebra *HEB 360* enaka *360 mm*, izberemo ojačitveni obroč dimenzij ***360 x 20 mm***. Cilinder se dotika ojačitvenega obroča na razdalji *180 mm* od notranjega roba obroča (*Slika 56*). Notranji radij je tako enak:

$$r_s = 1600\,mm - 180\,mm = 1420\,mm \qquad \text{(II.187)}$$

## 11.4.2 Vertikalne ojačitve

Izberimo dimenzije vertikalnih ojačitev:

$$b_V / t_V = 100 / 15\ mm = 10 / 1.5\ cm \qquad \text{(II.188)}$$



**Slika 56: Detajl ojačitvenega obroča in vertikalnih ojačitev**

**Kontrola varnosti proti uklonu:**

Uporabljena je procedura iz *EN 1993-1-6:2004*, *Poglavje 8.5* in *Priloga D*.

Predpostavimo, da je redukcijski uklonski faktor $\chi$ enak:

$$\chi_a = 0.60 \tag{II.189}$$

Potreben prerez vertikalne ojačitve znaša:

$$A_{req} = \frac{R_{c1}}{\chi_a \cdot f_{yd}} = \frac{495.5}{0.60 \cdot 27.5} = 30.3\ cm^2 \cong 30\ cm^2 \tag{II.190}$$

Širina podpor:

$$\text{Steber } HEB\ 360 \rightarrow \text{širina } d = 30\ cm \tag{II.191}$$

Debelina cilindra ob vertikalnih ojačitvah:

$$t = 8\ mm = 0.8\ cm \tag{II.192}$$

Dejanski prečni prerez:

$$A_{act} = d \cdot t + b_V \cdot t_V = 30 \cdot 0.8 + 10 \cdot 1.5 = 39\ cm^2 \tag{II.193}$$

Kontrola velikosti prečnega prereza:

$$A_{req} = 30\ cm^2 \le A_{act} = 39\ cm^2 \tag{II.194}$$

Maksimalna napetost v vertikalni ojačitvi**:**

$$\sigma_{z,Ed} = \frac{R_{C1}}{A_{act}} = \frac{495.5}{39} = 12.7\ kN/cm^2 \tag{II.195}$$

Uporabljena je bila maksimalna sila v podporah $R_{C1}$, enačba (II.130).

Brezdimenzijski parameter dolžine $\omega$:

$$\omega = \frac{l}{\sqrt{r \cdot t}} = \frac{1}{\sqrt{1.6 \cdot 0.008}} = 8.84 \tag{II.196}$$

$$1.7 \le \omega = 8.84 \le 0.5\frac{r}{t} = 0.5\frac{1.6}{0.008} = 100$$

$$\rightarrow\ C_x = 1.0 \tag{II.197}$$

Kritična uklonska napetost:

$$\sigma_{z,Rcr} = 0.605 \cdot 21000 \cdot 1.0 \cdot \frac{0.008}{1.6} = 63.5\ kN / cm^2 \tag{II.198}$$

Relativna vitkost:

$$\bar{\lambda}_z = \sqrt{\frac{27.5}{63.5}} = 0.66 \tag{II.199}$$

Razred kakovosti izdelave:

$$Razred\ B \qquad \rightarrow \qquad Q = 25 \tag{II.200}$$

Amplituda nepopolnosti:

$$\Delta w_k = \frac{1}{25}\sqrt{\frac{1600}{8}} \cdot 8 = 4.53\ mm \tag{II.201}$$

Faktor nepopolnosti:

$$\alpha_x = \frac{0.62}{1 + 1.91(4.53/8)^{1.44}} = 0.337 \tag{II.202}$$

Faktor plastičnosti:

$$\beta = 0.6 \tag{II.203}$$

Vitkost na meji plastičnosti:

$$\bar{\lambda}_p = \sqrt{\frac{\alpha_x}{1-\beta}} = \sqrt{\frac{0.337}{1-0.6}} = 0.92 \tag{II.204}$$

Redukcijski faktor $\chi$ je enak:

$$\bar{\lambda}_z = 0.66 \le \bar{\lambda}_p = 0.92 \rightarrow$$

$$\chi = 1 - \beta \cdot \frac{\bar{\lambda}_z - 0.2}{\bar{\lambda}_p - 0.2} = 1 - 0.6 \cdot \frac{0.66 - 0.2}{0.92 - 0.2} = 0.62 \qquad \text{(II.205)}$$

Odpornost vertikalne ojačitve proti uklonu:

$$\sigma_{z,Rd} = \frac{\chi \cdot f_{yk}}{\gamma_{M1}} = \frac{0.62 \cdot 27.5}{1.1} = 15.50\,kN/cm^2 \qquad \text{(II.206)}$$

Kontrola napetosti:

$$\sigma_{z,Ed} = 12.7\ kN/cm^2 \le \sigma_{z,Rd} = 15.50\,kN/cm^2 \qquad \text{(II.207)}$$

→ Kontrola napetosti je izpolnjena – do uklona vertikalne ojačitve ne pride.

## 11.5 Kontrola pomikov

Maksimalna vrednost horizontalnega pomika na vrhu silosa:

$$w_{max} = 0.02 \cdot (\,9\,m + 4.4\,m + 5.6\,m\,) = 0.38\,m = 38\,cm \qquad \text{(II.208)}$$

### 11.5.1.1 Kontrola pomika zaradi delovanja vetra

Projektna vrednot sile vetra je bila izračunana v poglavju *10.2*. Maksimalni vodoravni pomik na vrhu silosa v smeri *x*, zaradi delovanja vetra znaša:

$$w_{Wx} = W_d \left( \frac{1}{k_{vx}} + \frac{h_b{}^2}{2\,k_{my}} \right)$$

$$w_{Wx} = 4.38 \cdot \left( \frac{1}{6666.7} + \frac{9^2}{2 \cdot 157133.9} \right)$$

$$w_{Wx} = 0.0018\,m = 1.8\,mm \qquad \text{(II.209)}$$

Maksimalni vodoravni pomik na vrhu silosa v smeri *x*, zaradi delovanja vetra znaša:

$$w_{Wy} = W_d \left( \frac{1}{k_{vy}} + \frac{h_b{}^2}{2\,k_{mx}} \right)$$

$$w_{Wy} = 4.38 \cdot \left( \frac{1}{1760.6} + \frac{9^2}{2 \cdot 365363.5} \right)$$

$$w_{Wy} = 0.003\,m = 3.0\,mm \qquad \text{(II.210)}$$

Kontrola pomikov:

$$w_{Wy} = 0.30\ cm \le w_{max} = 38\ cm \qquad \text{(II.211)}$$

→ Kontrola pomikov je izpolnjena.

### 11.5.1.2 Kontrola pomikov zaradi delovanja potresne obremenitve

Projektna vrednot sile vetra je bila izračunana v poglavju *10.2*. Maksimalna vrednost pomika na v smeri *x*, na vrhu silosa, zaradi delovanja potresa znaša:

$$w_{Fe,x} = F_{ed,x} \cdot q \cdot \left( \frac{1}{k_{vx}} + \frac{h_b^{\,2}}{2\,k_{my}} \right)$$

$$w_{Fe,x} = 110.36 \cdot 1.25 \cdot \left( \frac{1}{6666.7} + \frac{9^2}{2 \cdot 157133.9} \right)$$

$$w_{Fe,x} = 0.056\,m = 5.6\,cm \tag{II.212}$$

Maksimalna vrednost pomika na v smeri *y*, na vrhu silosa, zaradi delovanja potresa znaša:

$$w_{Fe,y} = F_{ed,y} \cdot q \cdot \left( \frac{1}{k_{vy}} + \frac{h_b^{\,2}}{2\,k_{mx}} \right)$$

$$w_{Fe,y} = 48.7 \cdot 1.25 \cdot \left( \frac{1}{1760.6} + \frac{9^2}{2 \cdot 365363.5} \right)$$

$$w_{Fe,y} = 0.041 m = 4.1 cm \tag{II.213}$$

Kontrola pomikov:

$$w_{Fe,y} = 5.6\ cm \le w_{max} = 38\ cm \tag{II.214}$$

→ Kontrola pomikov je izpolnjena.

# III. DEL - RAČUNALNIŠKA PROGRAMA

## 12 Program »Stored Solid Load on Cylindrical Silos«

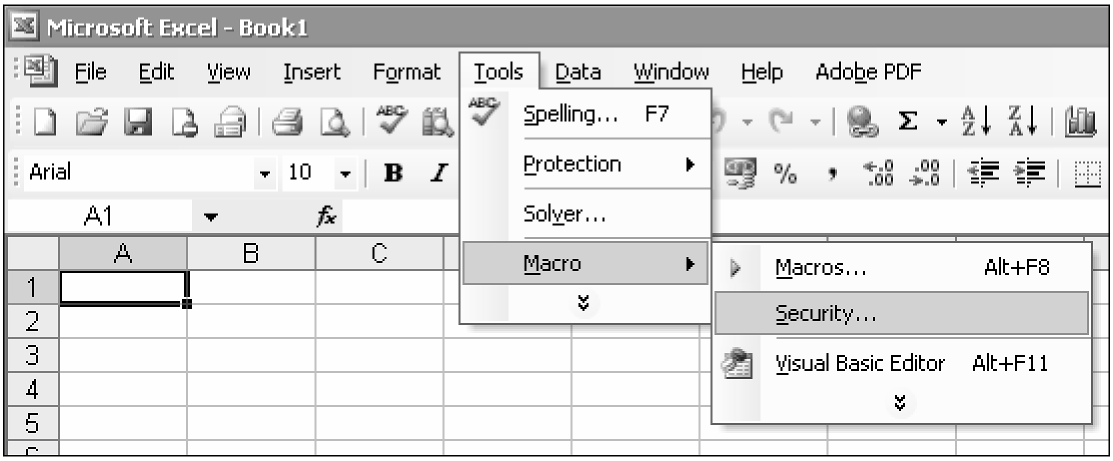
Program omogoča izračun osno simetričnih pritiskov in notranjih sil, ki se pojavijo v silosu med polnjenjem in praznjenjem shranjenega materiala. Na voljo je v angleškem jeziku. Veljajo predpostavke, ki so podane v poglavju *1.3*. Program pri izračunu uporablja postopek, ki je predstavljen v poglavju *3.1.1*. Notranje sile so izračunane po izrazih, ki so podani v poglavju *3.3.1*.

### *Zagon programa*

Program se nahaja na zgoščenki, ki je priložena na zadnji platnici. Zažene se ga tako, da se odpre datoteko z imenom »*Stored Solid Load on Cylindrical Silos.xls*«.

Za zagon programa potrebujete *Microsoft Excel 2003* (ali novejši).

Da bo program deloval, morajo biti v programu *Excel* omogočeni makroji. Njihova varnost mora biti nastavljena na *Srednjo* (ang. »*Medium*«) oz. *Nizko* (ang. »*Low*«) stopnjo. To storimo tako, da odpremo *Excel* in izberemo meni *Tools/Macro/Security*. V novem okno nato nastavimo varnost na »*Medium*« ali »*Low*« in nato zaženemo program.



Slika 57: Spreminjanje varnostnih nastavitev za makroje v programu Microsoft Excel 2003

○ Very High. Only macros installed in trusted locations will be allowed to run. All other signed and unsigned macros are disabled.

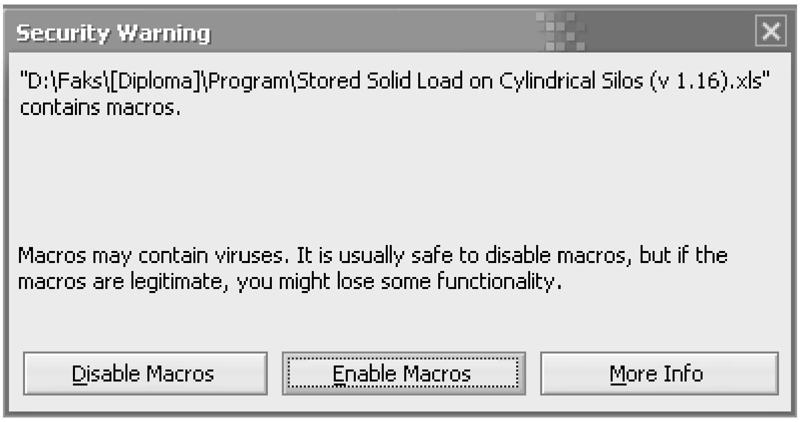
○ High. Only signed macros from trusted sources will be allowed to run. Unsigned macros are automatically disabled.

● Medium. You can choose whether or not to run potentially unsafe macros.

○ Low (not recommended). You are not protected from potentially unsafe macros. Use this setting only if you have virus scanning software installed, or you have checked the safety of all documents you open.

Slika 58: Nastavitev varnosti za makroje

Če je varnost makrojev nastavljena na srednjo (»*Medium*«) stopnjo, se bo ob odpiranju programa odprlo pogovorno okno, na katerem moramo izbrati možnost »*Enable macros*«.



"D:\Faks\[Diploma]\Program\Stored Solid Load on Cylindrical Silos (v 1.16).xls" contains macros.

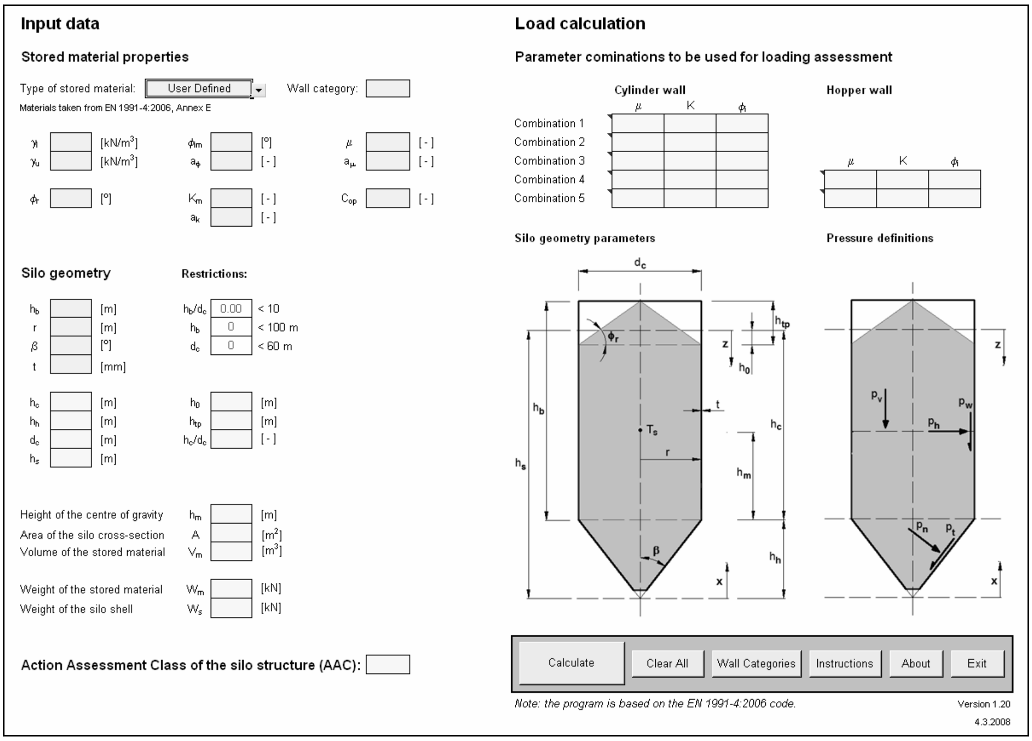
Macros may contain viruses. It is usually safe to disable macros, but if the macros are legitimate, you might lose some functionality.

Slika 59: Pogovorno okno za omogočanje makrojev

## Uporaba programa

### Vnos podatkov

Ko se program odpre, se pojavi delovni list za vnos podatkov. Tukaj vnesemo vse podatke, ki jih potrebujemo pri izračunu. Ustrezne podatke je potrebno vnesti samo v celice pobarvane z *zeleno* barvo.



Type of stored material: User Defined

$h_b/d_c$ 0.00 < 10
$h_b$ 0 < 100 m
$d_c$ 0 < 60 m

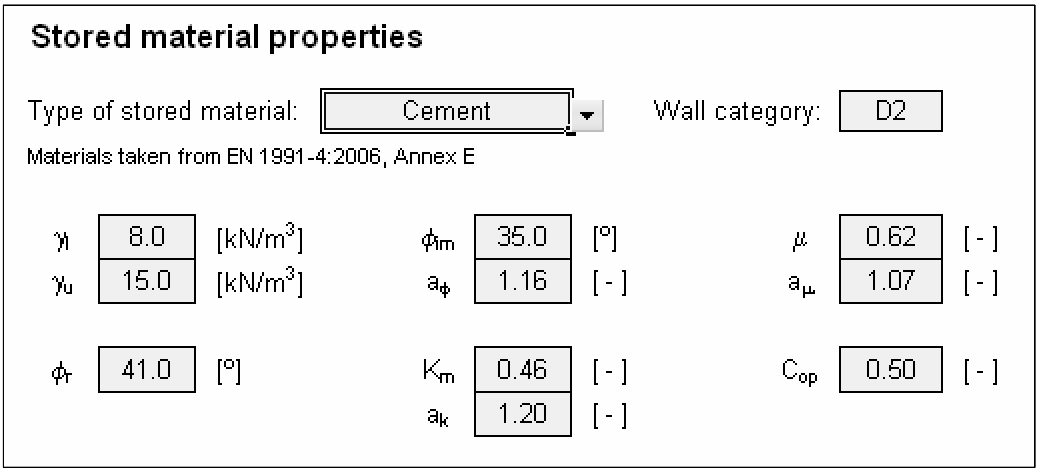
*Note: the program is based on the EN 1991-4:2006 code.*

Version 1.20
4.3.2008

Slika 60: Delovni list za vnos podatkov

### Lastnosti shranjenega materiala

Najprej je potrebno vnesti podatke o shranjenem materialu. Program že vsebuje spisek materialov, ki so vzeti iz preglednice v *Prilogi C* , oz. iz *EN 1991-4:2006, Preglednica E.1*.



**Stored material properties**

Type of stored material: Cement

Wall category: D2

Materials taken from EN 1991-4:2006, Annex E

$\gamma_l$ 8.0 [kN/m$^3$]

$\gamma_u$ 15.0 [kN/m$^3$]

$\phi_r$ 41.0 [°]

$\phi_{im}$ 35.0 [°]

$a_\phi$ 1.16 [ - ]

$K_m$ 0.46 [ - ]

$a_K$ 1.20 [ - ]

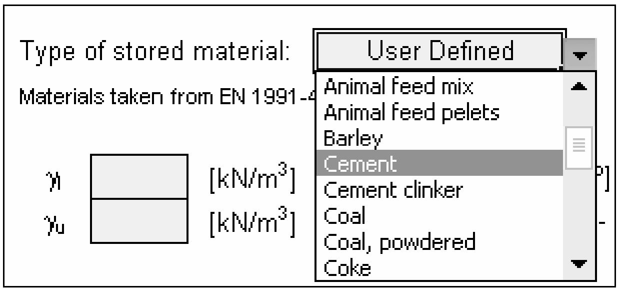
$\mu$ 0.62 [ - ]

$a_\mu$ 1.07 [ - ]

$C_{op}$ 0.50 [ - ]

Slika 61: Vnos podatkov o materialu

Spisek materialov se nahaja v drsnem seznamu. Ko izberemo nek material, se ustrezne vrednosti samodejno vnesejo.



Type of stored material: User Defined

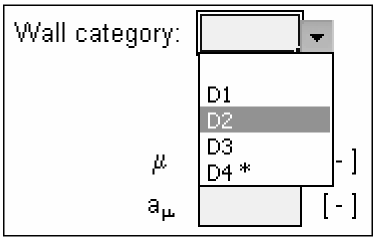
Materials taken from EN 1991-4

$\gamma_l$ [kN/m$^3$]

$\gamma_u$ [kN/m$^3$]

Slika 62: Drsni seznam prednastavljenih materialov

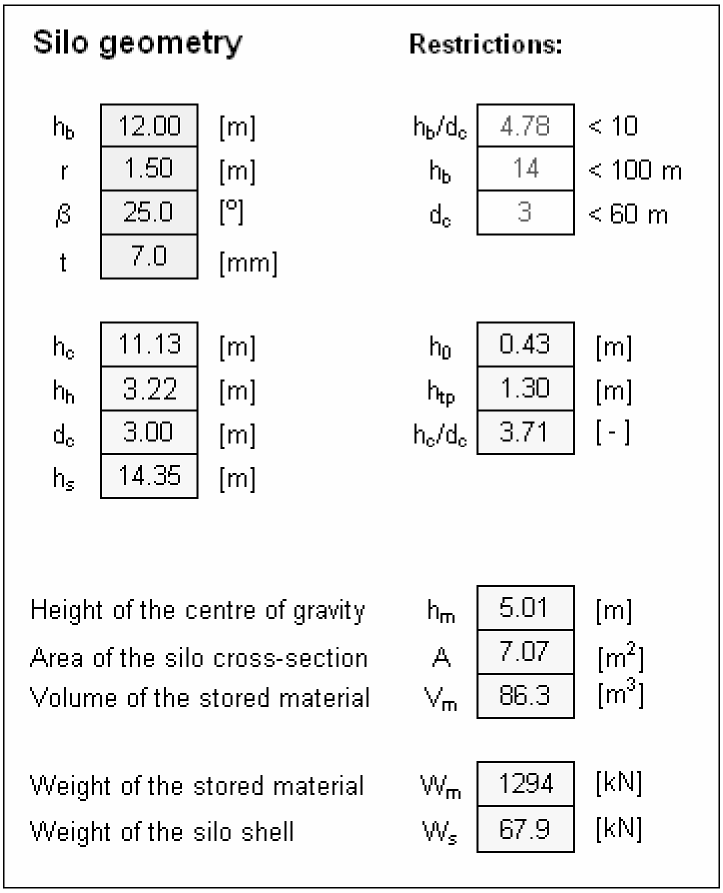
Za vsak material je potrebno izbrati ustrezno kategorijo stene, ki se tudi nahaja v drsnem seznamu. Imena kategorij ustrezajo tistim, ki jih podaja *Preglednica 8*.



**Slika 63: Drsni seznam kategorij sten**

Če želimo sami podati nek poljuben material, izberemo iz drsnega seznama za material možnost »*User defined*« in vnesemo ustrezne vrednosti za ta material. V tem primeru ni potrebno podajati kategorije stene.

## Geometrija silosa



**Silo geometry**

$h_b$ 12.00 [m]
r 1.50 [m]
β 25.0 [°]
t 7.0 [mm]

$h_c$ 11.13 [m]
$h_h$ 3.22 [m]
$d_c$ 3.00 [m]
$h_s$ 14.35 [m]

**Restrictions:**

$h_b/d_c$ 4.78 < 10
$h_b$ 14 < 100 m
$d_c$ 3 < 60 m

$h_0$ 0.43 [m]
$h_{tp}$ 1.30 [m]
$h_c/d_c$ 3.71 [ - ]

Height of the centre of gravity $h_m$ 5.01 [m]
Area of the silo cross-section A 7.07 [$m^2$]
Volume of the stored material $V_m$ 86.3 [$m^3$]

Weight of the stored material $W_m$ 1294 [kN]
Weight of the silo shell $W_s$ 67.9 [kN]

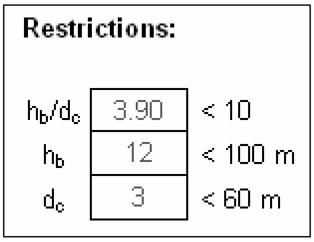
**Slika 64: Vnos geometrije silosa**

Potrebno je podati samo 4 geometrijske parametre. Ti parametri so:

- $h_b$, ki predstavlja celotno višino cilindra,
- $r$ , ki predstavlja radij cilindra,
- $\beta$, ki predstavlja kot lijaka glede na simetrijsko os in
- $t$ , ki predstavlja debelino stene. Če je silos sestavljen iz več delov, različne debeline, potem je potrebno podati povprečno debelino.

Vsi ostali geometrijski parametri se določijo samodejno, z uporabo enačb podanih v poglavju *3.1.1.2*.

Program tudi preveri, če se geometrija ujem v skladu z zahtevami, določenimi v *EN 1991-4:2006, 1.1.2(3)*. Te zahteve so v programu predstavljene kot omejitve.



**Restrictions:**

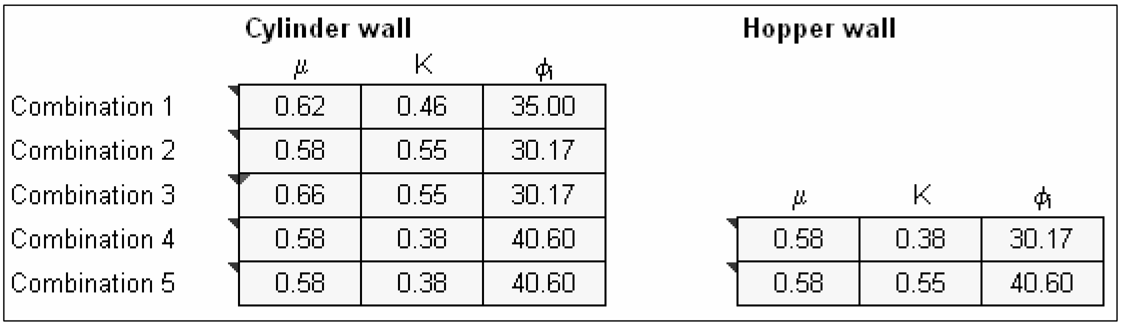
| | | |
|---|---|---|
| $h_b/d_c$ | 3.90 | < 10 |
| $h_b$ | 12 | < 100 m |
| $d_c$ | 3 | < 60 m |

**Slika 65: Omejitve geometrije**

Če je določen pogoj izpolnjen, potem je v celici izpiše z zeleno barvo, v nasprotnem primeru se izpiše z rdečo. V tem primeru je potrebno geometrijo silosa spremeniti, tako da so vsi trije pogoji izpolnjeni.

## Račun obtežbe

Program najprej določi ustrezne minimalne, maksimalne in srednje vrednosti parametrov $\mu$, $K$ in $\phi_i$, glede na kombinacije, ki jih podajata *Preglednica 11* in *Preglednica 12*. Te vrednosti se bodo nato uporabile pri računu.



| | Cylinder wall | | | Hopper wall | | |
|---|---|---|---|---|---|---|
| | $\mu$ | K | $\phi_i$ | $\mu$ | K | $\phi_i$ |
| Combination 1 | 0.62 | 0.46 | 35.00 | | | |
| Combination 2 | 0.58 | 0.55 | 30.17 | | | |
| Combination 3 | 0.66 | 0.55 | 30.17 | | | |
| Combination 4 | 0.58 | 0.38 | 40.60 | 0.58 | 0.38 | 30.17 |
| Combination 5 | 0.58 | 0.38 | 40.60 | 0.58 | 0.55 | 40.60 |

**Slika 66: Kombinacije parametrov**

Izračun zaženemo tako, da pritisnemo na gumb »*Calculate*«, ki se nahaja v orodni vrstici.

## Upravljanje programa

Program se upravlja z gumbi iz orodne vrstice, ki se nahaja v desnem spodnjem kotu delovnega lista.

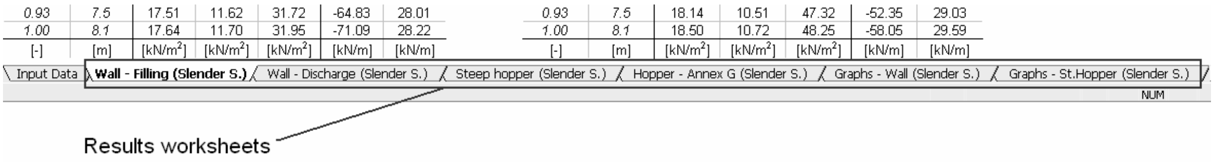
**Slika 67: Orodna vrstica**

Funkcije posameznih gumbov so:

- **Calculate** – izvrši izračun,
- **Clear All** – pobriše vse rezultate in vhodne podatke,
- **Wall Categories** – prikaže definicije za kategorije sten,
- **Instructions** – prikaže kratka navodila za uporabo,
- **About** – prikaže informacije v zvezi z avtorjem in licenco programa,
- **Exit** – izhod iz programa.

## Ogled rezultatov

Rezultati izpišejo na ločenih delovnih listih, ki se pojavijo po izvršenem izračunu.



| | | | | | | |
|---|---|---|---|---|---|---|
| 0.93 | 7.5 | 17.51 | 11.62 | 31.72 | -64.83 | 28.01 |
| 1.00 | 8.1 | 17.64 | 11.70 | 31.95 | -71.09 | 28.22 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.93 | 7.5 | 18.14 | 10.51 | 47.32 | -52.35 | 29.03 |
| 1.00 | 8.1 | 18.50 | 10.72 | 48.25 | -58.05 | 29.59 |
| [-] | [m] | [kN/m$^2$] | [kN/m$^2$] | [kN/m$^2$] | [kN/m] | [kN/m] |

**Slika 68: Delovni listi z rezulati**

Rezultati za *vitke silose*, *silose srednje vitkosti* in *plitve silose* so podani na naslednjih delovnih listih:

- **Wall – Filling:** pritiski in notranje sile na steno cilindra silo med polnjenjem in hranjenjem materiala,
- **Wall – Discharge:** pritiski in notranje sile na steno cilindra silo med praznjenjem materiala,
- **Steep/Shallow Hopper:** pritiski in notranje sile na lijak med polnjenjem in praznjenjem materiala,
- **Hopper – Annex G:** pritiski in notranje sile na lijak med polnjenjem in praznjenjem materiala izračunani po določilih, ki jih podaja *EN 1991-4, Priloga G*[9],
- **Graphs – Wall:** grafična ponazoritev pritiskov po steni cilindra,
- **Graphs – Steep/Shallow Hopper:** grafična ponazoritev pritiskov po steni lijaka.

Rezultati za *zadrževalne silose* imajo samo dva delovna lista:

- **Wall:** pritiski in notranje sile na steno cilindra silo med polnjenjem in hranjenjem materiala,
- **Graphs – Wall:** grafična ponazoritev pritiskov po steni cilindra.

---

[9] V Prilogi G standarda EN 1991-4 je podan postopek, ki temelji na nemških predpisih. EN 1991-4 dovoljuje tudi uporabo tega postopka za določitev obtežbe na stene lijaka.

Vsi rezultati za pritiske in notranje sile so podani v tabelarični obliki, pri čemer je višina cilindra vedno razdeljena na 16 enakih delov (višin), višina lijaka pa na 11 delov (višin). Rezultati so izračunani za vsako višino posebej. Rezultate na vmesnih višinah se lahko izračuna z linearno interpolacijo rezultatov za sosednji višini.

## *Informacije o licenci*

# 13 Program »Wind Load on Cylindrical Silos«

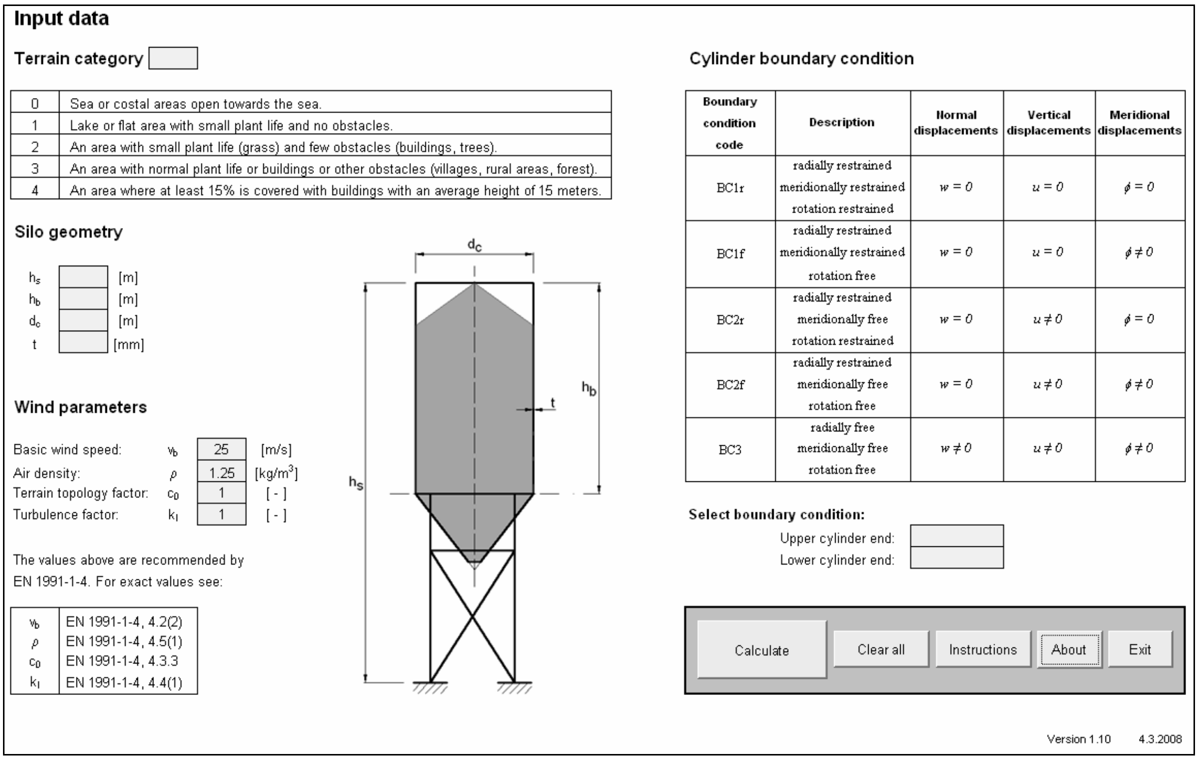
Program izračuna razporeditev pritiskov zaradi delovanja vetra in celotno silo, ki jo povzroča veter na cilindru silosa. Progam upošteva določila, ki so podana v poglavju *4.2*, pri čemer so v program vgrajene vse tri krivulje, ki jih prikazuje slika v poglavju *4.2.1*.

## Zagon programa

Program se nahaja na zgoščenki, ki je priložena na zadnji platnici. Zažene se ga tako, da se odpre datoteko z imenom »*Wind Load on Cylindrical Silos.xls*«.

Dalje postopamo enako, kot pri programu iz poglavja *12*.

## Uporaba programa



**Input data**

Terrain category

| | |
|---|---|
| 0 | Sea or costal areas open towards the sea. |
| 1 | Lake or flat area with small plant life and no obstacles. |
| 2 | An area with small plant life (grass) and few obstacles (buildings, trees). |
| 3 | An area with normal plant life or buildings or other obstacles (villages, rural areas, forest). |
| 4 | An area where at least 15% is covered with buildings with an average height of 15 meters. |

**Silo geometry**

$h_s$ [m]
$h_b$ [m]
$d_c$ [m]
$t$ [mm]

**Wind parameters**

| | | | |
|---|---|---|---|
| Basic wind speed: | $v_b$ | 25 | [m/s] |
| Air density: | $\rho$ | 1.25 | [kg/m$^3$] |
| Terrain topology factor: | $c_0$ | 1 | [ - ] |
| Turbulence factor: | $k_I$ | 1 | [ - ] |

The values above are recommended by EN 1991-1-4. For exact values see:

| | |
|---|---|
| $v_b$ | EN 1991-1-4, 4.2(2) |
| $\rho$ | EN 1991-1-4, 4.5(1) |
| $c_0$ | EN 1991-1-4, 4.3.3 |
| $k_I$ | EN 1991-1-4, 4.4(1) |

**Cylinder boundary condition**

| Boundary condition code | Description | Normal displacements | Vertical displacements | Meridional displacements |
|---|---|---|---|---|
| BC1r | radially restrained<br>meridionally restrained<br>rotation restrained | $w = 0$ | $u = 0$ | $\phi = 0$ |
| BC1f | radially restrained<br>meridionally restrained<br>rotation free | $w = 0$ | $u = 0$ | $\phi \neq 0$ |
| BC2r | radially restrained<br>meridionally free<br>rotation restrained | $w = 0$ | $u \neq 0$ | $\phi = 0$ |
| BC2f | radially restrained<br>meridionally free<br>rotation free | $w = 0$ | $u \neq 0$ | $\phi \neq 0$ |
| BC3 | radially free<br>meridionally free<br>rotation free | $w \neq 0$ | $u \neq 0$ | $\phi \neq 0$ |

**Select boundary condition:**

Upper cylinder end:
Lower cylinder end:

Version 1.10 4.3.2008

**Slika 69: Delovni list z vhodnimi podatki**

## Vhodni podatki

Ob zagonu programa se pokaže delovni list za vnos vhodnih podatkov. Podatke je potrebno vnesti v vse celice pobarvane z *zeleno* barvo.

### Geometrija silosa

Podatki, ki se nanašajo na geometrijo silosa, so:

- $h_s$ – celotna višina silosa (cilinder in podporna konstrukcija),
- $h_b$ – višina cilindra,
- $d_c$ – premer silosa,
- $t$ – debelina stene. Če je silos sestavljen iz več delov, različne debeline, potem je potrebno podati povprečno debelino.

### Parametri, ki določajo lastnosti vetra

Ti parametri vključujejo:

- $v_b$ – osnovna hitrost vetra,
- $\rho$ – gostota zraka,
- $c_0$ – faktor topologije terena,
- $k_I$ – faktor vetrne turbulence.

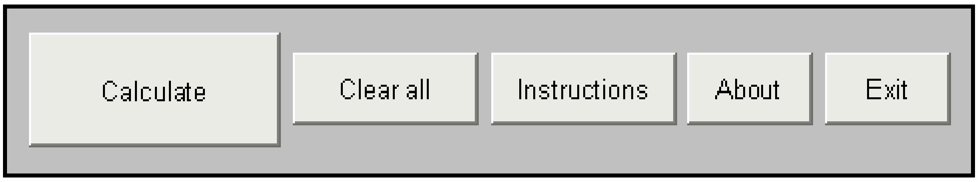
Program že sam poda priporočene vrednosti, ki jih podaja *EN 1991-1-4.* Možno je vnesti tudi točne vrednosti in sicer tako, da se nadomesti priporočeno vrednost.

### Robni pogoji za cilinder

Potrebno je izbrati robni pogoj cilindra za spodnji in zgornji konec.

## Upravljanje programa

Program se upravlja z gumbi iz orodne vrstice, ki se nahaja v desnem spodnjem kotu delovnega lista.



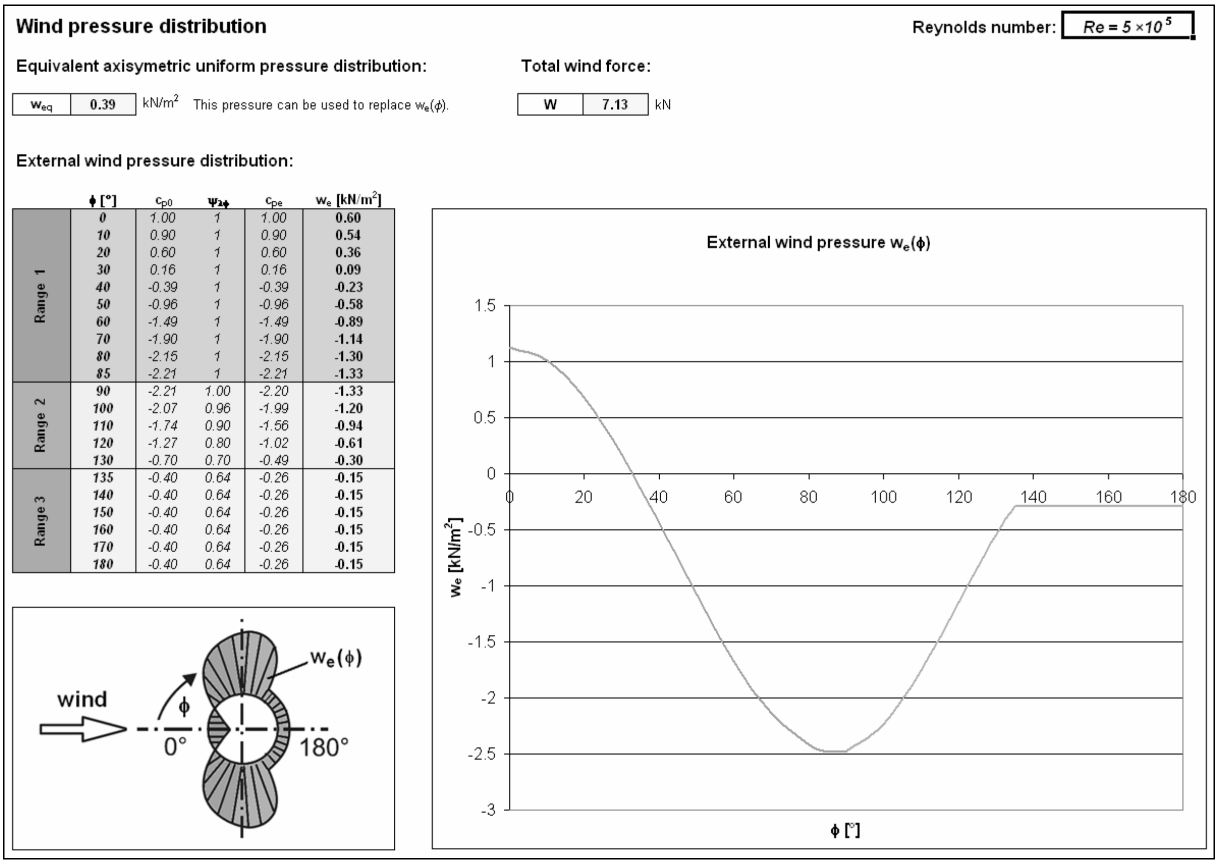
**Slika 70: Orodna vrstica**

Funkcije posameznih gumbov so:

- **Calculate** – izvrši izračun,
- **Clear All** – pobriše vse rezultate in vhodne podatke,
- **Instructions** – prikaže kratka navodila za uporabo,
- **About** – prikaže informacije v zvezi z avtorjem in licenco programa,
- **Exit** – izhod iz programa.

## Ogled rezultatov

Rezultati izpišejo na ločenih delovnih listih, ki se pojavijo po izvršenem izračunu.



**Wind pressure distribution**

Reynolds number: $Re = 5 \times 10^5$

Equivalent axisymetric uniform pressure distribution:

$w_{eq}$ 0.39 kN/m$^2$ This pressure can be used to replace $w_e(\phi)$.

Total wind force:

W 7.13 kN

External wind pressure distribution:

| | $\phi$ [°] | $c_{p0}$ | $\psi_{\lambda\phi}$ | $c_{pe}$ | $w_e$ [kN/m$^2$] |
|---|---|---|---|---|---|
| Range 1 | 0 | 1.00 | 1 | 1.00 | 0.60 |
| | 10 | 0.90 | 1 | 0.90 | 0.54 |
| | 20 | 0.60 | 1 | 0.60 | 0.36 |
| | 30 | 0.16 | 1 | 0.16 | 0.09 |
| | 40 | -0.39 | 1 | -0.39 | -0.23 |
| | 50 | -0.96 | 1 | -0.96 | -0.58 |
| | 60 | -1.49 | 1 | -1.49 | -0.89 |
| | 70 | -1.90 | 1 | -1.90 | -1.14 |
| | 80 | -2.15 | 1 | -2.15 | -1.30 |
| | 85 | -2.21 | 1 | -2.21 | -1.33 |
| Range 2 | 90 | -2.21 | 1.00 | -2.20 | -1.33 |
| | 100 | -2.07 | 0.96 | -1.99 | -1.20 |
| | 110 | -1.74 | 0.90 | -1.56 | -0.94 |
| | 120 | -1.27 | 0.80 | -1.02 | -0.61 |
| | 130 | -0.70 | 0.70 | -0.49 | -0.30 |
| Range 3 | 135 | -0.40 | 0.64 | -0.26 | -0.15 |
| | 140 | -0.40 | 0.64 | -0.26 | -0.15 |
| | 150 | -0.40 | 0.64 | -0.26 | -0.15 |
| | 160 | -0.40 | 0.64 | -0.26 | -0.15 |
| | 170 | -0.40 | 0.64 | -0.26 | -0.15 |
| | 180 | -0.40 | 0.64 | -0.26 | -0.15 |

Slika 71: Delovni list z rezultati

Rezultati za zunanji pritisk vetra $w_e(\phi)$ so podani v tabelarični obliki. Zaradi simetrije, je ta pritisk podan samo od kota $\phi = 0°$ do $\phi = 180°$.

# Informacije o licenci

Wind Load on Cylindrical Silos

Izvod Splošnega dovoljenja GNU (v angleškem izvirniku) se nahaja na zgoščenki, ki je priložena tej diplomski nalogi. Za več informacij o dovoljenju GNU, obiščite <http://www.gnu.org/licenses/>.

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Eurocode 3: Design of steel structures - Part 4-1: Silos, tanks and pipelines
Silos. prEN 1993-4-1, 2005.

Eurocode 8: Design of structures for earthquake resistance - Part 1: General rules, seismic actions and rules for buildings. EN 1998-1, 2004.

Eurocode 8: Design of structures for earthquake resistance - Part 4: Silos, tanks and pipelines. EN 1998-4, 2006.

# Priloga A: Angleški prevod I. dela z uvodom

# *Appendix A: English translation of Part I with introduction*

## Table of contents

**PART I - LOADS ON SILO STRUCTURES**

# 1 Introduction

## *1.1 Motivation*

Silo structures are widely used in industry and agriculture as they provide efficient short–term to mid–term storage of organic and inorganic particulate solid material. They vary in capacity, which can range over several orders of magnitude, from 10 tons to 10 000 tons. Silos usually have circular or rectangular cross-sections. The walls of the silo shell are mainly made of steel or reinforced concrete, although materials like aluminum and even timber can also be used. In this work we will focus only on thin-walled steel silo structures that have a circular cross-section.

The silo load assessment process is relatively complex, because of the granular structure of the filling material, with its diversities of grain properties. Specific considerations have to be made for the filling state and for the discharge (flow) state. Additionally, wind and seismic loads present extra dynamic problems in the silo design process. A full silo usually has a very large mass, which is supported above the ground on relatively flexible supports. Empty silo structures are susceptible for buckling and vibration effects.

The purpose of this work is to serve as means of assistance and guidance for designing thin-walled cylindrical silos in accordance with EN Eurocode building codes. It offers a compact overview of the silo design process, together with an example silo, which is analyzed and designed using the provisions from the EN standards and the guidelines from this work. Special emphasis is also given on how to design silos to withstand seismic loads.

## 1.2 Structure of the work

The work is divided in to three parts. In the first part of the work the loads on silo structures are classified and detailed load calculation guidelines of the most important load types are given. At the end of the first part some basic guidelines on silo design are also given, together with section forces for each load type, which are derived from the membrane theory of shells and can be used directly in the design.

In the second part of the work, an actual example silo is analyzed and designed using the guidelines from the first part.

In the third part of the work two computer programs, one for determining the loads on the silo walls due to effects of the stored solid material, the other for determining the wind loading, are presented. The programs were also developed in the scope of this work. They are included on the accompanying CD, which is located on the back cover.

## 1.3 Types of considered silo structures

Silo structures may vary in shape, size, the material from which they are made of and the material they store. We will consider only axisymetric thin-walled steel silo structures.

The following assumptions therefore apply for silo geometry and silo loading in this work:

- axisymetric global geometry (cylindrical barrel and conical hopper),
- the silo is thin walled and made of metal (stainless steel, aluminum),
- the filling inlet and discharge outlet lie on the axis of symmetry,
- the silo has only one cell compartment, where the material is stored.

A typical silo structure represented in this work consists of two parts – a cylindrical upper part (cylinder or barrel), where most of the material is stored, and a conical lower part (hopper), which is used to facilitate natural discharge by gravity action. Easy access beneath the silo structure is enabled by placing the silo in an elevated position. Ground support is then provided either by discrete equidistant column supports, or by continuous cylindrical skirts extending the barrel to the ground level.

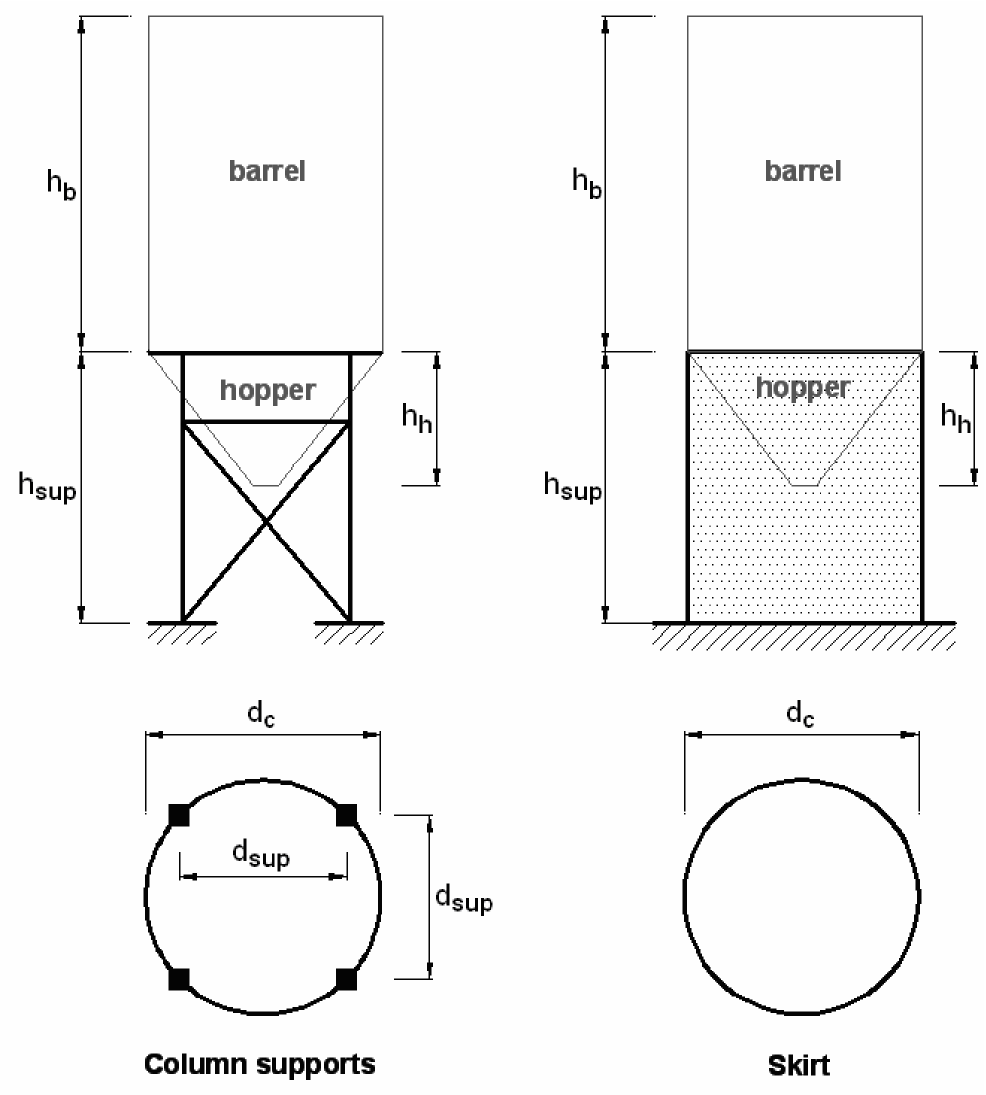


**Figure 1: Typical elevated thin-walled cylindrical silo structures and support structures**

## 1.4 Considered model building codes

The following EN building codes have been considered when writing this work:

**Table 1: Considered EN codes**

| Code | Description |
|---|---|
| **EN 1990:2000** | Eurocode 0: Basis of structural design |
| **EN 1991-1-4:2005** | Eurocode 1: Actions on structures, Part 1.4: General actions – Wind actions |
| **EN 1991-4:2006** | Eurocode 1: Actions on structures, Part 4: Silos and tanks |
| **EN 1993-1-1:2005** | Eurocode 3: Design of steel structures, Part 1-1 : General rules and rules for buildings |
| **prEN 1993-1-6:2004** | Eurocode 3: Design of steel structures, Part 1-6: Strength and Stability of Shell Structures |
| **prEN 1993-4-1:2005** | Eurocode 3: Design of steel structures, Part 4-1: Silos |
| **EN 1998-1:2004** | Eurocode 8: Design of structures for earthquake resistance, Part 1: General rules, seismic actions and rules for buildings |
| **EN 1998-4:2006** | Eurocode 8 : Design of structures for earthquake resistance, Part 4: Silos, tanks and pipelines |

## 1.5 Used symbols

The symbols listed below appear in the work and have a physical interpretation. Only one meaning is assigned to each symbol unless otherwise defined in the text where it occurs.

**Table 2: List of used symbols for silo geometry**

| Symbol | Meaning |
|---|---|
| $A$ | plan cross-section of the storage area of the silo |
| $\beta$ | hopper apex half angle |
| $d_c$ | diameter of the silo |
| $d_{sup}$ | distance between the supports of the supporting structure |
| $\phi$ | coordinate, representing the angle between a fixed direction and a chosen direction in the silo cross-section |
| $G_m$ | weight of the stored material |

| | |
|---|---|
| $G_s$ | weight of the silo shell |
| $h_0$ | value of z at the highest solid-wall contact |
| $h_b$ | total height of the silo cylinder |
| $h_c$ | depth of the silo base below the equivalent surface |
| $h_h$ | vertical height between the hopper apex and the transition |
| $h_m$ | silo centre of gravity |
| $h_s$ | total silo height |
| $h_{sup}$ | height of the supporting structure, relevant for the seismic calculation |
| $h_{tp}$ | is the total height of the top pile, defined as the vertical distance from lowest point on the wall that is not in contact with the stored solid to the highest stored particle |
| $\lambda$ | silo cylinder slenderness |
| $r$ | inner radius of the silo |
| $s$ | coordinate, running in the direction of the hopper wall |
| $t$ | thickness of the silo shell |
| $U$ | internal perimeter of the plan cross-section of the silo |
| $V$ | silo volume |
| $V_m$ | volume of the stored material |
| $\omega$ | the dimensionless length parameter |
| $x$ | vertical coordinate, running upwards from hopper apex |
| $\psi_{\lambda\phi}$ | slenderness factor |
| $z$ | coordinate, running from the equivalent surface of the solid towards the silo apex |
| $z'$ | coordinate, running from the top of the silo cylinder towards the silo apex |
| $z_s$ | depth below the highest stored solid contact with the wall |

**Table 3: List of used symbols for pressures and section forces**

| Symbol | Meaning |
|---|---|
| $\Delta_{ph,s}$ | additional normal pressure on the silo cylinder wall due to seismic load |
| $n_{\phi,ees}$ | circumferential section force in the empty silo cylinder wall due to seismic load |
| $n_{\phi,efs}$ | circumferential section force in the full silo cylinder wall due to seismic load |
| $n_{\phi,s}$ | circumferential section force in the empty silo cylinder wall due to permanent loads |
| $n_{\phi,s}$ | circumferential section force in the empty silo cylinder wall due to snow load |
| $n_{\phi,we}$ | circumferential section force in the empty silo cylinder wall due to wind load |
| $n_{\phi,wf}$ | circumferential section force in the full silo cylinder wall due to wind load |
| $n_{\phi,d}$ | design value circumferential section force in the silo cylinder wall |

| | |
|---|---|
| $n_{\phi,e}$ | circumferential section force in the silo cylinder wall due to discharge of the particulate solid |
| $n_{\phi,f}$ | circumferential section force in the silo cylinder wall due to filling of the particulate solid |
| $n_{s,ees}$ | section force in the empty silo, in direction s, in the hopper wall due to the seismic load |
| $n_{s,efs}$ | section force in the full silo, in direction s, in the hopper wall due to the seismic load |
| $n_{sd}$ | design value of the section force in direction s |
| $n_{se}$ | section force in direction s, in the hopper wall due to discharge of the particulate solid |
| $n_{sf}$ | section force in direction s the hopper wall due to filling of the particulate solid |
| $n_{z,ees}$ | vertical section force in the empty silo cylinder wall due to the seismic load |
| $n_{z,efs}$ | vertical section force in the full silo cylinder wall due to the seismic load |
| $n_{z,p}$ | vertical section force in the silo cylinder wall due to permanent loads |
| $n_{z,we}$ | vertical section force in the empty silo cylinder wall due to wind load |
| $n_{z,wf}$ | vertical section force in the full silo cylinder wall due to wind load |
| $n_{zd}$ | design value of vertical section force in the silo cylinder wall |
| $n_{ze}$ | vertical section force in the silo cylinder wall due to discharge of the particulate solid |
| $n_{zf}$ | vertical section force in the silo cylinder wall due to filling of the particulate solid |
| $p_{he}$ | horizontal pressure on the silo wall for the discharge load |
| $p_{hf}$ | horizontal pressure on the silo wall for the filling load |
| $p_{vb}$ | uniform component of vertical pressure |
| $p_{ve}$ | the vertical pressure for the discharge load |
| $p_{vf}$ | the vertical pressure for the filling load |
| $p_{vft}$ | mean vertical stress in the solid at the transition after filling |
| $p_{vho}$ | vertical pressure at the base of the top pile |
| $p_{we}$ | wall frictional traction on the silo wall for the discharge load |
| $p_{wf}$ | wall frictional traction on the silo wall for the filling load |
| $q_b$ | basic wind pressure |
| $q_p$ | peak wind pressure |
| $R_e$ | reaction in the silo support because of the earthquake load on the full silo |
| $R_s$ | reaction in the silo support because of the snow load |
| $R_{ss}$ | reaction in the silo support because of the stored solid load |
| $R_{sw}$ | reaction in the silo support because of the self-weight of the silo |
| $R_w$ | reaction in the silo support because of the wind load |
| $w_e$ | external wind pressure (characteristic value) |
| $w_{eq}$ | equivalent axisymetric wind pressure |

**Table 4: List of used symbols for silo loading calculation and material**

| Symbol | Meaning |
|---|---|
| $A$ | plan cross-section of the storage area of the silo |
| $a_g$ | design ground acceleration on type A ground |
| $\alpha_w$ | coefficient of thermal expansion of the silo wall |
| $C_b$ | bottom load magnifier to account for the possibility of larger loads being transferred to the hopper or bottom from the vertical walled segment discharge factor for horizontal pressure |
| $c_0$ | terrain topology factor |
| $c_e$ | exposure factor |
| $C_h$ | discharge factor for horizontal wall pressure |
| $C_{op}$ | patch load solid reference factor |
| $c_{pe}$ | external pressure coefficient |
| $c_r$ | roughness factor |
| $C_S$ | slenderness adjustment factor |
| $C_w$ | discharge factor for wall frictional traction |
| $\Delta T$ | temperature differential |
| $E_i$ | modulus of elasticity of element i |
| $E_{sU}$ | unloading effective elastic modulus of the stored solid |
| $E_w$ | elastic modulus of the silo wall |
| $F_b$ | the seismic base shear force |
| $F_e$ | characteristic value of the hopper pressure ratio for the discharge load |
| $F_f$ | characteristic value of the hopper pressure ratio for the filling load |
| $\phi_i$ | angle of internal friction of the stored solid (minimum, maximum or mean - depending on the load case) |
| $\phi_r$ | characteristic value of the angle of repose of the solid (minimum, maximum or mean - depending on the load case) |
| $\{\phi\}$ | vector representing the mode of vibration |
| $\varphi$ | rotation of the silo center of mass due to seizmic load |
| $\gamma$ | characteristic value of the unit weight (minimum, maximum or mean - depending on the load case) |
| $g$ | acceleration of gravity (g = 9.81 m/s$^2$) |
| $\Gamma$ | mass participation factor |
| $\gamma_u$ | upper (maximum) characteristic value for the unit weight of the stored particulate solid material |
| $\eta$ | damping correction factor |

| | |
|---|---|
| $K$ | characteristic value of the lateral pressure ratio (minimum, maximum or mean - depending on the load case) |
| $k_I$ | turbulence factor |
| $k_m$ | stiffness of the rotational spring |
| $k_r$ | terrain factor |
| $k_v$ | stiffness of the horizontal spring |
| $\mu$ | characteristic value of the wall friction coefficient for solid sliding on the vertical wall (minimum, maximum or mean - depending on the load case) |
| $m$ | mass of the silo contents |
| $m_{silo}$ | mass of the silo contents and silo shell structure |
| $m_{sup}$ | part of the mass of the supporting structure, relevant for the seismic analysis |
| $m_{sup,total}$ | total mass of the supporting structure |
| $q$ | behavior factor |
| $\rho$ | air density |
| $Re$ | Reynolds number |
| $S$ | soil factor |
| $S_d\,(T)$ | design value of the response spectrum |
| $S_e\,(T)$ | elastic response spectrum |
| $T$ | vibration period of a linear single-degree-of-freedom system |
| $T_1$ | fundamental period of vibration |
| $T_B$, $T_C$ | limits of the constant spectral acceleration branch |
| $T_D$ | value defining the beginning of the constant displacement response range of the spectrum |
| $\Theta_m$ | rotational mass of the supporting structure |
| $u$ | total displacement of the silo center of mass due to seismic load |
| $u_v$ | displacement of the silo center of mass due to the deformation of the horizontal spring |
| $u_z$ | displacement of the silo center of mass due to the deformation of the rotational spring |
| $v_b$ | basic wind speed |
| $\omega$ | angular frequency |

# PART I - Loads on silo structures

## 2 Types of loads

Loads on silos are determined by taking into account the silo structure, the terrain topography, the stored solid properties, and the discharge flow patterns that arise during the process of emptying. Additionally, external loads due to snow, wind and earthquake also have to be considered.

*Annex A* of *EN 1991-4:2006* defines the following load actions for design consideration. The loads, dealt with in this work, are written in bold and marked by grey shading.

**Table 5: Loads on silo structures**

| |
|---|
| **self-weight of the silo structure**, |
| **filling and storage of particulate solids (filling loads),** |
| **discharge of particulate solids (discharge loads),** |
| imposed loads, |
| **snow loads**, |
| **wind action when the silo is either full or empty**, |
| thermal loads, |
| imposed deformations (foundation settlement), |
| **seismic loads**, |
| dust explosion loads. |

In the following chapters particulate solid loads (filling and discharge), wind loads and seismic loads will be analyzed thoroughly due to special considerations that arise in the case of cylindrical silo structures.

# 3 Loads due to stored particulate solids

In silo structures different loading pressures apply for the stored particulate solid, depending on the filling and discharge process. Because we are dealing with axisymetric silo structures, the eccentricities that appear can be taken as small, because they arise only from accidental asymmetries of loading associated with eccentricities and imperfections in the filling process. Loads on the walls of silos due to filling and discharge with small eccentricities will be represented by an axisymetric load and by an unsymmetrical (patch) load.

## *3.1 Axisymetric load*

The axisymetric load on silos will be expressed in terms of a horizontal (circumferential) pressure $p_h$ on the inner surface of the vertical silo wall, a normal pressure $p_n$ on an inclined wall, tangential frictional tractions on the vertical wall $p_w$ and on the inclined wall $p_t$, and a uniform vertical pressure $p_v$ in the stored solid. The vertical pressure at the transition between the cylinder and the hopper will be denoted as $p_{vft}$.

The forces $G_c$ and $G_h$ in the figure below represent the weight of the stored solid in the cylinder and hopper.

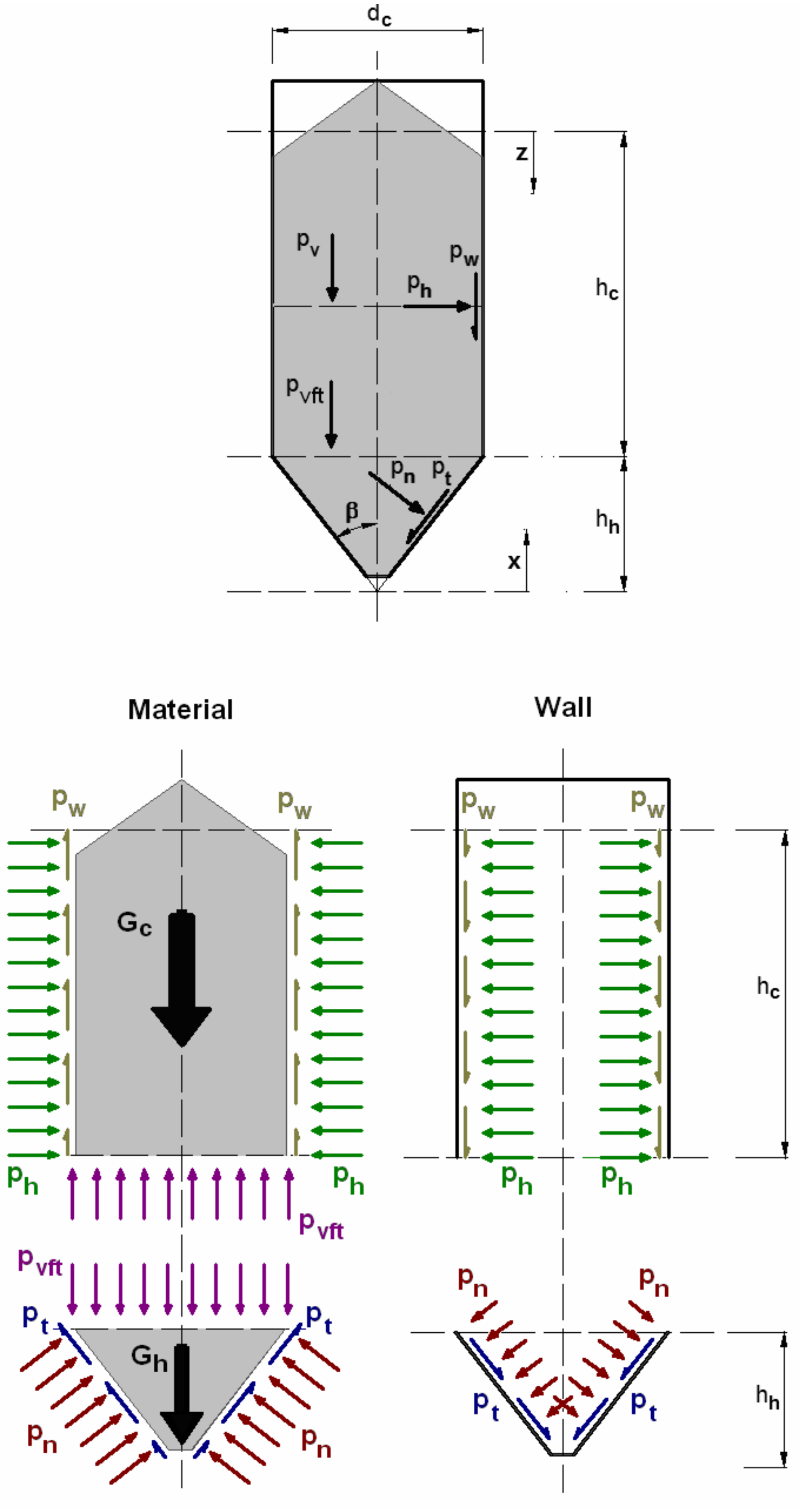


**Figure 2: Axisymetric pressures on the stored material and silo walls**

### 3.1.1 Load Assessment procedure

The procedure below is based on the provisions of the *EN 1991-4:2006* code and can be used for the assessment of axisymetric actions on silo walls due to the stored particulate solid material.

**Table 6: Overview of procedure steps**

| Step | Task | Task location in EN 1991 - 4 |
|---|---|---|
| 1 | Specify the stored material properties | Table E.1 |
| 2 | Specify silo geometry parameters | Figure 1.1 a |
| 3 | Silo cylinder and hopper type | Section 5.1(2)P and 6.1.1(2)P |
| 4 | Action assessment class | Table 2.1 |
| 5 | Relevant material parameter combinations | Table 3.1 |
| 6 | Loads on cylinder walls | Section 5 |
| 7 | Loads on hopper walls and flat silo bottoms | Section 6 |

### 3.1.1.1 Step 1: Stored material parameters

The material parameters of the stored material can be obtained from *Appendix C*.

**Table 7: Stored material parameters**

| Material parameter | Description |
|---|---|
| $\gamma_{min}$ | unit weight of the stored material (minimum and maximum) |
| $\gamma_{max}$ | |
| $\phi_r$ | angle of repose |
| $\phi_{i,min} = \phi_{im} / a_\phi$ | angle of internal friction (minimum, mean and maximum) |
| $\phi_{im}$ | |
| $\phi_{i,max} = \phi_{im} \cdot a_\phi$ | |
| $K_{min} = K_m / a_K$ | lateral pressure ratio (minimum, mean and maximum) |
| $K_m$ | |
| $K_{max} = K_m \cdot a_K$ | |
| $\mu_{min} = \mu_m / a_\mu$ | wall friction coefficient (minimum, mean and maximum) |
| $\mu_m$ | |
| $\mu_{min} = \mu_m \cdot a_\mu$ | |
| $C_{op}$ | patch load solid reference factor |

The values of parameters $\mu_m$, $K_m$, $\phi_{I,m}$ represent the mean values of parameters $\mu$, $K$, $\phi_I$. In order to calculate the maximum pressures on the cylinder and hopper, the proper combinations of the minimum and maximum values of these parameters need to be considered. These combinations will be presented in *Section 3.1.1.4*.

Additionally, values for the mean wall friction coefficient ($\mu_m$) must be determined concerning the proper wall surface category. The wall surfaces are grouped into four categories (*D1* to *D4*) regarding their friction. Different values of $\mu_m$ apply do different wall categories.

**Table 8: Wall surface category**

| Category | Friction | Typical wall material |
|---|---|---|
| **D1** | Low | Cold-rolled stainless steel,<br>polished stainless steel,<br>coated surface designed for low friction. |
| **D2** | Moderate | Smooth mild carbon steel,<br>mill finish stainless steel,<br>galvanized carbon steel. |
| **D3** | High | Aged (corroded) carbon steel,<br>abrasion resistant steel. |
| **D4** | Irregular | Horizontally corrugated walls,<br>profiled sheeting with horizontal ribs,<br>non-standard walls with large aberrations. |

If the wall category is *D4*, then the mean wall friction coefficient $\mu_m$ must be determined by using the provisions of *EN 1991-4.3, Annex D*.

### 3.1.1.2 Step 2: Silo geometry parameters

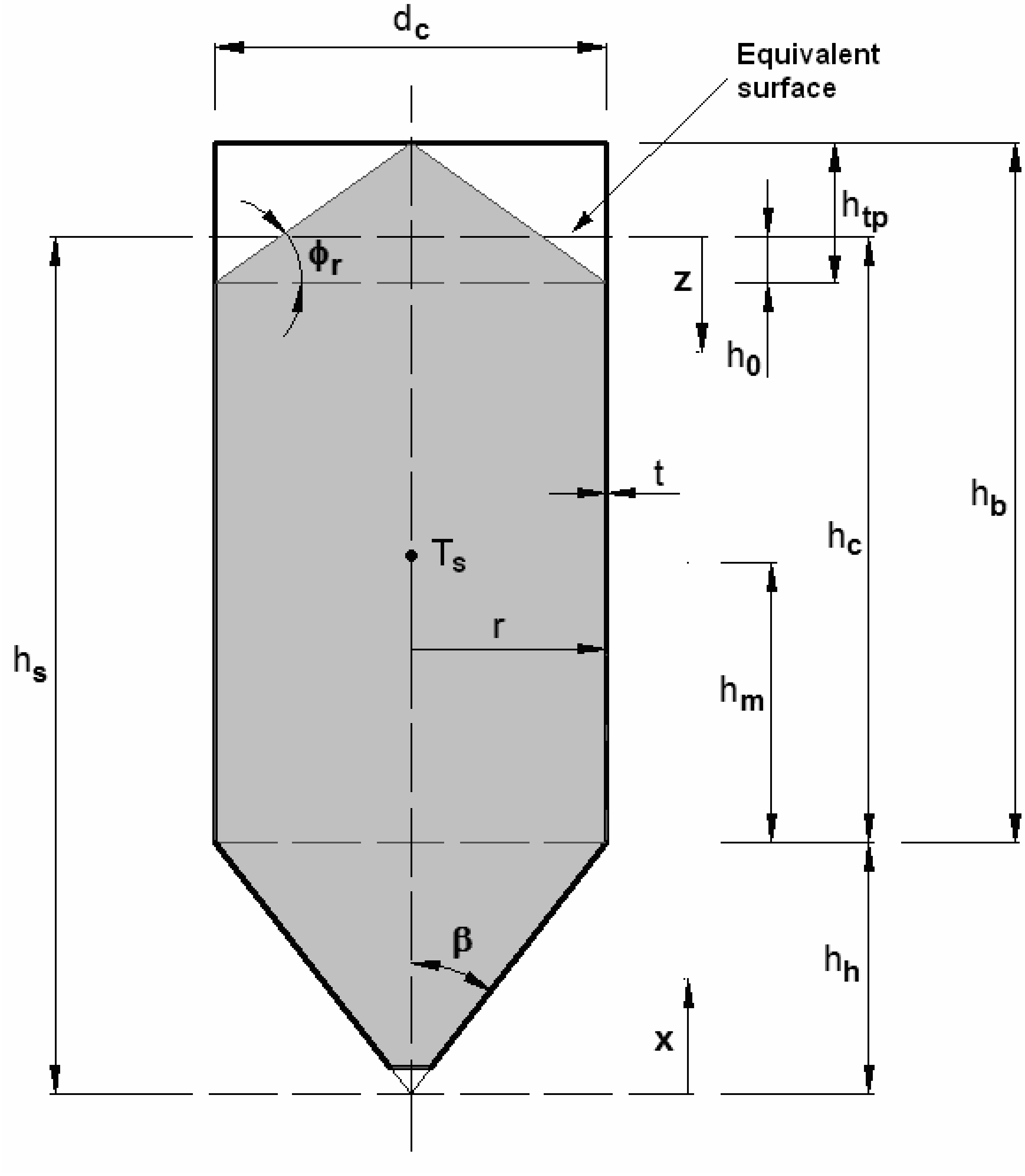


Figure 3: Geometrical parameters of a silo

**Independent parameters**

Height of the silo cylinder: $h_b$

Radius of the silo: $r$

Half-angle of the silo hopper: $\beta$

Thickness of the silo wall: $t$

**Dependent parameters**

Height of the hopper: $h_h = \dfrac{r}{tan\,\beta}$ (A.1)

Height of the top pile: $h_{tp} = r \cdot tan\,\phi_r$ (A.2)

Depth below the equivalent surface: $h_0 = \dfrac{1}{3} \cdot h_{tp}$ (A.3)

Equivalent height of the silo cylinder: $h_c = h_b - h_{tp} + h_0$ (A.4)

Equivalent height of the stored solid: $h_s = h_h + h_c < 100m$ (A.5)

Diameter of the silo structure: $d_c = 2 \cdot r < 60m$ (A.6)

Additional restriction: $h_s / d_c < 10$ (A.7)

Circumference of the silo cross-section: $U = \pi \cdot d_c$ (A.8)

Area of the silo storage cross-section: $A = \pi \cdot \dfrac{{d_c}^2}{4}$ (A.9)

Volume of the stored material: $V_m = A \cdot \left( h_c - h_0 + \dfrac{1}{3}\left(h_h + h_{tp}\right)\right)$ (A.10)

Weight of the stored material: $G_m = \gamma_u \cdot V_m$ (A.11)

Weight of the silo shell: $G_s = 2\pi t \left( r + \dfrac{t}{2}\right) \cdot \left( h_b + \dfrac{1}{3} \cdot h_h \right) \cdot \gamma_{steel}$ (A.12)

Silo centre of gravity: $h_m = \dfrac{1 + \dfrac{1}{18} \cdot \left(\dfrac{h_{tp}}{h_c}\right)^2 - \dfrac{1}{6} \cdot \left(\dfrac{h_h}{h_c}\right)^2}{1 + \dfrac{1}{3} \cdot \dfrac{h_h}{h_c}} \cdot \dfrac{h_c}{2}$ (A.13)

Moment of inertia of the silo cylinder: $$I = \frac{1}{8}\pi t \cdot (d_c + t) \cdot \left( d_c^{2} + 2 d_c t + 2t^2 \right) \qquad \text{(A.14)}$$

If the silo wall has a varying thickness, then the average wall thickness $\bar{t}$ should be considered for the calculation of $G_m$ and $I$.

### 3.1.1.3 Step 3: Silo cylinder and hopper type

Because the silo cylinder and hopper represent the main parts of a silo structure, they are classified into different types, regarding their geometry and incline of the wall.

**Silo cylinder type:**

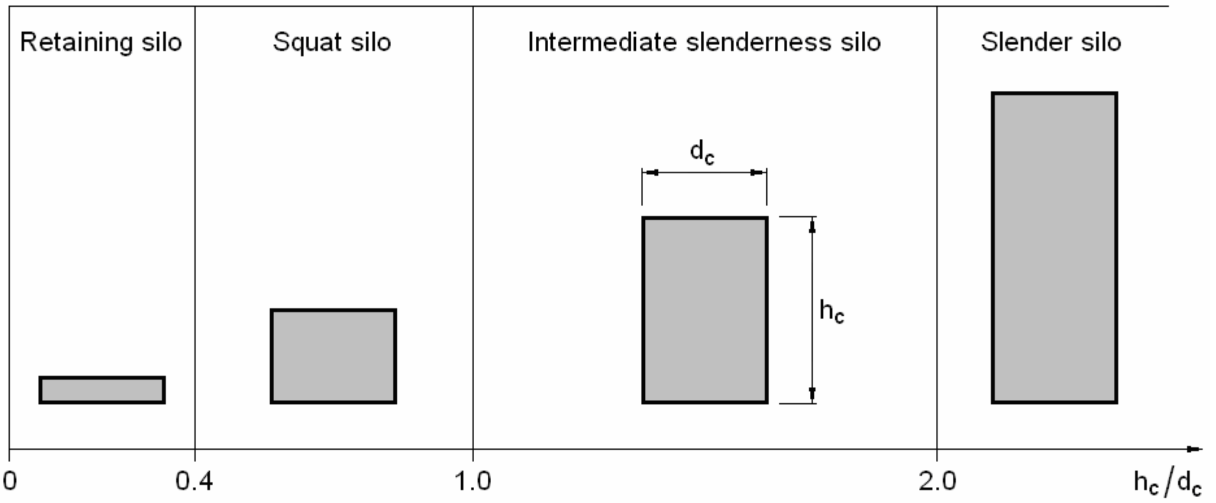


**Figure 4: Silo cylinder type**

**NOTE:** In the present context, the term “slenderness” does not relate to silo shell, but rather to the global geometry of the silo cylinder.

## Hopper type:

**Table 9: Hopper type**

| Hopper type | Condition | |
|---|---|---|
| Steep hopper | $K_{\min} <$ | $1-2\mu_{\min}\tan\beta$ (*see Figure 5*) |
| Shallow hopper | $K_{\min} \geq$ | |
| Flat bottom | $\beta \geq 85^{\circ}$ | |

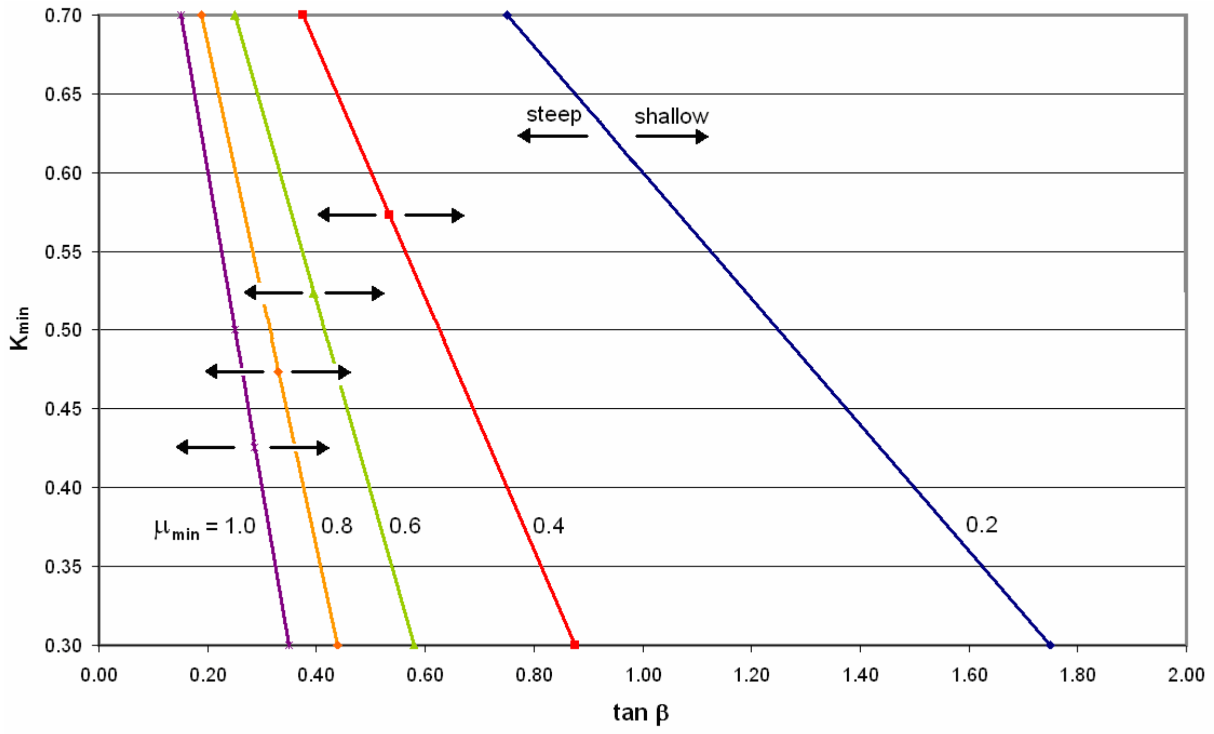


**Figure 5: Hopper type**

The figure above is used to determine the hopper type. First the proper curve has to be selected, regarding the minimum wall friction coefficient $\mu_{min}$ of the given stored solid material. If the minimum value of the lateral pressure ratio of the given stored solid material $K_{min}$ and the tangent value of the hopper apex half-angle $\tan\beta$ lie to the left of the selected curve, then the hopper is steep, otherwise it is shallow.

### 3.1.1.4 Step 4: Silo action assessment class

Silos are divided into three action assessment classes based on their capacity. This division is important, because certain simplifications apply to silos with a smaller capacity, when performing the silo loading calculation.

The silo capacity equals the mass of the stored solid:

$$m_{solid} = \frac{G_m}{g} \cong 0.1 \cdot G_m \tag{A.15}$$

The appropriate Action Assessment Class (AAC) can now be determined from the table below:

**Table 10: Action assessment class**

| Action assessment class (AAC) | $m_{solid}$ |
|---|---|
| 1 | below 100 tones |
| 2 | between 100 tones and 10 000 tones |
| 3 | more than 10 000 tones |

### 3.1.1.5 Step 5: Relevant material parameter combinations

The material parameters defined in *Table 7* are used for calculating the appropriate pressures on the walls of the silo structure. The maximum values of pressures are achieved by properly combining the minimum, maximum or mean values of parameters $\mu$, $K$ and $\phi_i$. The parameter $\gamma$ is always taken as the maximum value $\gamma_{max}$.

**Cylinder wall:**

For attaining maximum values of pressures ($p_h$ and $p_w$) on the cylinder wall during filling and discharge, the following parameter combinations have to be regarded.

**Table 11: Parameter combinations for maximum pressures on the cylinder wall**

| Parameter combination | AAC | Purpose | $\mu$ | K | $\phi_i$ |
|---|---|---|---|---|---|
| 1 | 1 | Maximum normal pressure ($p_h$) and maximum frictional traction ($p_w$) | MEAN | MEAN | MEAN |
| 2 | 2 and 3 | Maximum normal pressure ($p_h$) | MIN | MAX | MIN |
| 3 | | Maximum frictional traction ($p_w$) | MAX | MAX | MIN |

**Note:** the terms *mean*, *min* and *max* determine which value of a certain parameter (mean, minimum or maximum) we have to consider in the given parameter combination.

## Hopper wall:

When calculating the maximum values for pressures on hoppers, the combination of parameters accounts only for attaining the maximum pressures during filling and discharge. The vertical pressure in the silo cylinder $p_v$, evaluated at the level of the cylinder-cone transition junction, is transferred to the hopper and thus has an effect on the normal pressure $p_n$ and on tangential frictional traction $p_t$ of the inclined hopper wall. That is why the combination for the maximum vertical pressure in the silo cylinder has to be considered along with the combinations for the maximum filling and discharge pressures on the hopper.

**Table 12: Parameter combinations for maximum pressures on hopper walls**

| Parameter combination | AAC | Applies to | Purpose | $\mu$ | K | $\phi_i$ |
|---|---|---|---|---|---|---|
| 4 | 1, 2 and 3 | Cylinder wall | Maximum vertical pressure ($p_v$) | MIN | MIN | MAX |
|  |  | Hopper wall | Maximum pressures on filling | MIN | MIN | MIN |
| 5 |  | Cylinder wall | Maximum vertical pressure ($p_v$) | MIN | MIN | MAX |
|  |  | Hopper wall | Maximum pressures on discharge | MIN | MAX | MAX |

### 3.1.1.6 Step 6: Loads on the cylinder wall

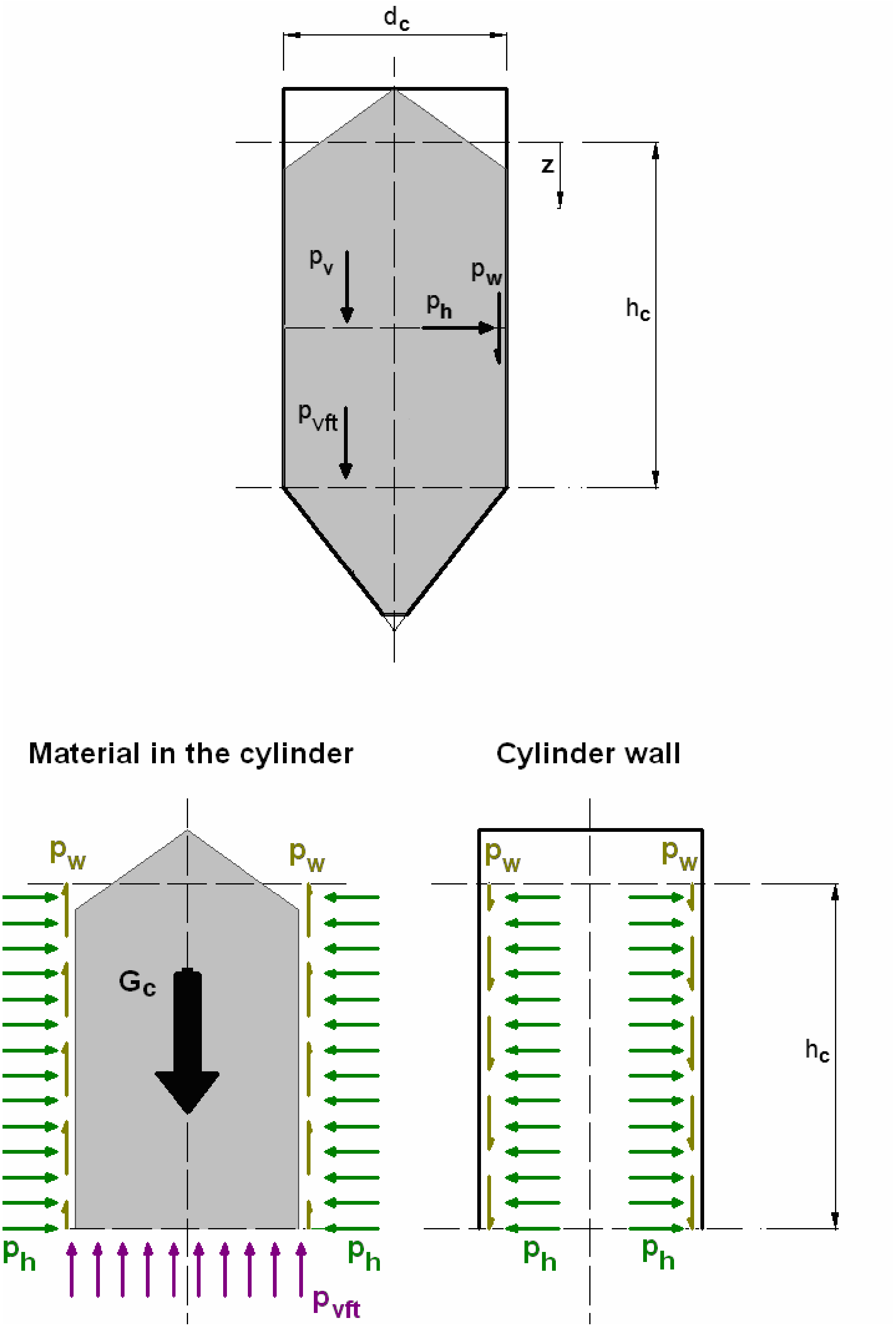
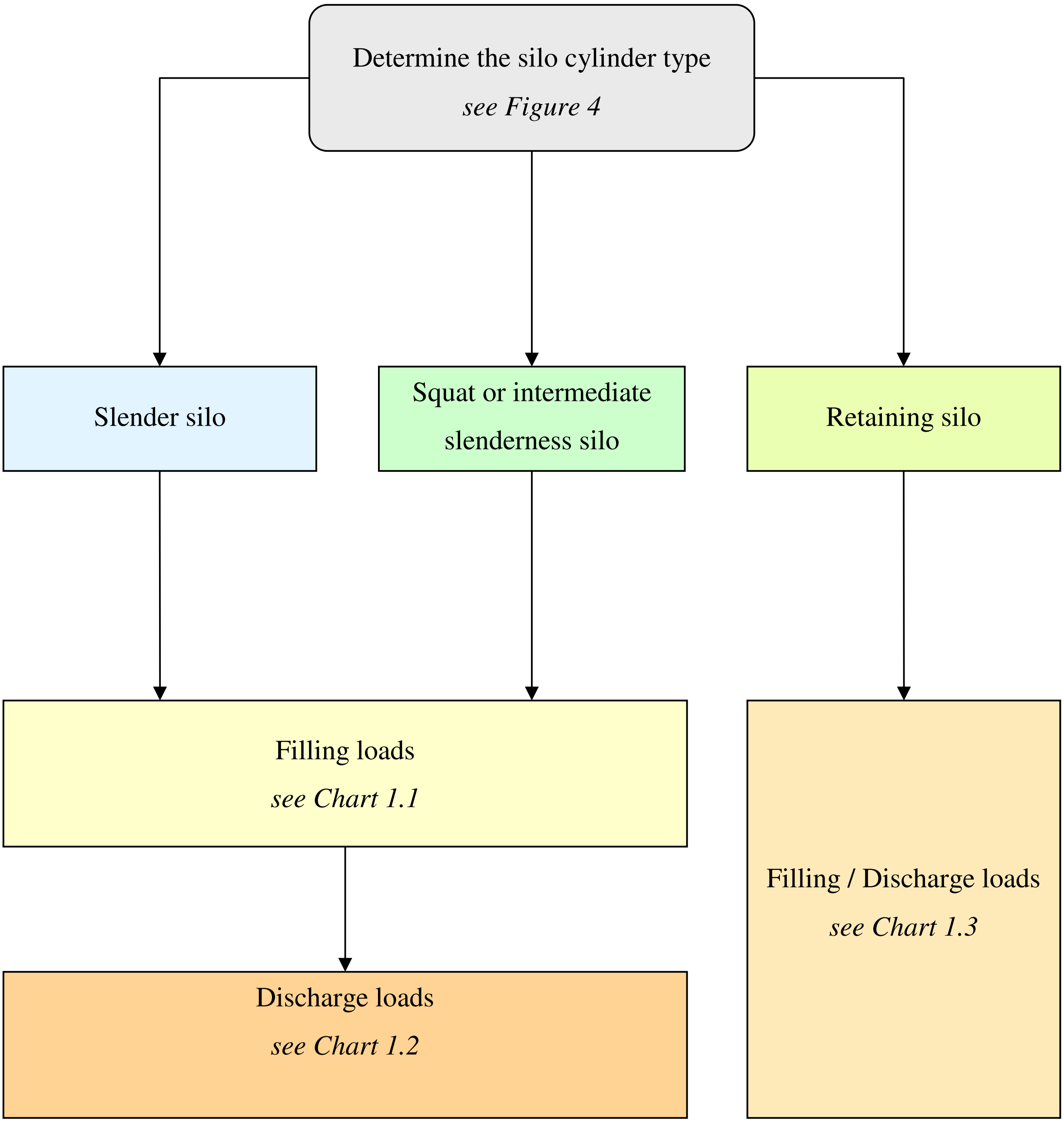


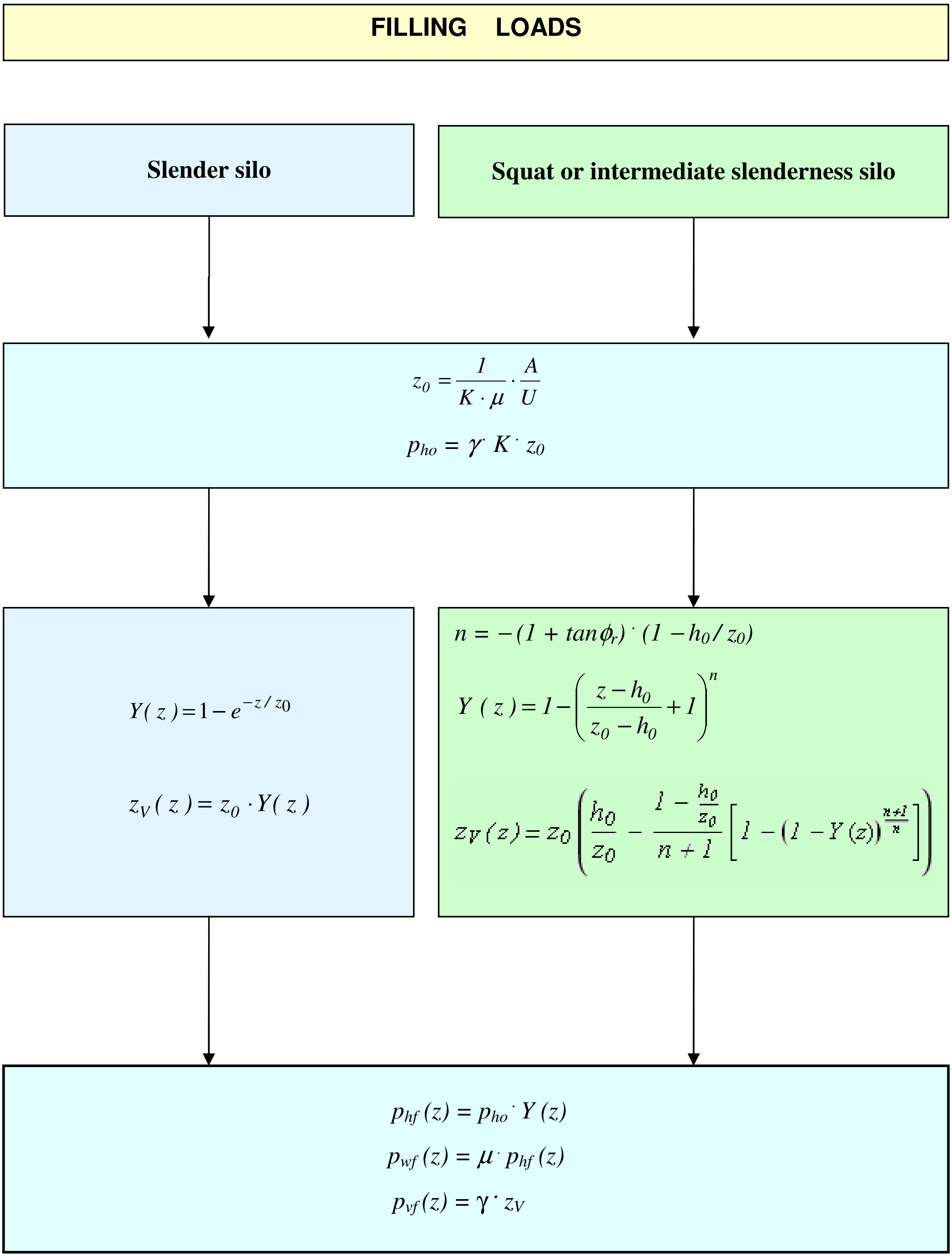
Figure 6: Pressures on the material in the cylinder and on the cylinder wall

In order to calculate the filling and discharge pressures on the cylinder wall, the procedure described in *Chart 1* has to be undertaken for each parameter combination described in *Table 11*, regarding the proper *Action Assessment Class* of the silo structure.

**Chart 1 - Loads on cylinder walls**



## Chart 1.1



$$z_0 = \frac{1}{K \cdot \mu} \cdot \frac{A}{U}$$

$$p_{ho} = \gamma \cdot K \cdot z_0$$

$$Y(z) = 1 - e^{-z/z_0}$$

$$z_V(z) = z_0 \cdot Y(z)$$

$$n = -(1 + \tan\phi_r) \cdot (1 - h_0 / z_0)$$

$$Y(z) = 1 - \left( \frac{z - h_0}{z_0 - h_0} + 1 \right)^n$$

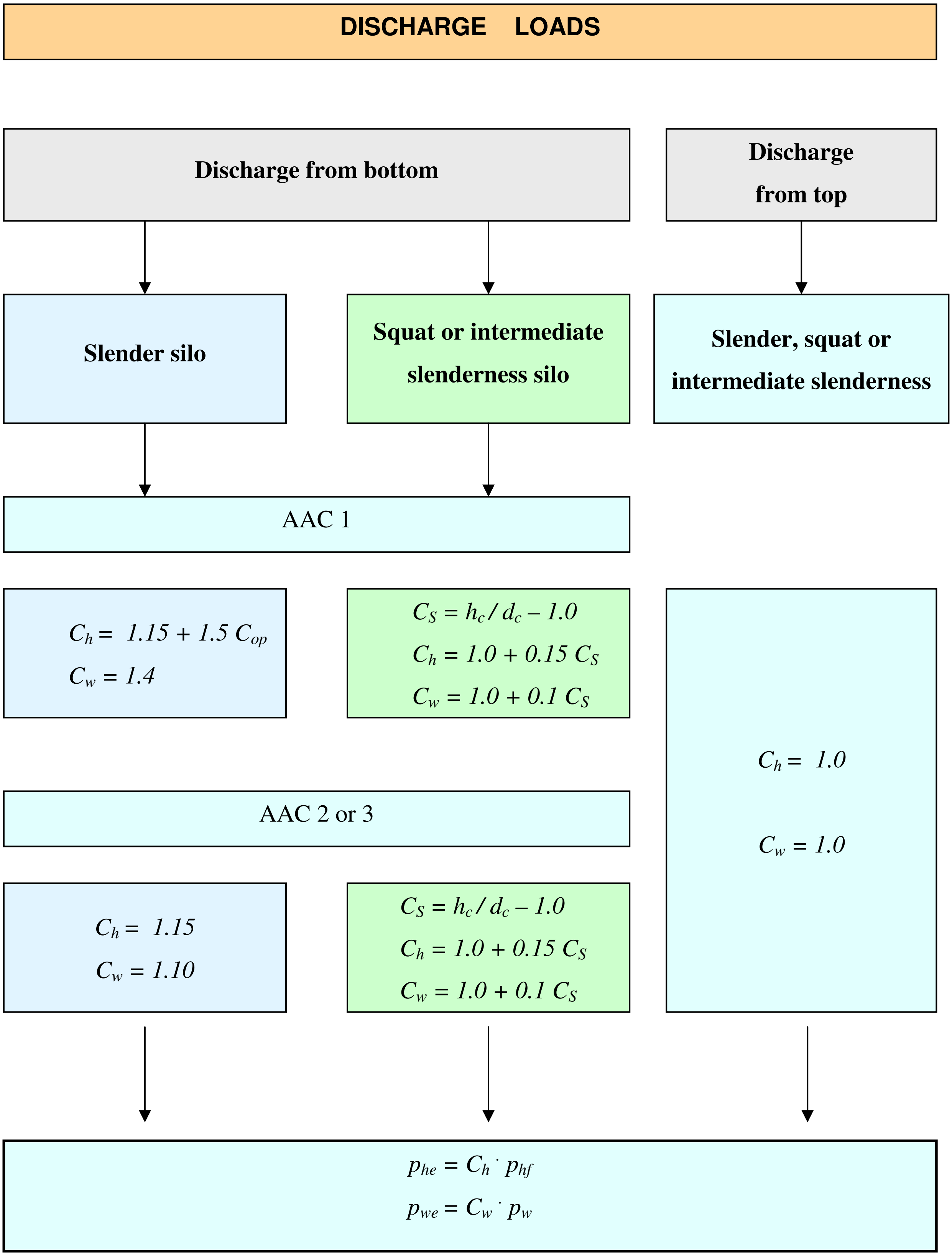
$$z_V(z) = z_0 \left( \frac{h_0}{z_0} - \frac{1 - \frac{h_0}{z_0}}{n + 1} \left[ 1 - (1 - Y(z))^{\frac{n+1}{n}} \right] \right)$$

$$p_{hf}(z) = p_{ho} \cdot Y(z)$$

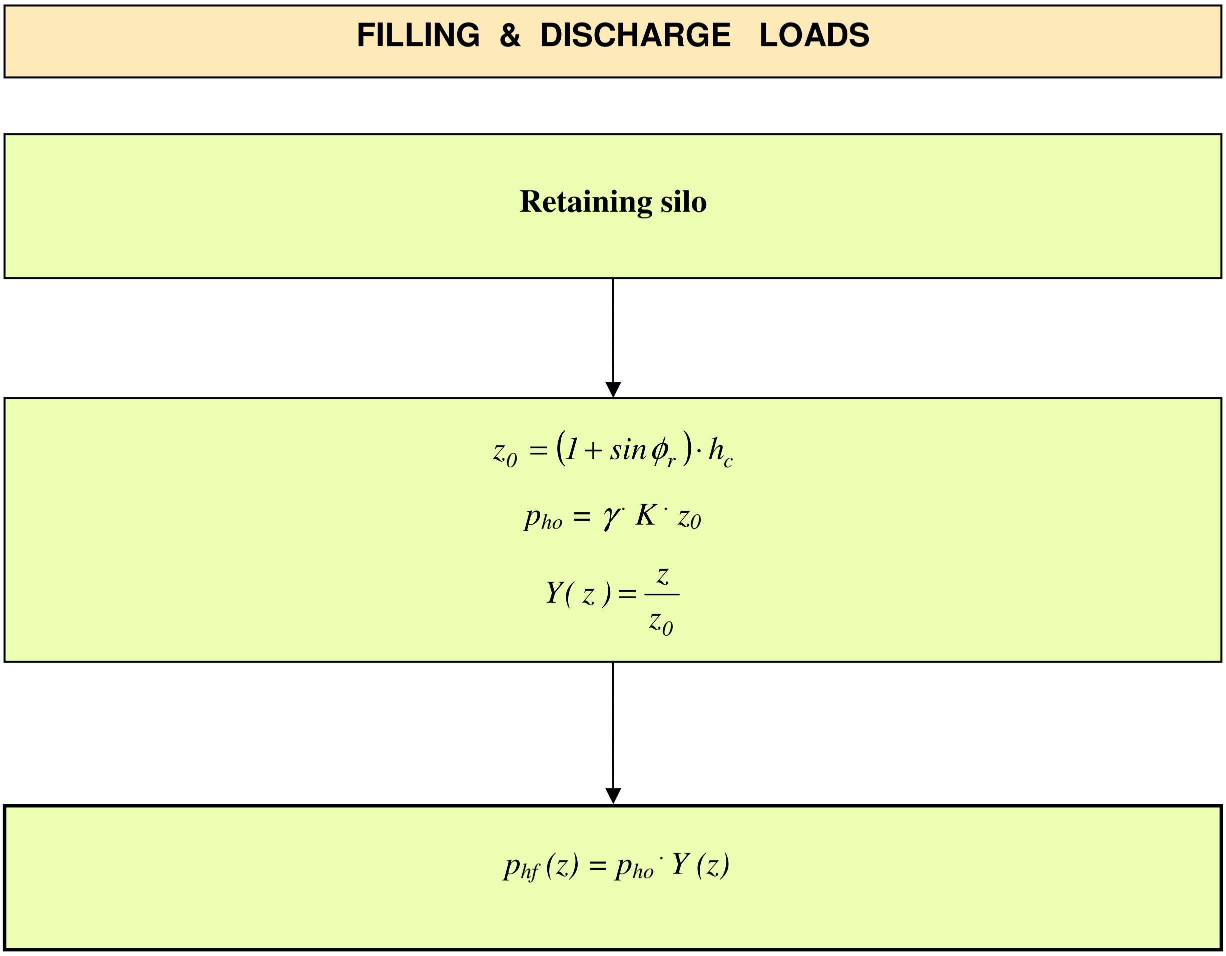
$$p_{wf}(z) = \mu \cdot p_{hf}(z)$$

$$p_{vf}(z) = \gamma \cdot z_V$$

**Chart 1.2**


DISCHARGE LOADS
Discharge from bottom
Discharge from top
Slender silo
Squat or intermediate slenderness silo
Slender, squat or intermediate slenderness
AAC 1
$C_h = 1.15 + 1.5\ C_{op}$
$C_w = 1.4$
$C_S = h_c / d_c - 1.0$
$C_h = 1.0 + 0.15\ C_S$
$C_w = 1.0 + 0.1\ C_S$
$C_h = 1.0$
$C_w = 1.0$
AAC 2 or 3
$C_h = 1.15$
$C_w = 1.10$
$C_S = h_c / d_c - 1.0$
$C_h = 1.0 + 0.15\ C_S$
$C_w = 1.0 + 0.1\ C_S$
$p_{he} = C_h \cdot p_{hf}$
$p_{we} = C_w \cdot p_w$

## Chart 1.3


FILLING & DISCHARGE LOADS
Retaining silo
$z_0 = (1 + \sin \phi_r) \cdot h_c$
$p_{ho} = \gamma \cdot K \cdot z_0$
$Y(z) = \frac{z}{z_0}$
$p_{hf}(z) = p_{ho} \cdot Y(z)$

### 3.1.1.7 Step 7: Loads on hopper wall

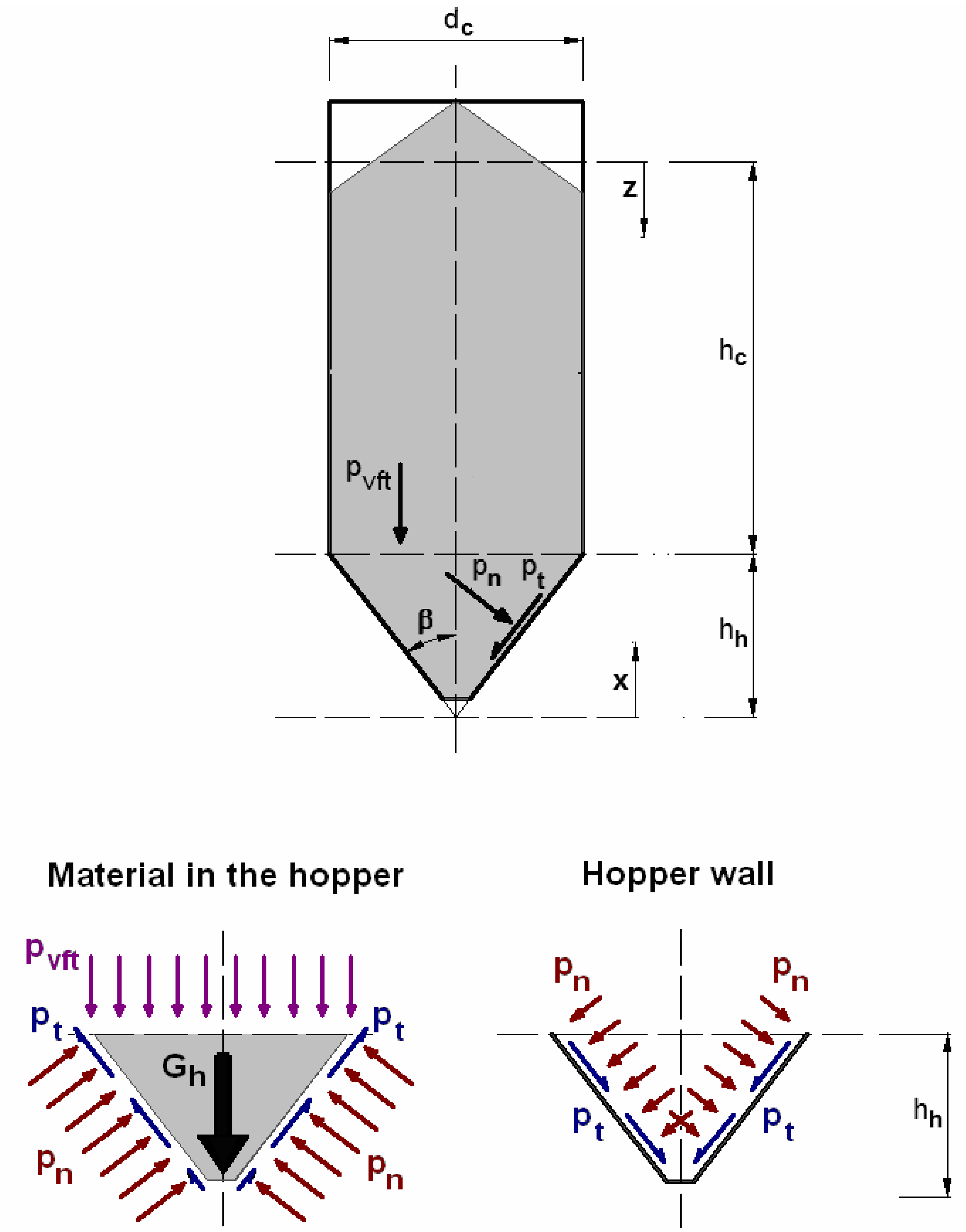


**Figure 7: Pressures on the material in the hopper and on the hopper wall**

In order to calculate the filling and discharge loads on the hopper wall, the procedure described in *Chart 2* has to be undertaken for each for each parameter combination described in *Table 12*.

The table below is used to determine the bottom load magnifier ($C_b$), which is needed in the procedure described in *Chart 2.1*.

**Table 13: Bottom load magnifier ($C_b$)**

| | AAC | $C_b$ |
|---|---|---|
| **STATIC or Standard** | 1 | 1.3 |
| | 2 and 3 | 1.0 |
| **DYNAMIC** | 1 | 1.6 |
| | 2 and 3 | 1.2 |

The dynamic load magnifier must be used where there is a significant probability that the stored solid can develop dynamic loading conditions. These conditions should be assumed to occur if either:

- a silo with a slender vertical walled section is used to store solids that cannot be classed as of low cohesion (see *EN 1991-4, 1.5.24*),
- the stored solid is identified as susceptible to mechanical interlocking (II.g. cement clinker).

## Chart 2 - Loads on hopper walls

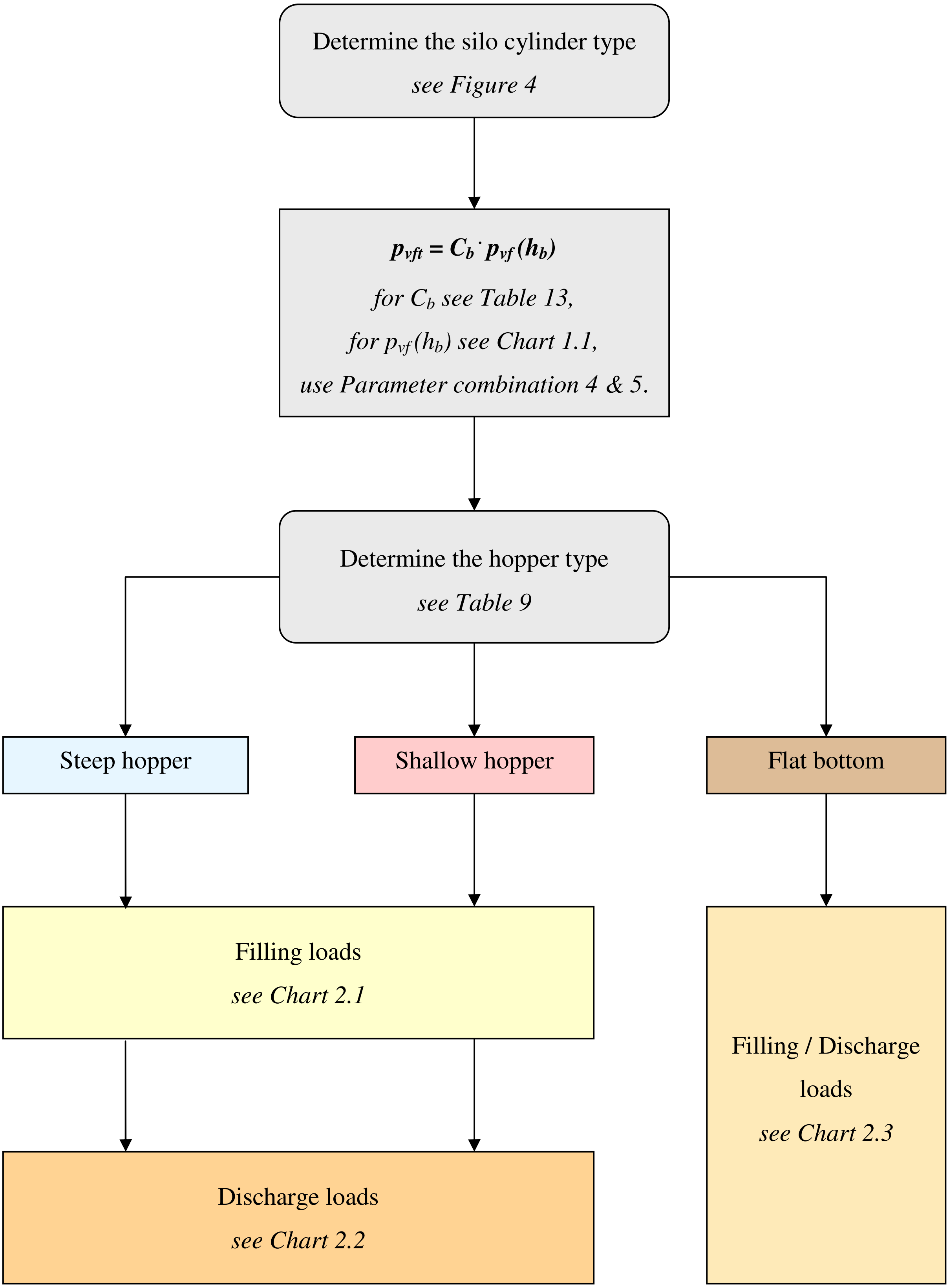
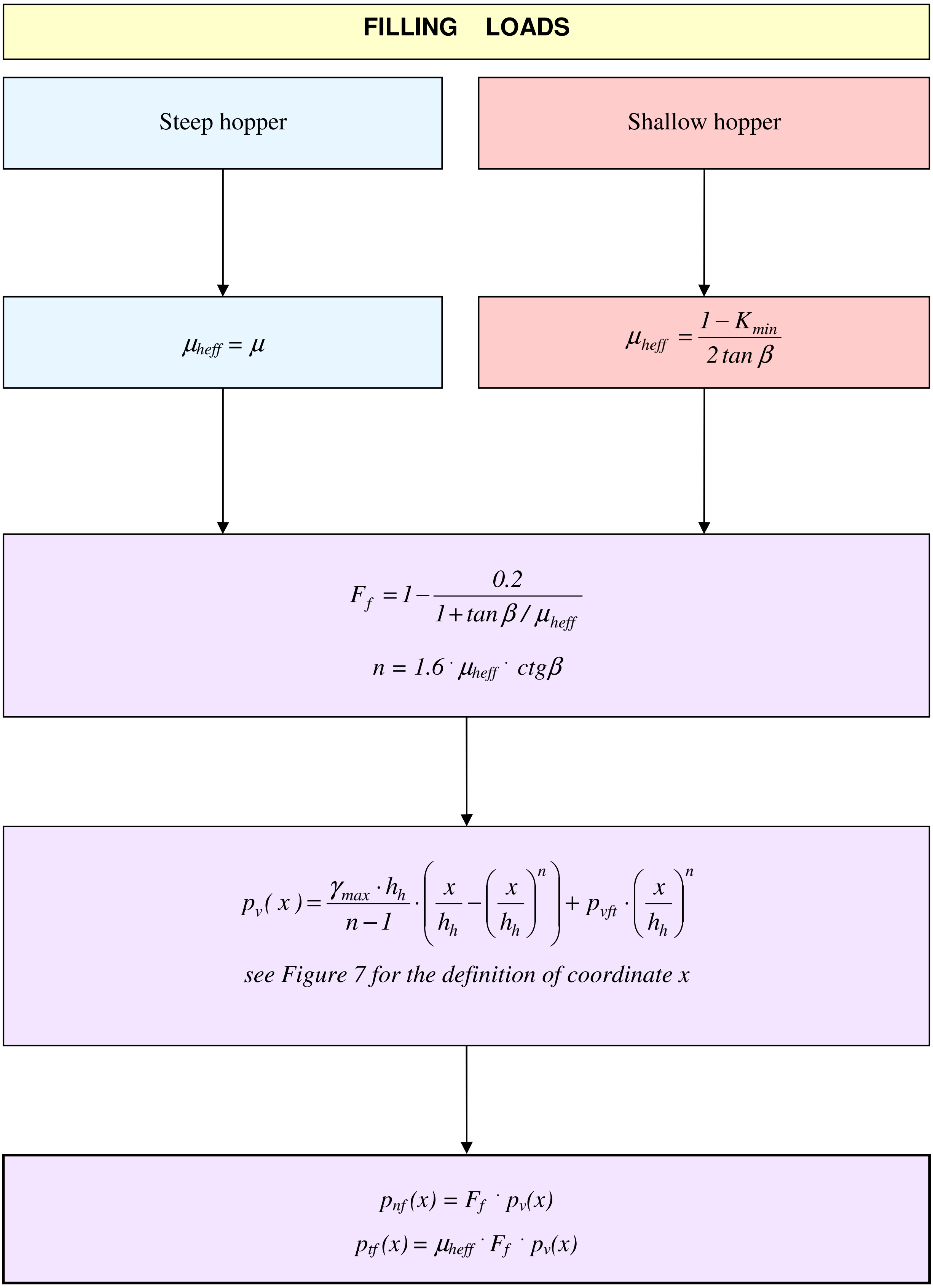


**Chart 2.1**



**FILLING LOADS**

Steep hopper

$$\mu_{heff} = \mu$$

Shallow hopper

$$\mu_{heff} = \frac{1 - K_{min}}{2\tan\beta}$$

$$F_f = 1 - \frac{0.2}{1 + \tan\beta / \mu_{heff}}$$

$$n = 1.6 \cdot \mu_{heff} \cdot ctg\beta$$

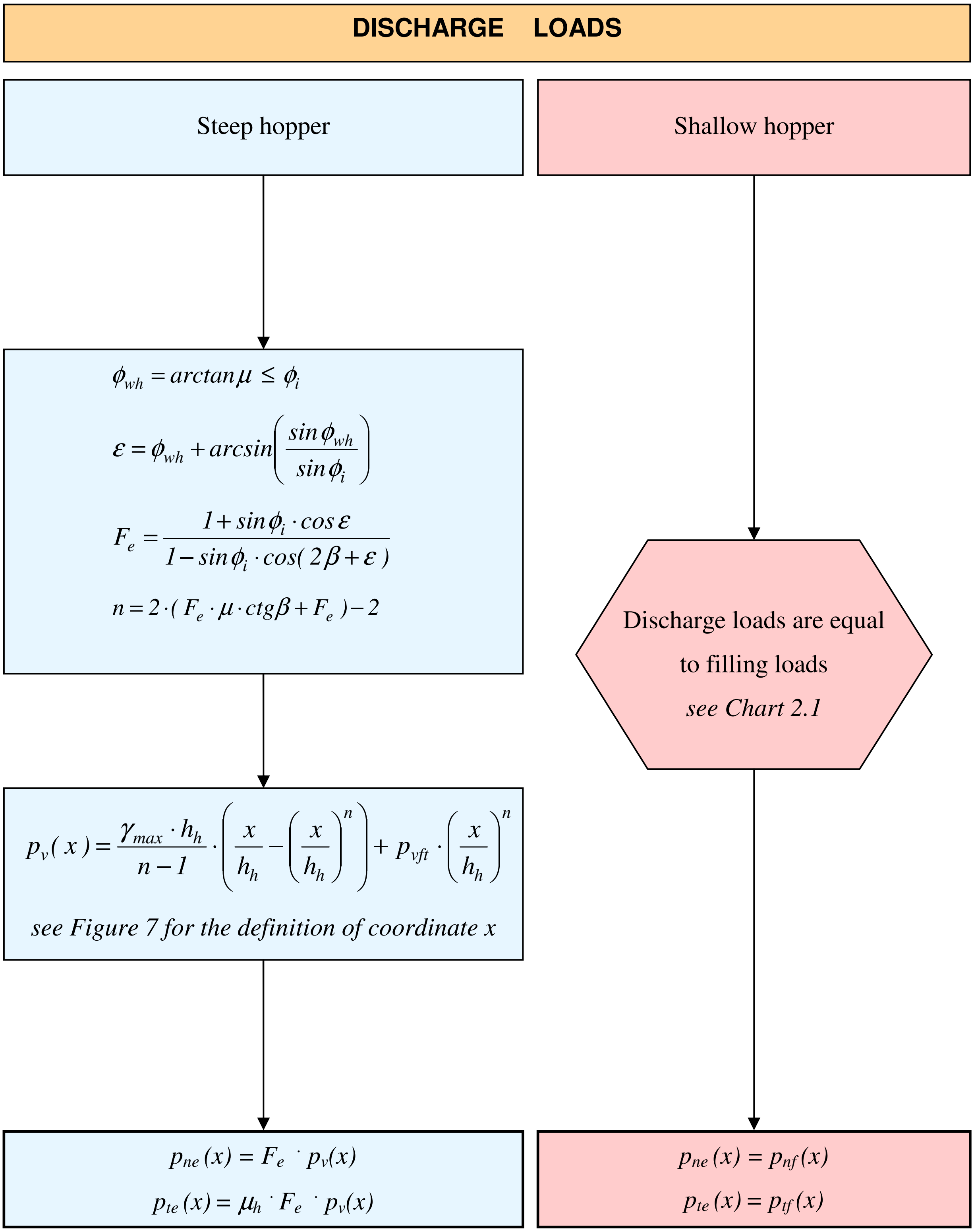
$$p_v(x) = \frac{\gamma_{max} \cdot h_h}{n-1} \cdot \left( \frac{x}{h_h} - \left( \frac{x}{h_h} \right)^n \right) + p_{vft} \cdot \left( \frac{x}{h_h} \right)^n$$

*see Figure 7 for the definition of coordinate x*

$$p_{nf}(x) = F_f \cdot p_v(x)$$

$$p_{tf}(x) = \mu_{heff} \cdot F_f \cdot p_v(x)$$

## Chart 2.2



**DISCHARGE LOADS**

Steep hopper

$$\phi_{wh} = arctan\,\mu \leq \phi_i$$

$$\varepsilon = \phi_{wh} + arcsin\left(\frac{sin\,\phi_{wh}}{sin\,\phi_i}\right)$$

$$F_e = \frac{1 + sin\,\phi_i \cdot cos\,\varepsilon}{1 - sin\,\phi_i \cdot cos(\,2\beta + \varepsilon\,)}$$

$$n = 2 \cdot (\,F_e \cdot \mu \cdot ctg\beta + F_e\,) - 2$$

$$p_v(\,x\,) = \frac{\gamma_{max} \cdot h_h}{n-1} \cdot \left(\frac{x}{h_h} - \left(\frac{x}{h_h}\right)^n\right) + p_{vft} \cdot \left(\frac{x}{h_h}\right)^n$$

*see Figure 7 for the definition of coordinate x*

$$p_{ne}(x) = F_e \cdot p_v(x)$$

$$p_{te}(x) = \mu_h \cdot F_e \cdot p_v(x)$$

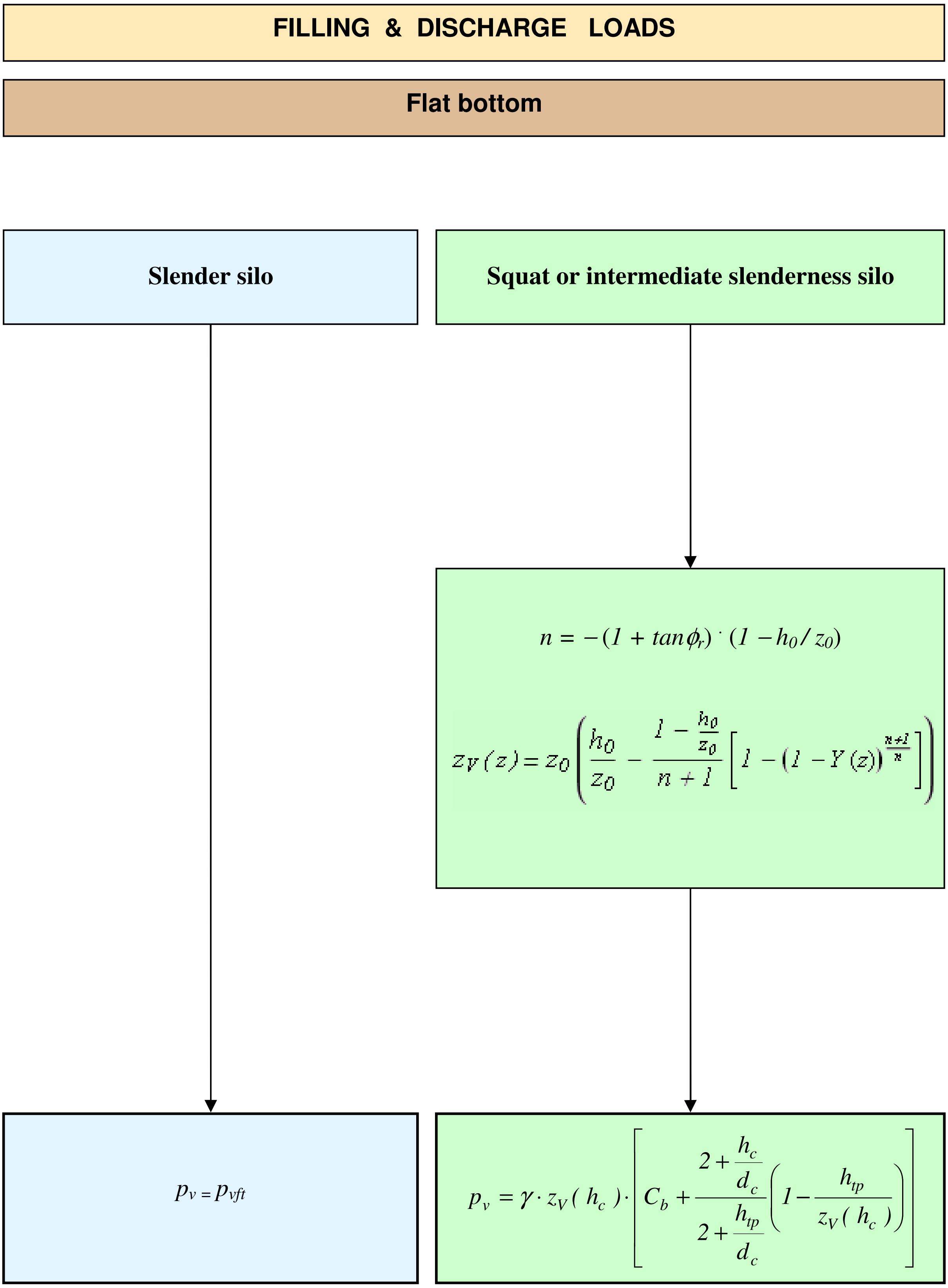
Shallow hopper

Discharge loads are equal
to filling loads
*see Chart 2.1*

$$p_{ne}(x) = p_{nf}(x)$$

$$p_{te}(x) = p_{tf}(x)$$

**Chart 2.3**



**FILLING & DISCHARGE LOADS**

**Flat bottom**

**Slender silo**

**Squat or intermediate slenderness silo**

$$n = -(1 + \tan\phi_r) \cdot (1 - h_0 / z_0)$$

$$z_V(z) = z_0 \left( \frac{h_0}{z_0} - \frac{1 - \frac{h_0}{z_0}}{n+1} \left[ 1 - \left(1 - Y(z)\right)^{\frac{n+1}{n}} \right] \right)$$

$$p_v = p_{vft}$$

$$p_v = \gamma \cdot z_V(h_c) \cdot \left[ C_b + \frac{2 + \frac{h_c}{d_c}}{2 + \frac{h_{tp}}{d_c}} \left( 1 - \frac{h_{tp}}{z_V(h_c)} \right) \right]$$

## 3.1.2 Graphical representation of pressures on silo walls

### 3.1.2.1 Pressures on cylinder wall

In this chapter we will make dimensionless plots of the pressures acting on silo cylinder walls ($p_h$ and $p_w$) and of the vertical pressure in the stored solid $p_v$, due to the filling and discharge of the stored solid material.

We can see from *Chart 1.1.*, that pressures on the cylinder wall due to the stored solid material can be determined as a product of the horizontal pressure depth variation function *Y(z)*, which represents the "shape" of the pressure distribution, and constant factors, which take into account the different stored granular material properties. Different pressure variation functions are defined for different silo cylinder types.

The vertical pressure in the stored solid $p_v$ is given as a product of the vertical pressure depth variation function $z_V(z)$ and constant material parameters.

#### *3.1.2.1.1 Slender silo*

In slender silos, the pressure depth variation function is called the *Janssen function* and equals:

$$Y(z) = 1 - e^{-z/z_0} \tag{A.16}$$

The parameter $z_0$ represents the characteristic depth and is given by equation:

$$z_0 = \frac{1}{K \cdot \mu} \cdot \frac{A}{U} \tag{A.17}$$

The pressure depth variation function $z_V(z)$ used for the calculation of the vertical pressure in the stored solid at any depth $z$ is given by:

$$z_V(z) = z_0 \cdot Y(z) \tag{A.18}$$

We see that in the case of slender silos, only the *Janssen pressure depth variation function* defines the "shape" of the pressures on the cylinder wall and on the stored solid.

The figure below represents parametrical plots of $Y(z/z_0)$. The function is plotted with the $z / z_0$ argument instead of just with $z$, because we are making the plot in dimensionless form. The coordinate $z$ runs from the equivalent surface ($z = 0$) to the equivalent height of the cylinder ($z = h_c$) (*Figure 3*).

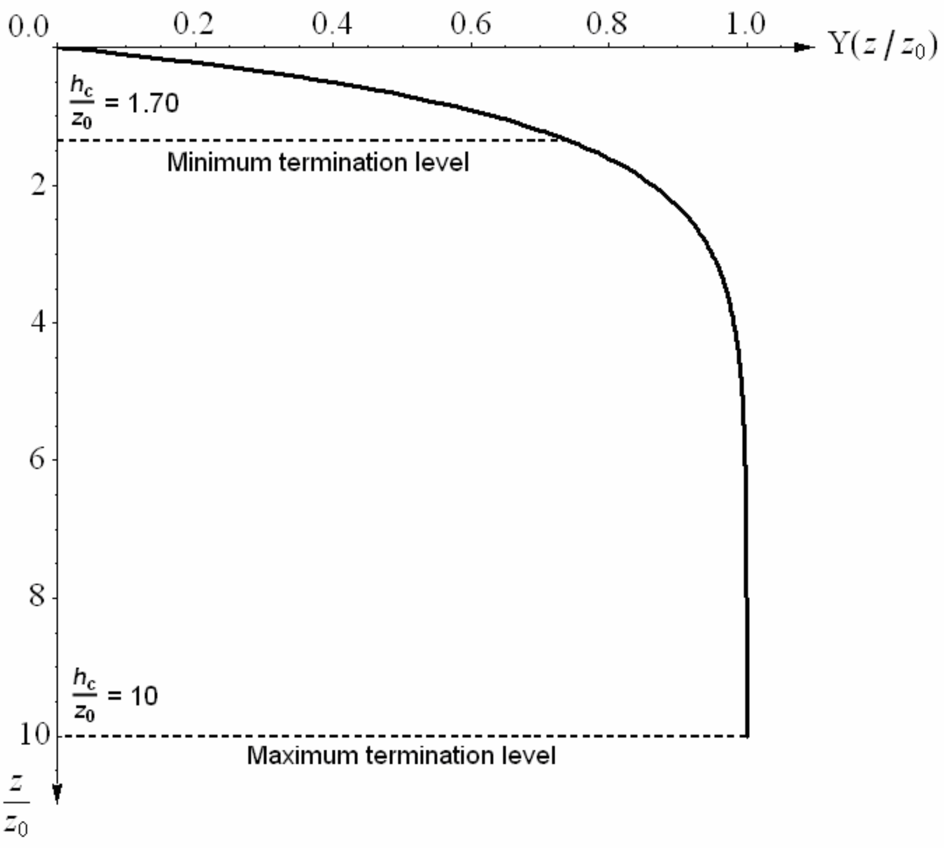


**Figure 8: Dimensionless plot of the Janssen pressure depth variation function *Y(z)* for slender silos**

The term "termination level" represents the limit value of the pressure depth variation function for a given silo cylinder. The limit value is attained, when the coordinate $z$ reaches the height $h_c$. For slender silos the minimum and maximum termination levels equal:

$$h_{c,min} = 1.7 \cdot z_0 \tag{A.19}$$

$$h_{c,max} = 10 \cdot z_0 \tag{A.20}$$

They are determined by considering the minimum silo height to diameter ratio for slender silos (*Figure 4*) and the various stored material properties.

#### 3.1.2.1.2 Intermediate slenderness and squat silo

In intermediate slenderness silos and squat silos, the pressure depth variation function equals:

$$Y(z) = 1 - \left( \frac{z - h_0}{z_0 - h_0} + 1 \right)^n \tag{A.21}$$

The parameter $z_0$ is the same as in slender silos (see equation (A.17)). The height $h_0$ represents the depth below the equivalent surface of the base of the top pile and is given in equation (A.3).

The parameter $n$ represents the exponent in the pressure function and is given by the following equation:

$$n = -(1 + \tan\phi_r) \cdot (1 - h_0/z_0) \tag{A.22}$$

The pressure depth variation function $z_V(z)$ used for the calculation of the vertical pressure in the stored solid at any depth $z$ is given by:

$$z_V(z) = z_0 \left( \frac{h_0}{z_0} - \frac{1 - \frac{h_0}{z_0}}{n + 1} \left[ 1 - (1 - Y(z))^{\frac{n+1}{n}} \right] \right) \tag{A.23}$$

All three parameters $z_0$, $h_0$ and $n$ depend on the type of stored material. Let us choose five different ratios $h_0 / z_0$. The range is such that that it takes into account the variability of the properties of the various stored granular materials that occur in engineering practice. For the given range we now have to consider the corresponding values of parameter $h_c$ and parameter $n$, which are given in the table below.

**Table 14: Range of $z_0 / h_c$ ratio and corresponding values of $h_c$ and $n$**

| $h_0 / z_0$ | 0.08 | 0.09 | 0.10 | 0.11 | 0.12 |
|---|---|---|---|---|---|
| $h_c / z_0$ | 0.70 | 0.95 | 1.2 | 1.45 | 1.70 |
| n | -1.52 | -1.59 | -1.64 | -1.80 | -1.88 |

We can now make parametric plots of the $Y(z)$ function for squat and intermediate slenderness silos. To make the plots in dimensionless form, we will divide the function with $z_0$.

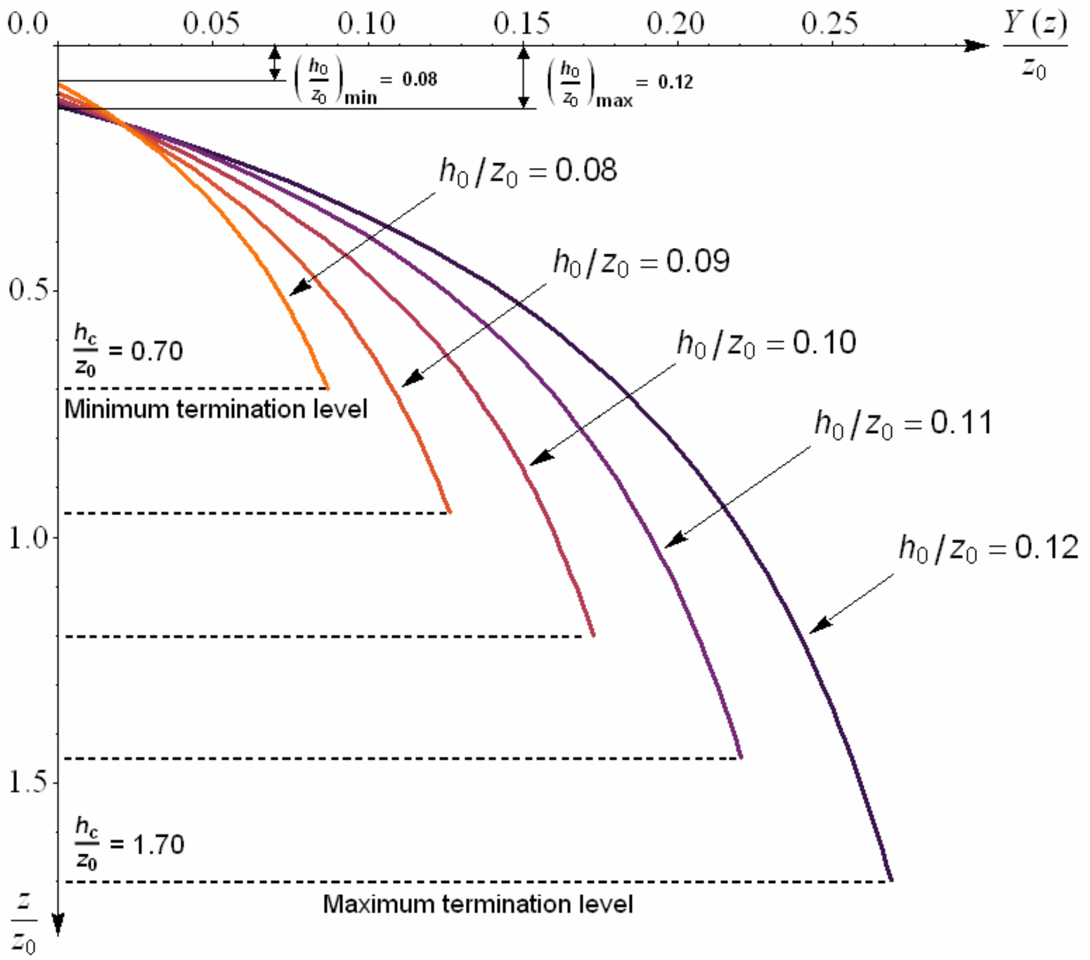


**Figure 9: Dimensionless plots of the pressure depth variation function $Y(z)$ for intermediate slenderness silos and squat silos**

Only positive values of the pressure variation function are plotted on the figure above. Negative values are not considered as the stored solid cannot cause negative pressures in the cylinder wall. We see that $Y(z) > 0$, when $z > h_0$. This means that in the case of squat and intermediate slenderness silos, we have to consider wall pressures only where the stored solid is in contact with the cylinder wall and not from the equivalent surface, like in slender silos.

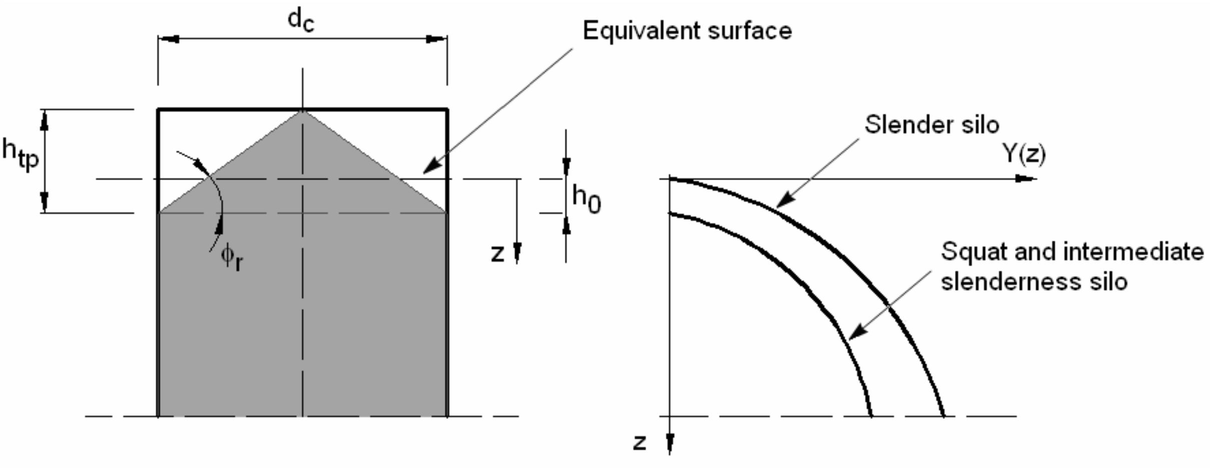


**Figure 10: Wall pressure distributions for slender silos, intermediate slenderness silos and squat silos**

As mentioned before, the parameter $h_0$ represents the distance between the base of the top pile and the equivalent surface, which is located in the centre of gravity of the top pile. We can see from *Figure 9*, that $h_0$ can be in the range *8%* to *12%* of the height $z_0$, depending of the type of stored solid material.

The minimum termination level in the case of intermediate slenderness and squat silos is given in *Table 14* and equals:

$$h_c = 0.7 \cdot z_0 \tag{A.24}$$

The maximum termination level in the case of intermediate slenderness and squat silos is also given in *Table 14*. It is the same as the minimum termination level in slender silos.

Let us now have a look at the vertical pressure depth variation function $z_V(z)$, which is used for the calculation of the vertical pressure in the stored solid. As before, we will divide the function with $z_0$ to get a dimensionless plot.

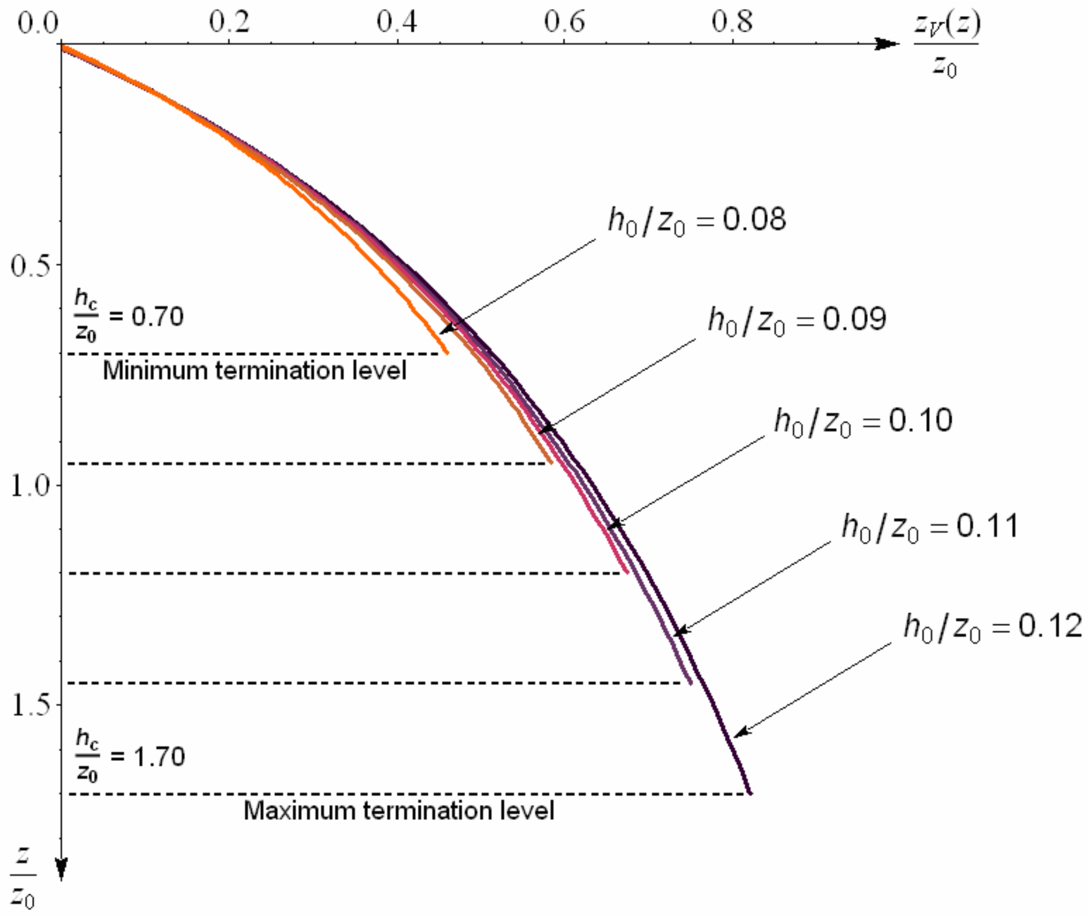


**Figure 11: Dimensionless plot of the pressure depth variation function** $z_V(z)$

If we compare the plots of the $z_V(z)$ function with $Y(z)$ function (*Figure 9*), we see that by increasing the $h_0 / z_0$ ratio, the value of the $z_V(z)$ function increases just slightly. We can conclude that the $h_0 / z_0$ ratio only has a considerable effect on the cylinder wall pressures, whereas its effects on the vertical pressures in the stored solid material are minimal.

### 3.1.2.2 Pressures on hopper wall

In this chapter parametrical plots of the mean vertical pressure $p_v(x)$, acting on the material stored in the hopper, will be presented.

The pressure $p_v(x)$ is given by the following equation:

$$p_v(x) = \frac{\gamma_{max} \cdot h_h}{n-1} \cdot \left( \frac{x}{h_h} - \left( \frac{x}{h_h} \right)^n \right) + p_{vft} \cdot \left( \frac{x}{h_h} \right)^n \tag{A.25}$$

The definition of coordinate $x$ is given in *Figure 7* and the values of parameter $n$ are given in *Charts 2.1*, *2.2* and *2.3*

Let us first divide the above equation with $p_{vft}$:

$$\frac{p_v(x)}{p_{vft}} = \frac{\gamma_{max} \cdot h_h}{(n-1) \cdot p_{vft}} \cdot \left( \frac{x}{h_h} - \left( \frac{x}{h_h} \right)^n \right) + \left( \frac{x}{h_h} \right)^n \tag{A.26}$$

The new equation can be simplified, if we define:

$$\alpha(n) = \frac{\gamma_{max} \cdot h_h}{(n-1) \cdot p_{vft}} \tag{A.27}$$

$$\xi = \frac{x}{h_h} \tag{A.28}$$

Equation (A.26) can now be written as:

$$\frac{p_v(\xi)}{p_{vft}} = \alpha(n) \cdot \left( \xi - \xi^n \right) + \xi^n \tag{A.29}$$

We see that the equation for the mean vertical pressure in the material stored in the hopper can be written as a sum of two parts. The left part of the equation represents contribution of the weight of the material stored in the hopper. The right part of the equation represents the contribution of the mean vertical pressure coming from the cylinder.

Let us denote the left part of equation (A.29) with a new function called $\kappa_{hop}(\xi)$ and the right part of the equation with a new function called $\kappa_{cyl}(\xi)$:

$$\kappa_{hop}(\xi) = \alpha(n) \cdot \left(\xi - \xi^n\right) \tag{A.30}$$

$$\kappa_{cyl}(\xi) = \xi^n \tag{A.31}$$

Equation (A.29) can then be written as:

$$\frac{p_v(\xi)}{p_{vft}} = \kappa_{hop}(\xi) + \kappa_{cyl}(\xi) \tag{A.32}$$

We will now make parametric plots of functions $\kappa_{hop}(\xi)$, $\kappa_{cyl}(\xi)$ and of the combined function $p_v(\xi)/p_{vft}$ to see to what extent the weight of the material stored in the cylinder of the silo contributes to the vertical pressure $p_v(\xi)$.

The default material presented in section 3.1.2.1 will be used, together with parameter *n* ranging from *0.5* to *4.5*. The given range of *n* is such, that it takes into account all of the silo and hopper types (bigger *n* equals steeper hopper). As $\alpha(n)$ is also a function of *n*, the corresponding values need to be considered. The considered values for *n* and its correlating values for the hopper apex half angle $\beta$ and $\alpha(n)$ are given in the following table:

**Table 15: Values of $\beta$ and $\alpha(n)$**

| n | Approximate value of $\beta$ | $\alpha$(n) |
|---|---|---|
| 0.5 | 70° | -1.40 |
| 1.5 | 45° | 3.00 |
| 2.5 | 30° | 0.85 |
| 3.5 | 15° | 0.60 |
| 4.5 | 15° | 0.40 |

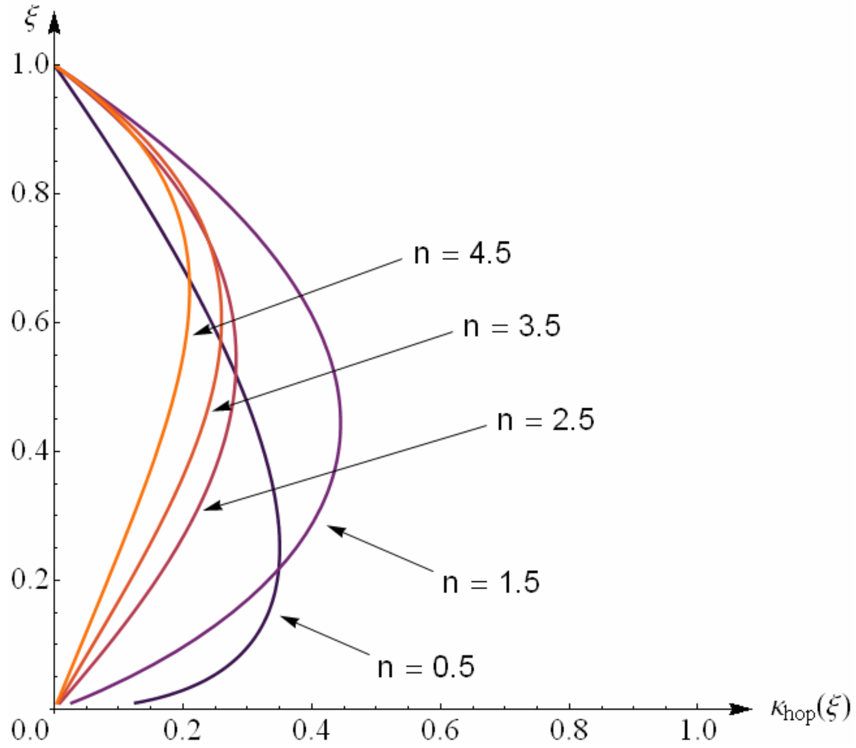


**Figure 12: Contribution of the material stored in the hopper to the vertical pressure $p_v(\xi)$**

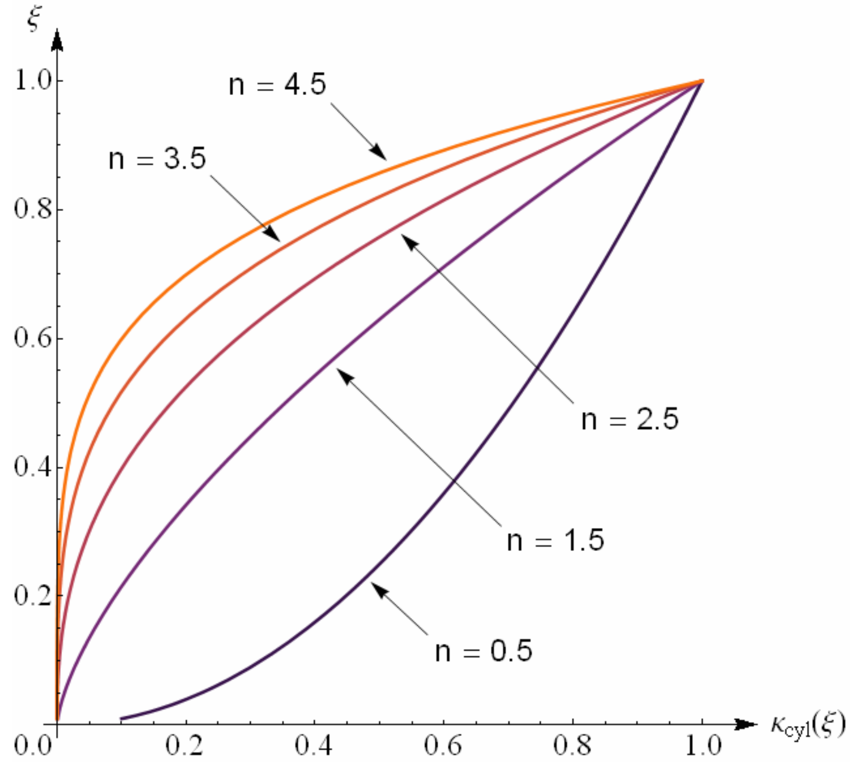


**Figure 13: Contribution of the material stored in the cylinder to the vertical pressure $p_v(\xi)$**

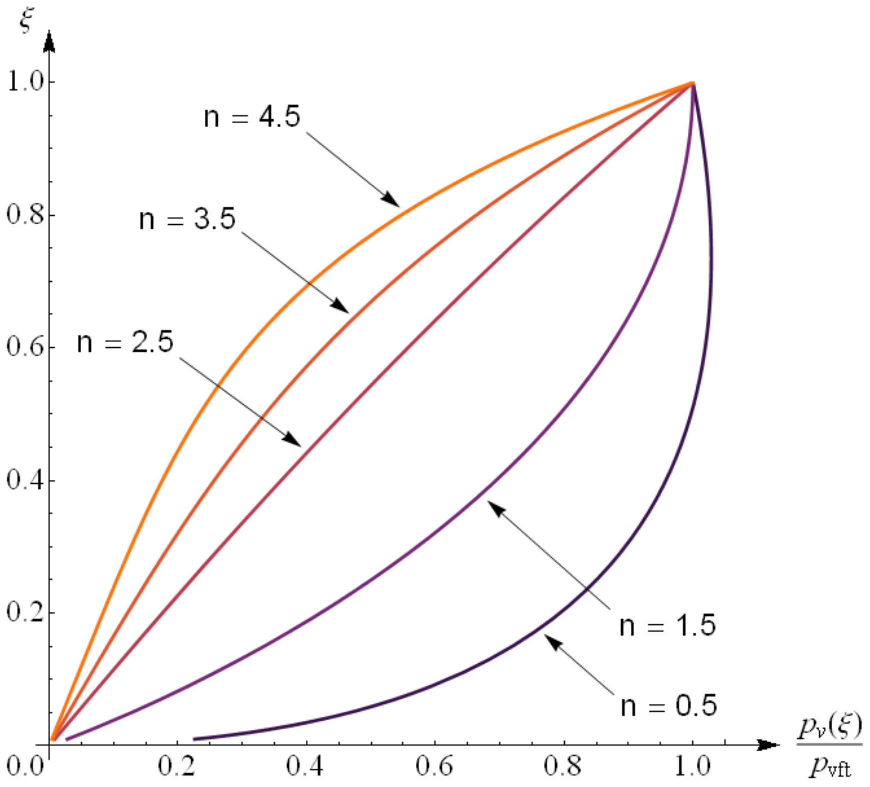


**Figure 14: The distribution of the vertical pressure $p_v(\xi)$ in the hopper**

We can see from *Figure 12* that the biggest contribution of the material stored in the hopper to the vertical pressure $p_v(\xi)$ appears in the range of $\xi$ from *0.2* to *0.7*, depending on parameter *n*. The contribution is highest when $n = 1.5$. In this case it represents a maximum of about 65% percent of the total $p_v(\xi)/p_{vft}$ ratio. This contribution is lower, when *n* is bigger. Its lowest point represents a maximum of *35%* of the total pressure ratio and is obtained when $n = 4.5$.

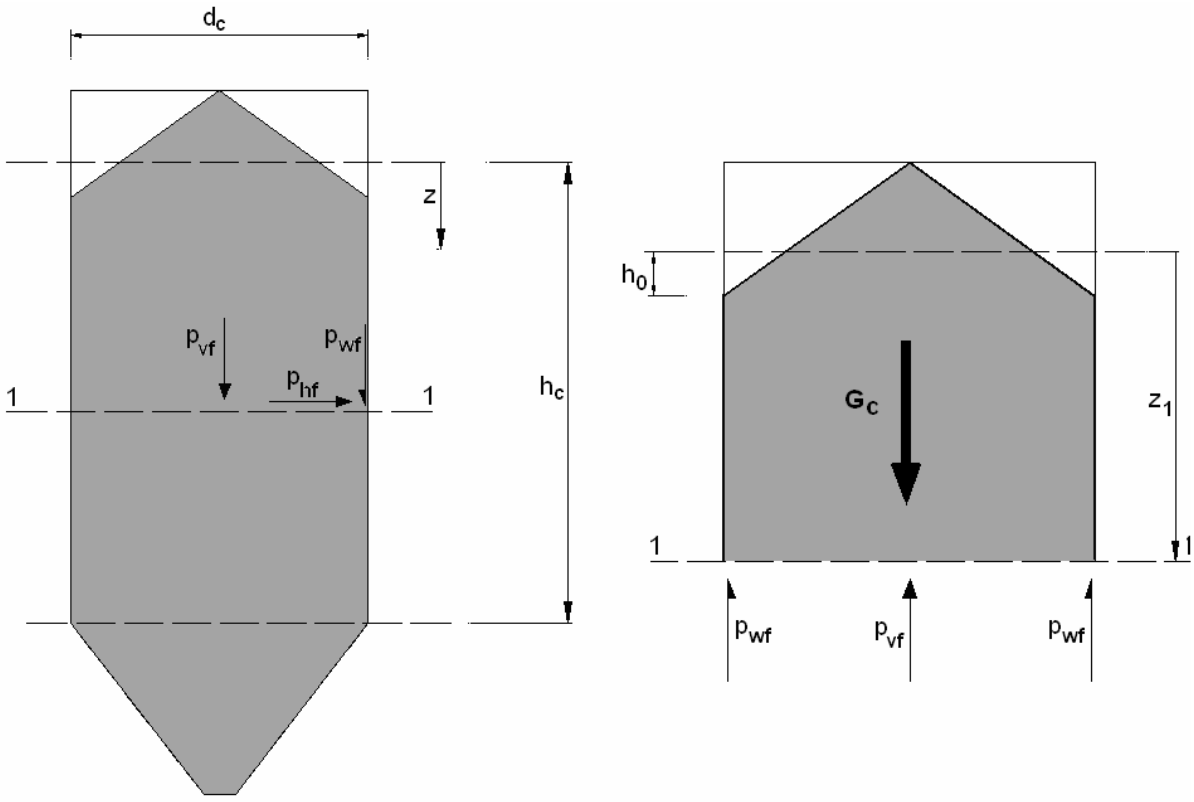
The contribution of the material stored in the cylinder is higher, when *n* is smaller. The pressure contribution increases exponentially from *0%* at the bottom of the hopper, to *100%* at the top.

We can conclude that for flat or shallow hoppers, where the values of parameter *n* are low ($n < 1$), the material stored in the silo cylinder has a greater effect on the vertical pressure in the hopper. It amounts to a maximum contribution of about *65%* to the total vertical pressure ratio. In the cases of steep hoppers, the contribution of the material stored in the cylinder is smaller and can represent up to *35%* of the total vertical pressure ratio. $p_v(\xi)/p_{vft}$.

## 3.1.3 Silo equilibrium checks

### 3.1.3.1 Silo cylinder

The values of the vertical pressure $p_{vf}$ and the wall frictional traction $p_{wf}$ at any depth after filling and during storage have to be in equilibrium with the weight of the stored solid $G_c$ at that depth.



(a) (b)

**Figure 15: Filling pressures on silo cylinder wall (a) and the equilibrium condition (b)**

### *3.1.3.1.1 Slender silo*

The values for these pressures for slender silos are:

$$p_{wf}(z) = \mu \cdot \gamma \cdot K \cdot z_o \cdot Y_J(z) \tag{A.33}$$

$$p_{vf}(z) = \gamma \cdot z_o \cdot Y_J(z) \tag{A.34}$$

The functions $Y_J(z)$ and $z_0$ are defined in equations (A.16) and (A.17).

The total wall friction force $P_w$ at height $z_1$ equals:

$$P_w = \int_0^{2\pi} \int_0^{z_1} p_{wf}(z)\, dz\, r\, d\phi \tag{A.35}$$

Considering equation (A.33) with (A.35) we get:

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \int_0^{2\pi} \int_0^{z_1} (1 - e^{-z/z_0})\, dz\, r\, d\phi$$

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \cdot 2 \cdot \pi \cdot r \int_0^{z_1} (1 - e^{-z/z_0})\, dz$$

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \cdot U \cdot \left(z_1 - z_0 + z_0 \cdot e^{-z_1/z_0}\right) \tag{A.36}$$

The total vertical force $P_v$ at height $z_1$ equals:

$$P_v = A \cdot p_{vf}$$

$$P_v = A \cdot \gamma \cdot z_0 \cdot \left(1 - e^{-z_1/z_0}\right) \tag{A.37}$$

The weight of the stored solid in the silo cylinder $G_c$ at height $z_1$ equals:

$$G_c = A \cdot \gamma \cdot z_1 \tag{A.38}$$

The system is in equilibrium, when the weight $G_c$ is equal to the sum of the vertical force $P_v$ and the wall friction force $P_w$.

**Equilibrium condition:** $P_v + P_w = G_c$ (A.39)

If we consider the equations (A.36) to (A.38) with the condition above, we get:

$$\mu \cdot \gamma \cdot K \cdot z_0 \cdot U \cdot \left( z_1 - z_0 + z_0 \cdot e^{-z_1/z_0} \right) + A \cdot \gamma \cdot z_0 \cdot \left( 1 - e^{-z_1/z_0} \right) = A \cdot \gamma \cdot z_1 \tag{A.40}$$

The product $K \cdot z_0$ simplifies into:

$$K \cdot z_0 = \frac{1}{\mu} \cdot \frac{A}{U} \tag{A.41}$$

Equation (A.40) can now be further simplified:

$$\mu \cdot \gamma \cdot \frac{1}{\mu} \cdot \frac{A}{U} \cdot U \cdot \left( z_1 - z_0 + z_0 \cdot e^{-z_1/z_0} \right) + A \cdot \gamma \cdot z_0 \cdot \left( 1 - e^{-z_1/z_0} \right) = A \cdot \gamma \cdot z_1$$

$$z_1 - z_0 + z_0 \cdot e^{-z_1/z_0} + z_0 - z_0 \cdot e^{-z_1/z_0} = z_1$$

$$\boxed{z_1 = z_1} \tag{A.42}$$

This means that the equilibrium condition is exactly fulfilled for any chosen height $z$.

### *3.1.3.1.2 Intermediate slenderness silo and squat silo*

The values of the vertical pressure $p_{vf}$ and wall frictional traction $p_{wf}$ at any depth after filling are determined as:

$$p_{vf}(z) = \gamma\, z_V \tag{A.43}$$

$$p_{wf}(z) = \mu \cdot p_{hf} \tag{A.44}$$

The function $z_v(z)$ is given in equation (A.23).

The horizontal pressure $p_{hf}$ is determined as:

$$p_{hf} = p_{ho} \cdot Y_R(z) \tag{A.45}$$

in which the functions $Y_J(z)$ and $z_0$ are defined in equations (A.16) and (A.17) and

$$p_{ho} = \gamma \cdot \frac{1}{\mu} \cdot \frac{A}{U} \tag{A.46}$$

Considering the expressions above, the total wall friction force $P_w$ at height $z_1$ can be obtained from equation (A.35). Here we have to integrate from $h_0$, because in the case of squat and intermediate silos, the pressures are calculated from the point $h_0$ onward.

$$P_w = \int_0^{2\pi} \int_{h_0}^{z_1} p_{wf}(z)\, dz\, r\, d\phi$$

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \cdot 2\pi \cdot r \int_{h_0}^{z_1} 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n dz$$

$$P_w = \mu \cdot \gamma \cdot K \cdot z_0 \cdot U \int_{h_0}^{z_1} Y_R(z)\, dz \tag{A.47}$$

The total vertical force $P_v$ at height $z_1$ equals:

$$P_v = A \cdot p_{vf}$$

$$P_v = A \cdot \gamma \cdot z_V \tag{A.48}$$

The weight of the stored solid in the silo cylinder $G_c$ at height $z_1$ is given in equation (A.38)(I.38).

The system is in equilibrium, when the weight $G_c$ is equal to the sum of the vertical force $P_v$ and the wall friction force $P_w$ at a certain height $z_1$, thus condition (A.39) applies:

$$P_v + P_w = G_c$$

$$A \cdot \gamma \cdot z_V + \mu \cdot \gamma \cdot K \cdot z_0 \cdot U \int_{h_0}^{z_1} Y_R(z)\, dz = A \cdot \gamma \cdot z_1 \tag{A.49}$$

If we consider equation (A.41), the equilibrium condition simplifies further:

$$A \cdot \gamma \cdot z_V + \mu \cdot \gamma \cdot \frac{1}{\mu} \cdot \frac{A}{U} \cdot U \int_{h_0}^{z_1} Y_R(z)\, dz = A \cdot \gamma \cdot z_1$$

$$z_V + \int_{h_0}^{z_1} Y_R(z)\, dz = z_1 \tag{A.50}$$

If we divide the equation above with $z_0$ and express it with $\frac{z_V}{z_0}$, we get:

$$\frac{z_V}{z_0} + \int_{h_0}^{z_1} \frac{Y_R(z)}{z_0}\, dz = \frac{z_1}{z_0}$$

$$\frac{z_V}{z_0} = \frac{z_1}{z_0} - \int_{h_0}^{z_1} \frac{Y_R(z)}{z_0}\, dz \tag{A.51}$$

Let us denote with *L* the left side of the equation above and with *R* the right side of the equation above. If the equilibrium condition is fulfilled, then the equality *L* = *R* must be true.

$$L = \frac{z_V}{z_0} = \frac{h_0 - \dfrac{1}{n+1}\left( z_0 - h_0 - \dfrac{(z_1 + z_0 - 2h_0)^{n+1}}{(z_0 - h_0)^n} \right)}{z_0}$$

$$L = \frac{h_0}{z_0} - \frac{1}{n+1}\left( 1 - \frac{h_0}{z_0} - \left[ \frac{z_1/z_0 - h_0/z_0}{1 - h_0/z_0} + 1 \right]^{n+1} \right)$$

$$L = \frac{h_0}{z_0} - \frac{1 - h_0/z_0}{n+1}\left( 1 - \frac{z_0}{z_0 - h_0}\left[ \frac{z_1/z_0 - h_0/z_0}{1 - h_0/z_0} + 1 \right]^{n+1} \right) \tag{A.52}$$

$$R = \frac{z_1}{z_0} - \int_{h_0}^{z_1} \frac{Y_R(z)}{z_0}\, dz = \frac{z_1}{z_0} - \int_{h_0}^{z_1} \frac{1 - \left( \dfrac{z - h_0}{z_0 - h_0} + 1 \right)^n}{z_0}\, dz$$

$$R = \frac{z_1}{z_0} - \left( \frac{z_1 - h_0}{z_0} - \frac{1}{n+1}\left( 1 - \frac{h_0}{z_0} - \left[ \frac{z_1/z_0 - h_0/z_0}{1 - h_0/z_0} + 1 \right]^{n+1} \right) \right)$$

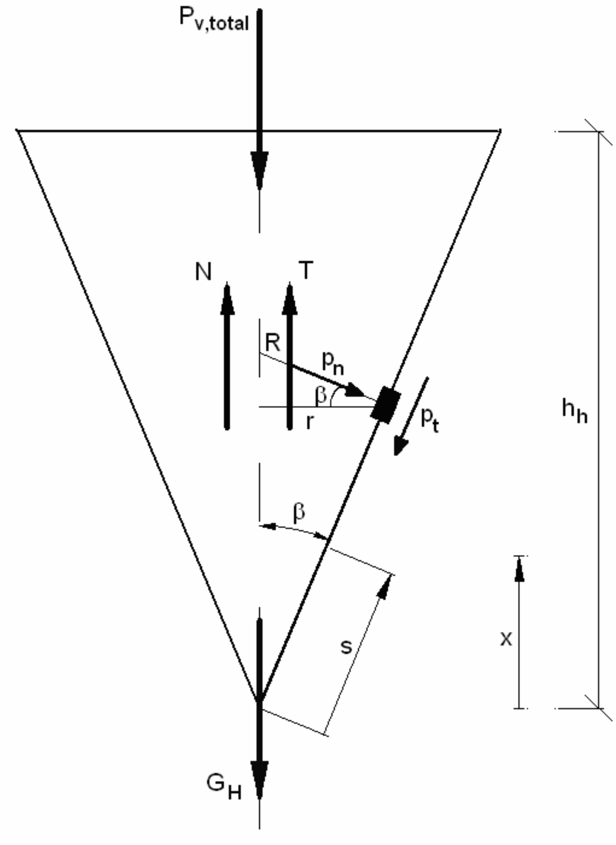
$$R = \frac{z_1}{z_0} - \int_{h_0}^{z_1} \frac{1}{z_0} - \left( \frac{z/z_0 - h_0/z_0}{1 - h_0/z_0} + \frac{1}{z_0} \right)^n dz$$

$$R = \frac{h_0}{z_0} - \frac{1 - h_0/z_0}{n+1}\left( 1 - \frac{z_0}{z_0 - h_0}\left[ \frac{z_1/z_0 - h_0/z_0}{1 - h_0/z_0} + 1 \right]^{n+1} \right)$$

$$\boxed{R = L} \tag{A.53}$$

We see that the equilibrium condition is exactly fulfilled for any chosen height *z*.

### 3.1.3.2 Conical hopper



$$\xi = \frac{x}{h_h} \tag{A.54}$$

$$r = x \cdot tan\,\beta \tag{A.55}$$

$$s = x \cdot cos\,\beta \tag{A.56}$$

**Figure 16: Hopper equilibrium condition for filling loads**

The total weight of the stored solid acting on the silo hopper $G_{h,total}$ has to be in equilibrium with the sum of the vertical components of the resulting normal force on the hopper wall $N$ and the vertical component of the resulting frictional force on the hopper wall $T$ after filling.

**Equilibrium condition:** $$G_{h,total} = N + T \tag{A.57}$$

On the other hand, the total weight of the stored solid acting on the silo hopper equals the sum of the vertical pressure resultant in the silo cylinder $P_{v,total}$ and the weight of the stored material in the hopper $G_h$.

$$G_{h,total} = P_{v,total} + G_h \tag{A.58}$$

The forces $P_{v,total}$ and $G_h$ equal:

$$P_{v,total} = \pi \cdot r^2 \cdot p_{vft} \tag{A.59}$$

$$G_h = \frac{1}{3}\pi \cdot r^2 \cdot \gamma_U \cdot h_h \tag{A.60}$$

The vertical pressure at the transition between the vertical walled segment and the hopper or on the silo bottom $p_{vft}$ is determined as:

$$p_{vft} = C_b \cdot p_{vf}\,(z = h_c) \tag{A.61}$$

where $C_b$ is a bottom load magnifier, which accounts for the possibility of larger loads being transferred to the hopper from the vertical walled segment. It is given in *Table 13*.

The total weight of the stored material in the hopper $G_{h,total}$ then equals:

$$G_{h,total} = \pi \cdot r^2 \cdot p_{vft} + \frac{1}{3}\pi \cdot r^2 \cdot \gamma \cdot h_h$$

$$G_{h,total} = \pi \cdot r^2 \cdot \left( p_{vft} + \frac{1}{3} \cdot \gamma \cdot h_h \right) \tag{A.62}$$

The resulting normal force on the hopper wall in the vertical direction $N$ and the resulting frictional force on the hopper wall in the vertical direction $T$ can be calculated by integrating the vertical components of the normal pressure $p_n$ and of the frictional traction $p_t$ over the area of the hopper wall:

$$N = \iint_A p_n(x) \cdot \sin\beta \; dA \tag{A.63}$$

$$T = \iint_A p_t(x) \cdot \cos\beta \; dA \tag{A.64}$$

The normal pressure $p_n$ and frictional traction $p_t$ at any point on the wall of the hopper after filling are determined as:

$$p_n(x) = F_f \cdot p_v(x) \tag{A.65}$$

$$p_t(x) = \mu_{heff} \cdot F \cdot p_v(x) \tag{A.66}$$

Symbol $F_f$ represents the characteristic value of the hopper pressure ratio after filling and symbol $\mu_{heff}$ represents the effective or mobilized wall friction coefficient for the hopper. They can be calculated by using formulas from in *Chart 2.1*.

The pressure $p_v(x)$ is the mean vertical stress in the solid at height $x$ and is given by equation (A.25).

The sum of forces $N$ and $T$ represents the reaction of the hopper wall due to the weight of the stored solid. Let us denote it with $R_w$ and further expand it using the definitions in equations (A.63) to (A.66).

$$\begin{aligned} R_w &= N + T \\ R_w &= \iint_A \left(p_n \cdot sin\,\beta + p_t \cdot cos\,\beta\right) dA \end{aligned} \tag{A.67}$$

The integral over the area of the silo hopper wall translates into two integrals – one over the silo circumference with the differential $d\phi$ and the other over the silo height in the direction of coordinate $s$ with the differential $ds$:

$$R_w = F \cdot \int_0^{2\pi} \int_0^{h_h'} \left(sin\,\beta + \mu_{heff} \cdot cos\,\beta\right) p_v \; ds \;\; r\, d\phi \tag{A.68}$$

Let us change the integration over height in the direction of coordinate *x* and take into account the expressions (A.54) to (A.56):

$$R_w = F_f \cdot sin\,\beta \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right) \cdot 2\,\pi \cdot tan\,\beta \cdot \frac{1}{cos\,\beta} \int_0^{h_h} p_v \cdot x\,dx$$

$$R_w = F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right) \cdot 2\,\pi \cdot tan^2\,\beta \cdot {h_h}^2 \int_0^1 p_v \cdot \xi\,d\xi$$

$$R_w = F_f \cdot \left(sin\,\beta + \mu_{heff} \cdot cos\,\beta\right) \cdot 2\,\pi \int_0^{h_h} p_v \cdot x \cdot tan\,\beta \cdot \frac{1}{cos\,\beta} dx$$

$$R_w = \pi \cdot r^2 \cdot 2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right) \int_0^1 p_v \cdot \xi\,d\xi \qquad \text{(A.69)}$$

We will calculate the integral $\int_0^1 p_v \cdot \xi\,d\xi$ separately, using the definition of $p_v(x)$ from equation (A.25):

$$\int_0^1 p_v \cdot \xi\,d\xi = \int_0^1 \left[\frac{\gamma \cdot h_h}{n-1}\left(\xi - \xi^n\right) + C_b \cdot p_{vft} \cdot \xi^n\right] \xi\,d\xi$$

$$\int_0^1 p_v \cdot \xi\,d\xi = \int_0^1 \frac{\gamma \cdot h_h}{n-1}\left(\xi - \xi^n\right)\xi\,d\xi + C_b \cdot p_{vft} \int_0^1 \xi^n \xi\,d\xi$$

$$\int_0^1 p_v \cdot \xi\,d\xi = \frac{\gamma \cdot h_h}{n-1} \cdot \left(\left[\frac{\xi^3}{3}\right]_0^1 + \left[\frac{\xi^{n+2}}{n+2}\right]_0^1\right) + C_b \cdot p_{vft}\left[\frac{\xi^{n+2}}{n+2}\right]_0^1$$

$$\int_0^1 p_v \cdot \xi\,d\xi = \frac{\gamma \cdot h_h}{n-1} \cdot \left(\frac{1}{3} - \frac{1}{n+2}\right) + C_b \cdot p_{vft} \cdot \frac{1}{n+2}$$

$$\int_0^1 p_v \cdot \xi\,d\xi = \frac{\gamma \cdot h_h}{n-1} \cdot \frac{n-1}{3(n+2)} + C_b \cdot p_{vft} \cdot \frac{1}{n+2}$$

$$\int_0^1 p_v \cdot \xi\,d\xi = \frac{1}{n+2} \cdot \left(\frac{\gamma \cdot h_h}{3} + C_b \cdot p_{vft}\right) \qquad \text{(A.70)}$$

The value of $R_w$ now equals:

$$R_w = \frac{\pi \cdot r^2 \cdot 2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n+2} \cdot \left(\frac{\gamma \cdot h_h}{3} + C_b \cdot p_{vft}\right) \tag{A.71}$$

If we consider the equilibrium condition from equation (A.57), we get:

$$W_s = R_w$$

$$\pi \cdot r^2 \cdot \left(C_b \cdot p_{vf} + \frac{1}{3} \cdot \gamma \cdot h_h\right) = \frac{\pi \cdot r^2 \cdot 2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n+2} \cdot \left(\frac{\gamma \cdot h_h}{3} + C_b \cdot p_{vft}\right)$$

$$1 = \frac{2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n+2} \tag{A.72}$$

Let us now check if the right part of the above equation really equals 1. First we need to introduce the definitions for $F_f$ and $n$.

The value $F_f$ denotes the characteristic value of the hopper pressure ratio and is defined as:

$$F_f = 1 - \frac{0.2}{1 + \frac{tan\,\beta}{\mu_{heff}}} \tag{A.73}$$

For conical hoppers the value $n$ is defined as follows:

$$n = 1.6 \cdot \frac{\mu_{heff}}{tan\,\beta} \tag{A.74}$$

If we use the above definitions in the right side of equation (A.72) we get:

$$\frac{2\,F_f \cdot \left(1 + \mu_{heff} \cdot ctg\beta\right)}{n+2} = \frac{2 \cdot \left[1 - 0.2 \Big/ \left(1 + \frac{tan\,\beta}{\mu_{heff}}\right)\right] \cdot \left[1 + \mu_{heff} \cdot ctg\beta\right]}{1.6 \cdot \frac{\mu_{heff}}{tan\,\beta} + 2}$$

$$\frac{2\,F_f \cdot \left(1+\mu_{heff} \cdot ctg\beta\right)}{n+2} = \frac{\left[1-0.2\Big/\left(1+\dfrac{\tan\beta}{\mu_{heff}}\right)\right]\cdot\left[1+\mu_{heff}\cdot ctg\beta\right]}{0.8\cdot\dfrac{\mu_{heff}}{\tan\beta}+1} \tag{A.75}$$

Let us denote the numerator on the right side of equation (A.75) with *U* and consider it separately:

$$U = \left[1-0.2\Big/\left(1+\frac{\tan\beta}{\mu}\right)\right]\cdot\left[1+\mu_{heff}\cdot ctg\beta\right]$$

$$U = 1+\mu_{heff}\cdot ctg\beta - \frac{0.2}{1+\dfrac{\tan\beta}{\mu_{heff}}} - \frac{0.2\cdot\mu_{heff}\cdot ctg\beta}{1+\dfrac{\tan\beta}{\mu_{heff}}}$$

$$U = \frac{1.8\,\mu_{heff}+0.8\mu_{heff}{}^2\,ctg\beta+\tan\beta}{\mu_{heff}+\tan\beta} = \frac{\left(1+\mu_{heff}\cdot ctg\beta\right)\cdot\left(0.8\mu_{heff}+\tan\beta\right)}{\mu_{heff}+\tan\beta}$$

$$U = \frac{\left(1+\mu_{heff}\cdot ctg\beta\right)\cdot\tan\beta\cdot\left(0.8\mu_{heff}\cdot ctg\beta+1\right)}{\mu_{heff}+\tan\beta}$$

$$U = \frac{\left(\tan\beta+\mu_{heff}\right)\cdot\left(0.8\mu_{heff}\cdot ctg\beta+1\right)}{\mu_{heff}+\tan\beta}$$

$$U = 0.8\mu_{heff}\cdot ctg\beta+1 \tag{A.76}$$

If we take equation (A.76) back into equation (A.75), we get:

$$\frac{2\,F_f \cdot \left(1+\mu_{heff} \cdot ctg\beta\right)}{n+2} = \frac{U}{0.8\cdot\dfrac{\mu_{heff}}{\tan\beta}+1}$$

$$\frac{2\,F_f \cdot \left(1+\mu_{heff} \cdot ctg\beta\right)}{n+2} = \frac{0.8\mu_{heff}\cdot ctg\beta+1}{0.8\cdot\dfrac{\mu_{heff}}{\tan\beta}+1}$$

$$\frac{2\,F_f \cdot \left(1+\mu_{heff} \cdot ctg\beta\right)}{n+2} = \frac{0.8\cdot\dfrac{\mu_{heff}}{\tan\beta}+1}{0.8\cdot\dfrac{\mu_{heff}}{\tan\beta}+1} = 1 \tag{A.77}$$

From equation (A.77) we see that the right side of equation really does equal 1, which means that $G_{h,\, total} = R_w$ and the equilibrium condition is exactly fulfilled.

This equilibrium check is valid for both hopper types – steep and shallow. The difference between the two hopper types is in the definition of the mobilized (effective) wall friction coefficient $\mu_{heff}$.

For **steep hoppers** $\mu_{heff}$ equals the lower characteristic value of wall friction coefficient in the hopper $\mu_{min}$:

$$\mu_{heff} = \mu_{min} \tag{A.78}$$

For **shallow hoppers** $\mu_{heff}$ equals:

$$\mu_{heff} = \frac{1 - K_{min}}{2 \tan \beta} \tag{A.79}$$

As the value $\mu_{heff}$ in equation (A.77) cancels out, this means that the equilibrium condition is independent of the parameter $\mu_{heff}$ and is thus valid for both hopper types.

### 3.1.3.3 Discrepancy of the total vertical load on the hopper and total vertical loads on silo supports

We have seen in section *3.1.3.2*, equation (A.61), that a bottom load magnifier $C_b$ is used to multiply the transitional vertical pressure $p_{vf}$ to account for the possibility of larger loads being transferred to the hopper from the vertical walled segment.

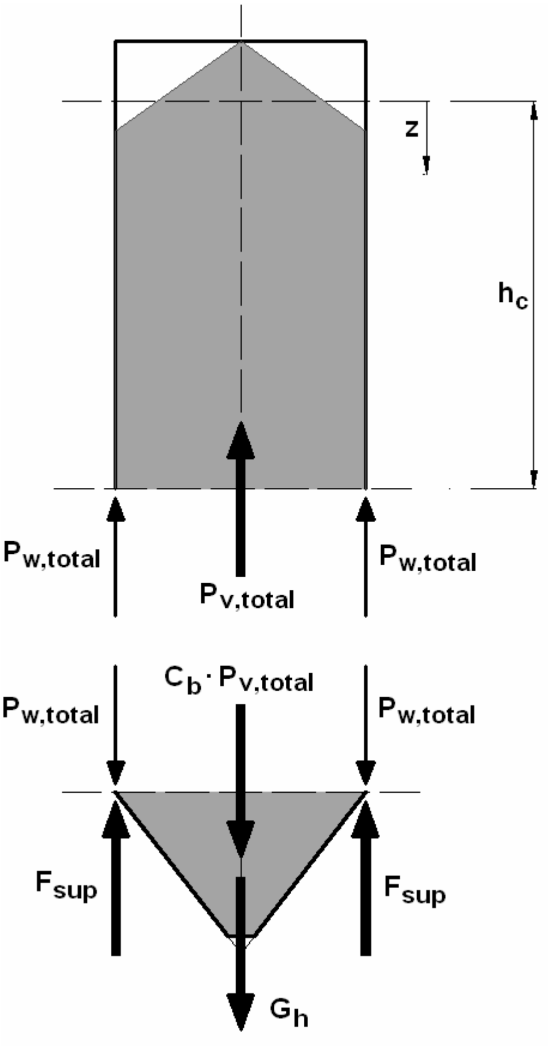


**Figure 17: Discrepancy of the total vertical load**

Let us look at the force in silo supports:

$$F_{sup} = P_{w,total} + C_b \cdot P_{v,total} + G_h \tag{A.80}$$

$P_{w,total}$ and $P_{v,total}$ are the total wall frictional and vertical force calculated at height $z = h_c$ and $G_h$ equals the weight of the stored material in the hopper (see equation (A.60)). The equilibrium condition given in equation (A.39) then yields:

$$P_{v,total} + P_{w,total} = G_c\,, \tag{A.81}$$

where $G_c$ equals the weight of the stored solid in the silo cylinder at height $z = h_c$.

If we use $G_m$ to denote the total weight of the stored material in the silo ($G_m = G_c + G_h$), we can now write equation (A.80) as:

$$F_{sup} = G_c + G_h + (1 - C_b) \cdot P_{v,total}$$

$$F_{sup} = G_m + (1 - C_b) \cdot P_{v,total} \tag{A.82}$$

If we look at *Table 13*, we see that for silos in *Action Assessment Class 2* or *3*, the static bottom load magnifier equals $C_b = 1.0$.

The force in supports then equals:

$$F_{sup} = G_m \tag{A.83}$$

For silos in *Action Assessment Class 1*, the bottom load magnifier equals $C_b = 1.3$.
In this case the force in supports equals:

$$F_{sup} = G_m + 0.3 \cdot P_{v,total} \tag{A.84}$$

The support force to be resisted is $\frac{0.3 \cdot P_{v,total}}{G_m}$ times bigger than $G_m$ and is thus violating the requirement of global equilibrium.

Therefore, the question remains, if (for silos in *Action Assessment Class 1*) the support force should be taken as $G_m$, or if it should be taken as an increased value, corresponding to $C_b = 1.3$.

**Proposed solution:**

- In static conditions the support force equals the total weight of the stored material ($F_{sup} = G_m$), as $C_b$ is only relevant for the design of the hopper and hopper connections.

- If dynamic effects take place, the support forces have to be considered with the increased values.

### 3.1.4 Computer program

A computer program has been written in order to aid the calculation of the pressures on silo walls due to the particulate stored solid material. It uses the procedure described in *Section 3.1.1*.

The program can be found on the CD accompanying this work. The CD is located on the inside of the back cover. To start the program open the *Stored Solid Load on Cylindrical Silos.xls* file.

In order to run the program *Microsoft Excel 2003* (or newer) needs to be installed. Macros have to be enabled and macro security has to be set to medium or low.

Detailed instructions on how to use the program can be found in *Appendix B*, *Section 1*.

## *3.2 Patch load*

The patch load is used to represent accidental asymmetries of loading associated with eccentricities and imperfections in the filling or discharge process. It will only appear in the silo cylinder.

The same patch load applies for the following types of silos:

- slender silo
- intermediate slenderness silo
- squat silo

The patch load may be ignored if:

- the Action Assessment Class is 1 (*AAC 1*)
- a retaining silo is considered

### 3.2.1 Patch load assessment procedure

Different procedures apply for silos in *AAC 2* and *AAC 3*.

#### 3.2.1.1 Action Assessment Class 2

This procedure may be used when a cylindrical silo is held at the top and bottom by a structurally connected roof or a ring stiffener. If this is not true, then the procedure for Action Assessment Class 3 should be used.

In this procedure the patch load may be represented as a uniform pressure increase of the total symmetrical horizontal pressure and the total symmetrical frictional traction for filling and discharge:

- Filling: $p_{hf,u} = p_{hf} \cdot (1 + 0.5 \cdot C_{pf})$ (A.85)
  $p_{wf,u} = p_{wf} \cdot (1 + C_{pf})$ (A.86)

- Discharge: $p_{he,u} = p_{he} \cdot (1 + 0.5 \cdot C_{pe})$ (A.87)
  $p_{we,u} = p_{we} \cdot (1 + C_{pe})$ (A.88)

For values $C_{pf}$ and $C_{pe}$ see the procedure for *AAC 3*.

### 3.2.1.2 Action Assessment Class 3

For silos in Action Assessment Class 3 the procedure described in the chart below should be used.

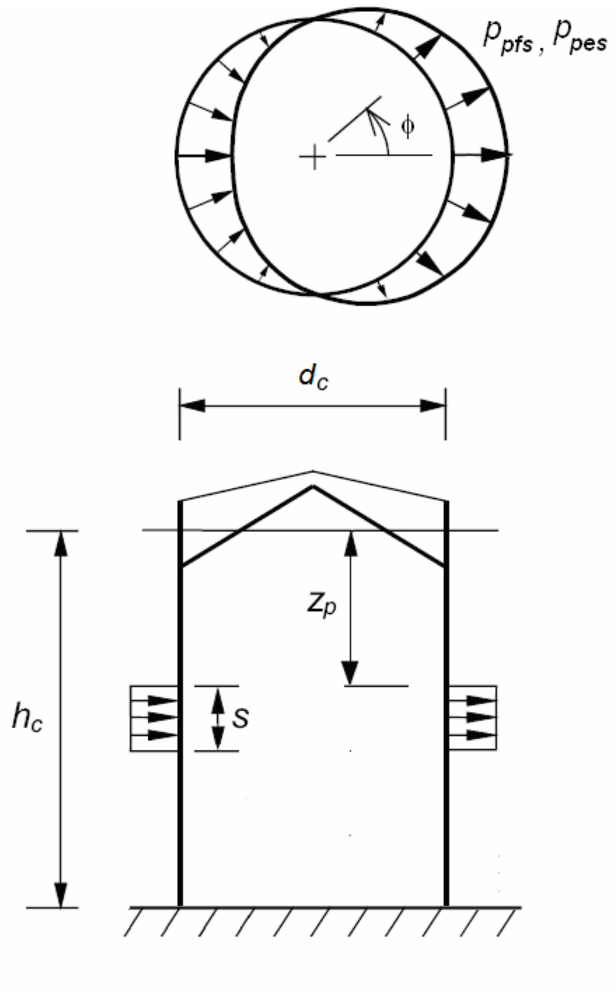


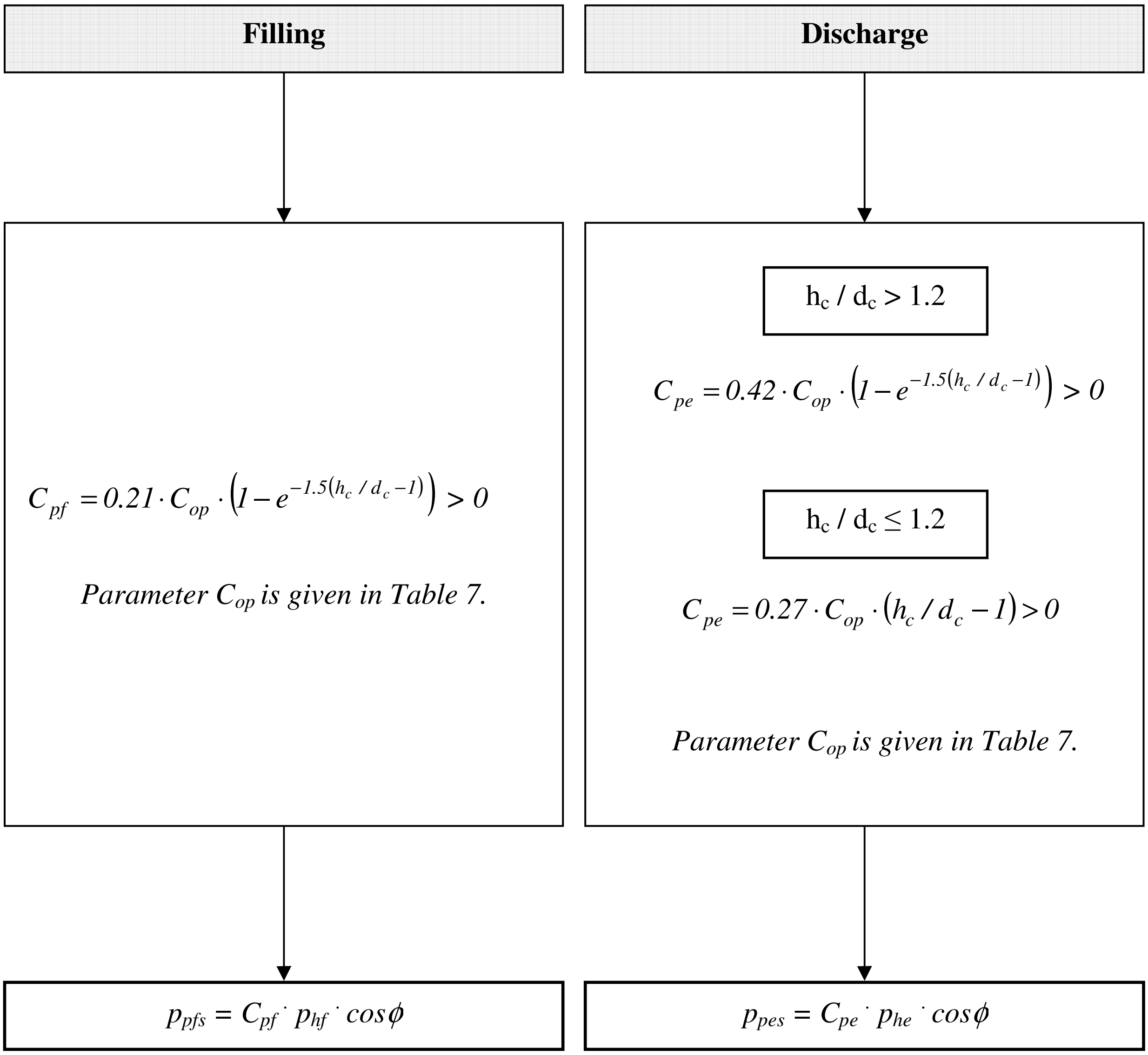
**Figure 18: Patch load parameters for AAC 3**

The height $s$ and $z_p$ in the figure above equal:

$$s \cong 0.2 \cdot d_c \tag{A.89}$$

$$z_p \cong 0.6 \cdot h_c \tag{A.90}$$

## Chart 3



**Filling**

$$C_{pf} = 0.21 \cdot C_{op} \cdot \left(1 - e^{-1.5(h_c / d_c - 1)}\right) > 0$$

*Parameter $C_{op}$ is given in Table 7.*

$$p_{pfs} = C_{pf} \cdot p_{hf} \cdot \cos\phi$$

**Discharge**

$h_c / d_c > 1.2$

$$C_{pe} = 0.42 \cdot C_{op} \cdot \left(1 - e^{-1.5(h_c / d_c - 1)}\right) > 0$$

$h_c / d_c \le 1.2$

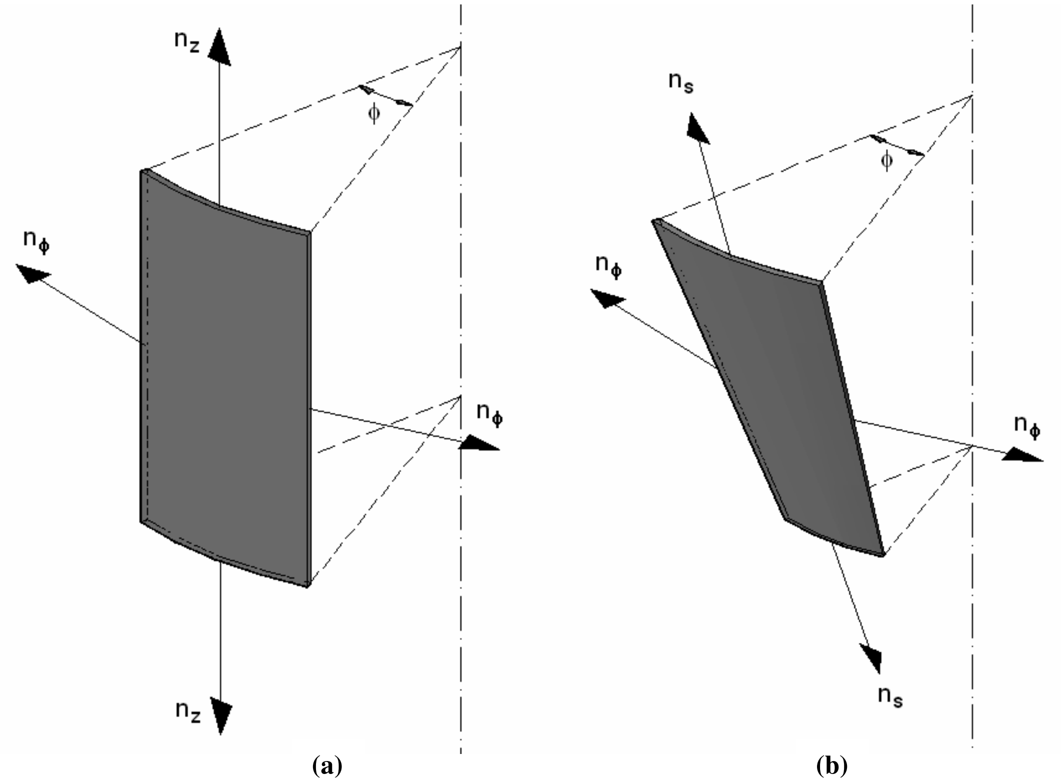
$$C_{pe} = 0.27 \cdot C_{op} \cdot (h_c / d_c - 1) > 0$$

*Parameter $C_{op}$ is given in Table 7.*

$$p_{pes} = C_{pe} \cdot p_{he} \cdot \cos\phi$$

## 3.3 Membrane forces

Only forces calculated using the membrane theory of shells will be considered. The following notation will be used (the positive directions are presented):



(a) (b)

**Figure 19: Membrane forces in silo cylinder (a) and hopper (b)**

The shear forces are present only in the case of unsymmetrical loads and are not presented on the figure.

## 3.3.1 Axisymetric load

### 3.3.1.1 Cylinder wall

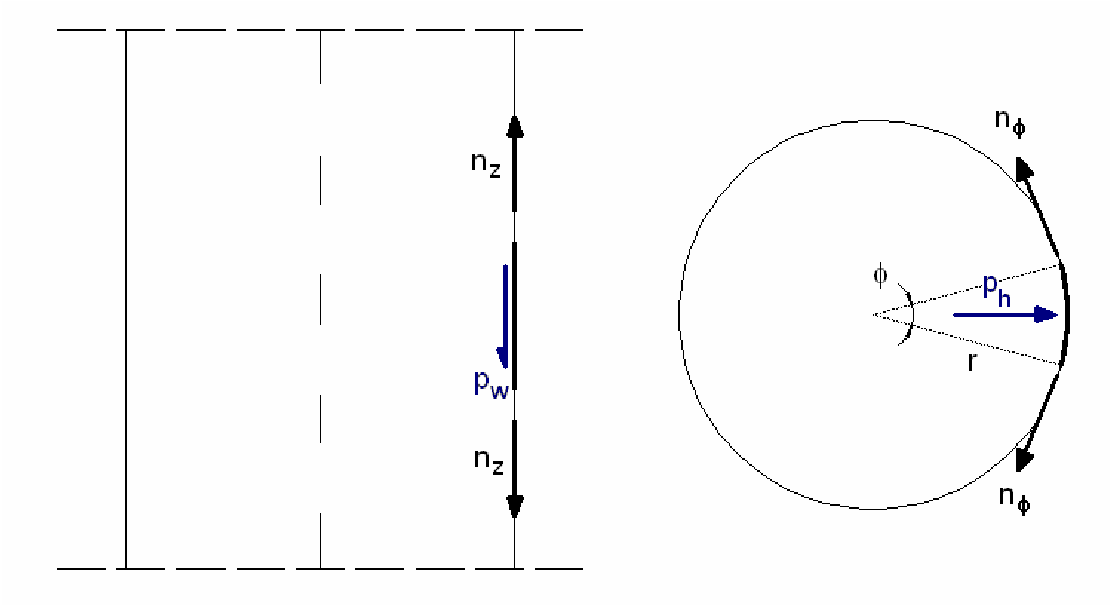


Figure 20: Pressures and section forces in silo cylinder wall

The **vertical section force** $n_z$ at depth $z = z_1$ is obtained by integrating the wall frictional traction $p_w$ over the height of the silo cylinder. In the silo filling and discharge process this force is compressive (negative).

$$n_z( z = z_1 ) = \int_0^{z_1} p_w( z )dz \tag{A.91}$$

The wall frictional traction $p_w$ can account either for filling ( $p_{wf}$ ) or discharge ( $p_{we}$ ).

The **circumferential force** $n_\phi$ is calculated by multiplying the horizontal pressure $p_h$ with the silo radius $r$. It is the same for any given angle $\phi$. In the silo filling and discharge process this force is tensile (positive).

$$n_\phi( z ) = p_h( z ) \cdot r \tag{A.92}$$

The horizontal wall pressure $p_h$ can account either or filling ( $p_{hf}$ ) and discharge ( $p_{he}$). The proper equations have to be considered regarding the slenderness of the silo. See the following sections for details.

### *3.3.1.1.1 Section forces in slender silos*

**Filling loads:**

The wall frictional traction for filling loads $p_{wf}$ in slender silos equals (see *Section 3.1.1.6, Chart 1.1* for details):

$$p_{wf}(z) = \mu \cdot p_{h0} \cdot Y_J(z) = \mu \cdot p_{h0} \cdot \left(1 - e^{-z/z_0}\right) \tag{A.93}$$

The vertical section force during filling at depth $z = z_1$ equals:

$$n_{zf}(z_1) = \int_0^{z_1} p_{wf}(z)dz = \int_0^{z_1} \mu \cdot p_{h0} \cdot \left(1 - e^{-z/z_0}\right) dz$$

$$n_{zf}(z_1) = \mu \cdot p_{h0} \cdot \left( [z]_0^{z_1} - z_0 \left(1 - e^{-z/z_0}\right)_0^{z_1} \right)$$

$$n_{zf}(z_1) = \mu \cdot p_{h0} \cdot \left(z_1 - z_0 \left(1 - e^{-z_1/z_0}\right)\right) \tag{A.94}$$

The vertical force $n_{zf}$ at depth $z$ then equals:

$$\boxed{n_{zf}(z) = \mu \cdot p_{h0} \cdot \left(z - z_0 \left(1 - e^{-z/z_0}\right)\right)} \tag{A.95}$$

The horizontal wall pressure for filling loads $p_{hf}$ in slender silos equals:

$$p_{hf} = p_{h0} \cdot Y_J(z) = p_{h0} \cdot \left(1 - e^{-z/z_0}\right) \tag{A.96}$$

The circumferential force during filling $n_\phi$ at depth $z$ equals:

$$\boxed{n_{\phi f}(z) = p_{h0} \cdot \left(1 - e^{-z/z_0}\right) \cdot r} \tag{A.97}$$

See equation (A.46) for the definition of $p_{h0}$. The depth $z_0$ is defined in equation (A.17).

**Discharge loads:**

In the case of discharge loads, the discharge pressures $p_{we}$ and $p_{he}$ are calculated by multiplying the filling pressures $p_{wf}$ and $p_{hf}$ with discharge factors $C_w$ and $C_h$ (see *Section 3.1.1.6*, *Chart 1.2* for details).

$$p_{we} = C_w \cdot p_{wf} \tag{A.98}$$

$$p_{he} = C_h \cdot p_{hf} \tag{A.99}$$

The vertical section force during discharge at depth $z$ equals:

$$\boxed{n_{ze}(z) = C_w \cdot \mu \cdot p_{h0} \cdot \left(z - z_0\left(1 - e^{-z/z_0}\right)\right)} \tag{A.100}$$

The circumferential force during discharge $n_\phi$ at depth $z$ equals:

$$\boxed{n_{\phi e}(z) = C_h \cdot p_{h0} \cdot \left(1 - e^{-z/z_0}\right) \cdot r} \tag{A.101}$$

### 3.3.1.1.2 Section forces in intermediate slenderness silos and squat silos

**Filling loads:**

The wall frictional traction for filling loads $p_{wf}$ in intermediate slenderness and squat silos equals (see *Section 3.1.1.6, Chart 1.1* for details):

$$p_{wf}(z) = \mu \cdot p_{h0} \cdot Y_R(z) = \mu \cdot p_{h0} \cdot \left[1 - \left(\frac{z - h_o}{z_o - h_o} + 1\right)^n\right] \tag{A.102}$$

The vertical section force during filling at depth $z = z_1$ equals:

$$n_{zf}(z_1) = \int_0^{z_1} p_{wf}(z)dz = \int_0^{z_1} \mu \cdot p_{h0} \cdot \left[1 - \left(\frac{z - h_o}{z_o - h_o} + 1\right)^n\right] dz$$

$$n_{zf}(z_1) = \mu \cdot p_{h0} \cdot \int_0^{z_1} 1 - \left(\frac{z - h_o}{z_o - h_o} + 1\right)^n dz$$

$$n_{zf}(z_1) = \mu \cdot p_{h0} \cdot \left[ z_1 - h_0 + \frac{1}{n+1}\left( z_0 - h_0 - \frac{(z_1 + z_0 - 2h_0)^{n+1}}{(z_0 - h_0)^n} \right)\right] \tag{A.103}$$

Let us denote:

$$z_V(z) = h_0 - \frac{1}{n+1}\left( z_0 - h_0 - \frac{(z + z_0 - 2h_0)^{n+1}}{(z_0 - h_0)^n} \right) \tag{A.104}$$

We can now write equation (A.103) for the vertical force at depth $z$ as:

$$\boxed{n_{zf}(z) = \mu \cdot p_{h0} \cdot \left( z - z_V(z) \right)} \tag{A.105}$$

See equation (A.46) for the definition of $p_{h0}$. The depth $z_0$ is defined in equation (A.17).

The horizontal wall pressure for filling loads $p_{hf}$ in intermediate slenderness and squat silos equals:

$$p_{hf}(z) = p_{h0} \cdot Y_R(z) = p_{h0} \cdot \left[ 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n \right] \tag{A.106}$$

The circumferential force during filling $n_\phi$ at depth $z$ equals:

$$\boxed{n_{\phi f}(z) = p_{h0} \cdot \left[ 1 - \left( \frac{z - h_o}{z_o - h_o} + 1 \right)^n \right] \cdot r} \tag{A.107}$$

**Discharge loads:**

In the case of discharge loads equations (A.98) and (A.99) apply.

The vertical section force during discharge at depth $z$ equals:

$$\boxed{n_{ze}(z) = C_w \cdot \mu \cdot p_{h0} \cdot \left(z_1 - z_V(z_1)\right)} \tag{A.108}$$

The circumferential force during discharge at depth $z$ equals:

$$\boxed{n_{\phi e}(z) = C_h \cdot p_{h0} \cdot \left[1 - \left(\frac{z - h_o}{z_o - h_o} + 1\right)^n\right] \cdot r} \tag{A.109}$$

### 3.3.1.1.3 *Section forces in retaining silos*

Horizontal wall pressure:

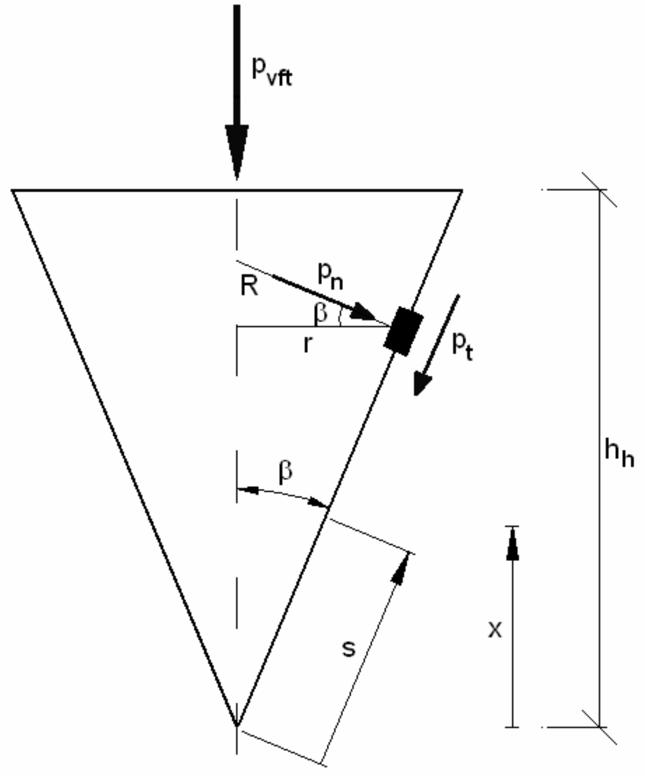
$$p_h(z) = \gamma K \,(1 + \sin\phi_r)\, z \tag{A.110}$$

Section forces for filling and discharge equal:

$$\boxed{n_z(z) = \gamma \cdot \frac{\mu \cdot K}{2} \cdot (1 + \sin\phi_r) \cdot z^2} \tag{A.111}$$

$$\boxed{n_\phi(z) = \gamma \cdot K \cdot (1 + \sin\phi_r) \cdot z \cdot r} \tag{A.112}$$

### 3.3.1.2 Hopper wall



$$r = x \cdot \tan\beta \tag{A.113}$$

$$R = r \cdot \cos\beta \tag{A.114}$$

$$s = x \cdot \cos\beta \tag{A.115}$$

$$\xi = \frac{x}{h_h} \tag{A.116}$$

**Figure 21: Pressures on the hopper wall because of the stored solid**

The total wall pressure in the direction of coordinate *s* at any height equals:

$$p_s(x) = p_t(x) + \tan\beta \cdot p_n(x) \tag{A.117}$$

The normal pressure $p_n$ and frictional traction $p_t$ at any point on the wall of the hopper are determined as:

$$p_t(x) = \mu_{heff} \cdot F \cdot p_v(x) \tag{A.118}$$

$$p_n(x) = F \cdot p_v(x) \tag{A.119}$$

Symbol $F_f$ represents the characteristic value of the hopper pressure ratio after filling ($F_f$) or discharge ($F_e$) and symbol $\mu_{heff}$ represents the effective or mobilized wall friction coefficient for the hopper.

Pressure $p_s$ can be then written as:

$$p_s(x) = F \cdot (\mu_{heff} + tan\,\beta) \cdot p_v(x) \tag{A.120}$$

The pressure $p_v(x)$ is the mean vertical stress in the solid at height $x$ and is given by equation (A.25).

Let us consider a small section of the hopper wall, with the height $ds$.

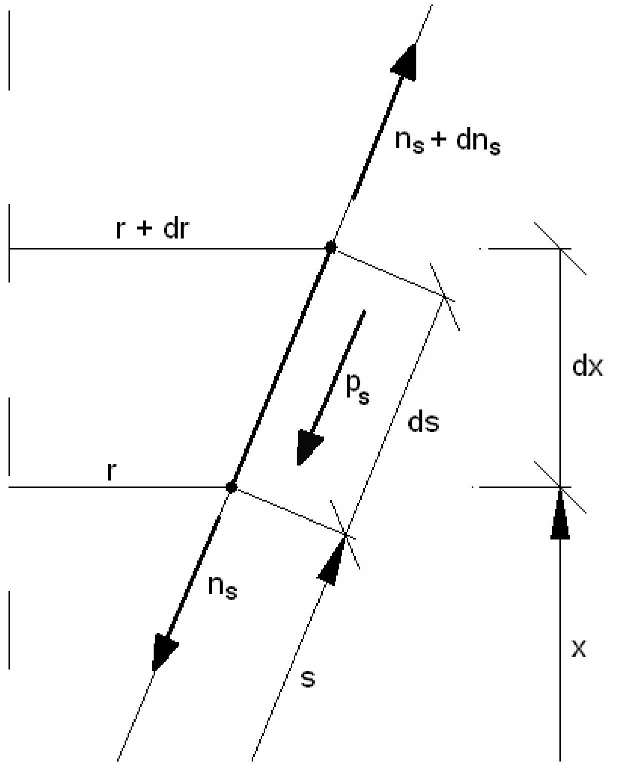


**Figure 22: Section of the hopper wall, with the height *ds***

The following equilibrium condition applies:

$$n_s \cdot r \cdot d\phi + p_s(x) \cdot r \cdot d\phi \cdot ds - (n_s + dn_s) \cdot (r + dr) \cdot d\phi = 0 \tag{A.121}$$

After simplifying, we get:

$$n_s \cdot r + p_s(x) \cdot r \cdot ds - n_s \cdot r - n_s \cdot dr - r \cdot dn_s - dr \cdot dn_s = 0$$

$$n_s \cdot dr + r \cdot dn_s = p_s(x) \cdot r \cdot ds$$

$$d(n_s \cdot r) = p_s(x) \cdot r \cdot ds \tag{A.122}$$

The total section force $n_s$ at height *s'* can then be calculated as:

$$n_s = \frac{1}{r} \int_0^{s'} p_s(x) \cdot r \, ds \tag{A.123}$$

If we consider equations (A.113) to (A.115) and rewrite the previous equation so that $n_s$ is calculated at height *x'* instead of *s'*, then we get:

$$n_s = \frac{1}{x \cdot \tan\beta} \int_0^{x'} p_s(x) \cdot \frac{\tan\beta \cdot x}{\cos\beta} \, dx$$

$$n_s = \frac{1}{x \cdot \cos\beta} \int_0^{x'} p_s(x) \cdot x \, dx \tag{A.124}$$

Considering also equations (A.116) and (A.120) we get:

$$n_s = \frac{1}{\xi \cdot h_h \cdot \cos\beta} \int_0^{\xi'} F \cdot (\mu_{heff} + \tan\beta) \cdot p_v(\xi) \cdot h_h^2 \cdot \xi \, d\xi$$

$$n_s = \frac{F \cdot (\mu_{heff} + \tan\beta)}{\cos\beta} h_h \cdot p_{vft} \cdot \frac{1}{\xi} \int_0^{\xi'} \frac{p_v(\xi)}{p_{vft}} \cdot \xi \, d\xi \tag{A.125}$$

Let us denote:

$$I = \int_0^{\xi'} \frac{p_v(\xi)}{p_{vft}} \cdot \xi \, d\xi \tag{A.126}$$

and

$$\boxed{\alpha = \frac{\gamma_{max} \cdot h_h}{(n-1) \cdot p_{vft}}} \tag{A.127}$$

Considering equations (A.126), (A.127) and (A.25) we get:

$$I = \int_0^{\xi'} \frac{p_v(\xi)}{p_{vft}} \cdot \xi \, d\xi = \int_0^{\xi'} \left[ \frac{\gamma_u \cdot h_h / (n-1)}{p_{vft}} \left( \xi - \xi^n \right) + \xi^n \right] \cdot \xi \, d\xi$$

$$I = \int_0^{\xi'} \left[ \alpha \cdot \left( \xi - \xi^n \right) + \xi^n \right] \cdot \xi \, d\xi$$

$$I = \alpha \int_0^{\xi'} \xi^2 \, d\xi - \alpha \int_0^{\xi'} \xi^{n+1} \, d\xi + \int_0^{\xi'} \xi^{n+1} \, d\xi \tag{A.128}$$

If we integrate equation (A.128) and write $\xi' = \xi$, we get:

$$I = \alpha \frac{\xi^3}{3} + \frac{(1-\alpha) \cdot \xi^{n+2}}{n+2} \tag{A.129}$$

Let us now denote:

$$f_s(\xi) = \frac{I}{\xi}$$

$$\boxed{f_s(\xi) = \frac{\xi}{3} \left( \alpha \cdot \xi + \frac{3 \cdot (1-\alpha) \cdot \xi^n}{n+2} \right)} \tag{A.130}$$

and

$$\boxed{n_{s0} = F \cdot \frac{(\mu + \tan\beta)}{\cos\beta} h_h \cdot p_{vft}} \tag{A.131}$$

The total hopper section force $n_s$ at the dimensionless height $\xi$ can be determined as:

$$\boxed{n_s(\xi)=n_{s0}\cdot f_s(\xi)} \tag{A.132}$$

A similar consideration is used when determining the total circumferential force on the hopper wall $n_\phi$ at height $\xi$.

$$n_\phi = p_n(\xi)\cdot R \tag{A.133}$$

Considering equations (A.133), (A.113), (A.114) and (A.116), we get:

$$n_\phi = p_n(\xi)\cdot\frac{r}{\cos\beta} = p_n\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot\xi \tag{A.134}$$

If we use the definitions in equation (A.25), (A.119) and (A.127) with equation (A.134), we get:

$$n_\phi = F\cdot p_v(\xi)\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot\xi$$

$$n_\phi = F\cdot p_{vft}\cdot\left[\alpha\cdot\left(\xi-\xi^n\right)+\xi^n\right]\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot\xi$$

$$n_\phi = F\cdot p_{vft}\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot\left[\alpha\cdot\xi^2+\left(1-\alpha\right)\cdot\xi^{n+1}\right] \tag{A.135}$$

Let us now denote:

$$\boxed{f_\phi(\xi)=\alpha\cdot\xi^2+\left(1-\alpha\right)\cdot\xi^{n+1}} \tag{A.136}$$

and

$$\boxed{n_{\phi 0} = F\cdot\frac{\tan\beta}{\cos\beta}h_h\cdot p_{vft}} \tag{A.137}$$

The total hopper circumferential section force $n_{\phi h}$ at height $\xi$ can be determined as:

$$\boxed{n_{\phi}(\xi) = n_{\phi 0} \cdot f_{\phi}(\xi)} \tag{A.138}$$

Filling and discharge pressures have to be considered separately. For the calculation of factors $p_{vft}$, $n$, $F$ for filling and discharge see *Chart 2.1* and *Chart 2.2* in *Section 3.1.1.7.*

## 3.3.2 Patch load

The following additional section forces appear at depth $z_p$, where the procedure for silos in *AAC 3* has been used (*Section 3.2.1.2*). In both cases the coordinate $z$ runs from $z_p$ to $z_p + s$ (*see Figure 18*).

**Filling loads:**

The additional vertical section force $n_{z,pfs}$ and circumferential section force $n_{\phi,\, pfs}$ on the silo during filling, due to the patch load, are given as:

$$\boxed{n_{z,pfs}(z,\phi) = \frac{p_{pfs}(\phi)}{d_c} \cdot z^2} \tag{A.139}$$

$$\boxed{n_{\phi,pfs}(\phi) = -p_{pfs}(\phi) \cdot r} \tag{A.140}$$

The shear force $n_{z\phi,pfs}$ equals:

$$\boxed{n_{z\phi,pfs}(z,\phi) = -p_{hf} \cdot sin\phi \cdot z} \tag{A.141}$$

**Discharge loads:**

The additional vertical section force $n_{z,pes}$ and circumferential section force $n_{\phi,\,pes}$ on the silo during discharge, due to the patch load, are given as:

$$\boxed{n_{z,pes}(z,\phi) = \frac{p_{pes}(\phi)}{d_c} \cdot z^2} \tag{A.142}$$

$$\boxed{n_{\phi,pes}(\phi) = -p_{pes}(\phi) \cdot r} \tag{A.143}$$

The shear force $n_{z\phi,pes}$ equals:

$$\boxed{n_{z\phi,pes}(z,\phi) = -p_{he} \cdot \sin\phi \cdot z} \tag{A.144}$$

# 4 Wind load on silos

## 4.1 Introduction and overview

In this chapter the wind loading on silo structures will be presented. First some information on how to determine the proper wind loading on a silo cylinder will be given. Formulas for determining the wind load on typical silo structures will be then derived and relevant design situations, which are needed in design, will be presented.

At the end of the work a computer program for determining the proper wind pressure and wind force will also given.

## 4.2 Wind pressure distribution

If a body of cylindrical shape is exposed to high velocity environmental airflow (wind), a specific pressure distribution arises around the circumference.

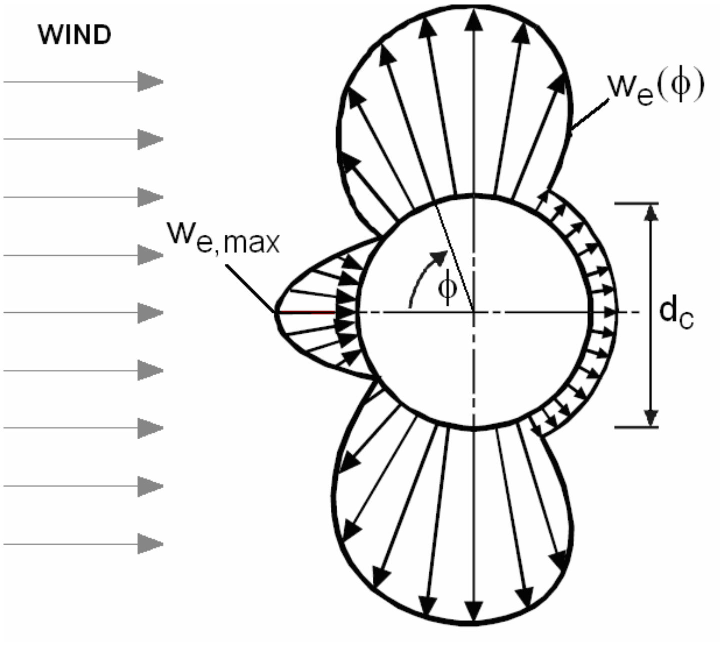


**Figure 23: Wind pressure distribution around the silo shell circumference**

We can see from the previous figure that the wind pressure is compressive only on the windward side. On the leeward side and on the sides of the cylinder the wind causes suction. The external wind pressure $w_e(\phi)$ is the same for all cross-sections along the height of the silo cylinder. It is given in the *EN 1991-1-4* by the equation:

$$w_e(\phi) = q_p \cdot c_{p0}(\phi) \cdot \psi_{\lambda\phi}(\phi) \qquad \text{(A.145)}$$

The symbol $q_p$ represents the peak wind pressure and should be calculated in accordance to *EN 1991-1-4*.

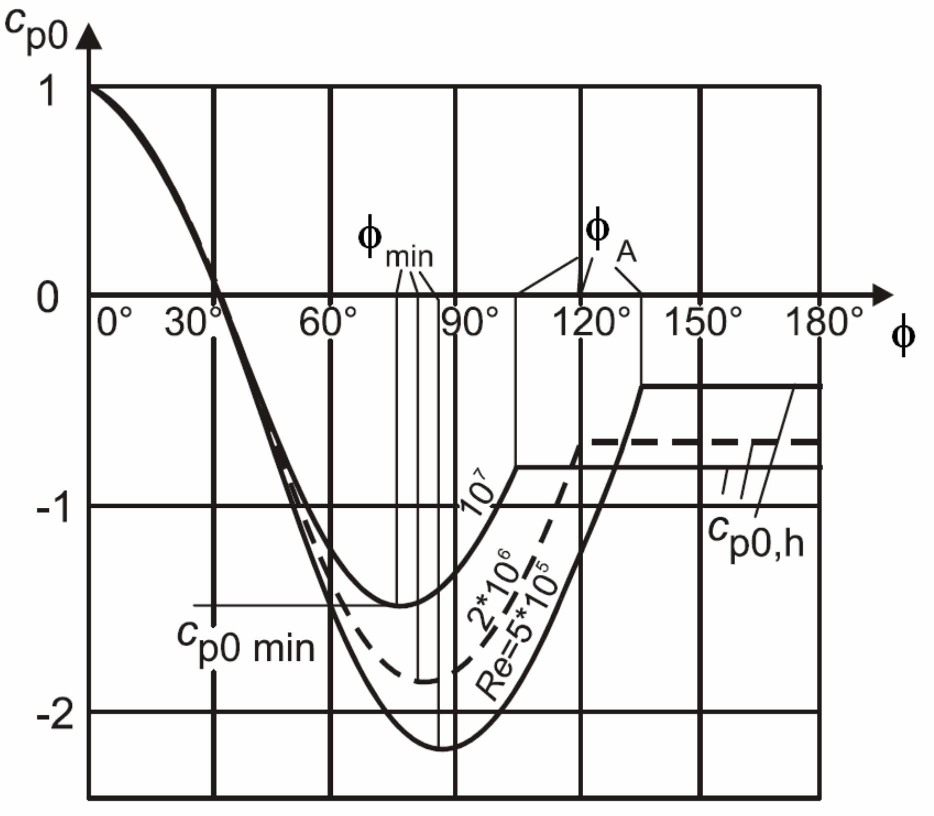
The function $c_{p0}(\phi)$ is called the external wind pressure coefficient and it represents the "shape" of the wind pressure distribution and it depends on the Reynolds number of the given silo cylinder.

The function $\psi_{\lambda\phi}(\phi)$ represents the slenderness function of the silo.

We will now consider the values of $c_{p0}(\phi)$ and $\psi_{\lambda\phi}(\phi)$ for typical silo structures.

### 4.2.1 External wind pressure coefficient for typical silo structures

In this chapter we will determine the formula for the $c_{p0}(\phi)$ curve, valid for typical silo structures. The following figure is taken from the code and represents the distribution of $c_{p0}(\phi)$ for three different Reynolds numbers.



**(Taken from EN 1991-1-4:2005, Figure 7.27)**

Typical silo structures have a Reynolds number of about $Re = 5 \cdot 10^5$, that is why we will consider only the curve corresponding to that Reynolds number. The *EN 1991-1-4* does not give a general equation for the curves on the figure above. Instead it gives only specific data points.

The data points for the curve typical for silo structures are given in the following table:

**Table 16: Data points for the $C_{p0}(\phi)$ curve**[10]

| $\phi$ [°] | 0 | 33 | 60 | 85 | 135 |
|---|---|---|---|---|---|
| $c_{p0}(\phi)$ | 1 | 0 | -1.45 | -2.2 | -0.4 |

Let us define the function $c_{p0}(\phi)$ as an infinite Fourier cosine series:

$$c_{p0}(\phi) = \frac{c_0}{2} + c_1 \cdot \cos\phi + c_2 \cdot \cos 2\phi + c_3 \cdot \cos 3\phi + \ldots$$

$$\boxed{c_{p0}(\phi) = \frac{c_0}{2} + \sum_{m=1}^{\infty} c_m \cdot \cos m\phi} \qquad \text{(A.146)}$$

For simplification reasons we can combine the coefficients $c_0$ and $c_m$, where $m \in [1, 2, \ldots, \infty]$, into one coefficient parameter $c_n$, where $n \in [0, 1, 2, \ldots, \infty]$.We will calculate these coefficients by using the least square method on the given data points.

As we will see in the following chapters, writing the $c_{p0}(\phi)$ as a Fourier cosine series is very useful, due to the orthogonality relationship of the cosine functions.

First we need to approximate the $c_{p0}(\phi)$ curve given on the figure on page *288* for $Re = 5 \cdot 10^5$, which runs through the data points specified in

. The nonlinear part of the can be approximated with a biquadratic polynomial[11], while in the constant part the function equals $-0.4$. The combined approximation function can then be then written as:

$$F(\phi) = \left\langle \begin{array}{cc} -0.462\,\phi^4 + 3.308\,\phi^3 - 5.711\,\phi^2 + 0.544\,\phi + 1 & ; \quad \phi < 135^\circ \\ -0.4 & ; \quad \phi \geq 135^\circ \end{array} \right\rangle \qquad \text{(A.147)}$$

---

[10] Values taken from EN 1991-1-4:2005, Figure 7.27 and Table 7.12

[11] The approximation was made using the Wolfram Mathematica 6.0 computer program

The figure below represents the plot of function $F(\phi)$. The angle $\phi$ is given in radians.

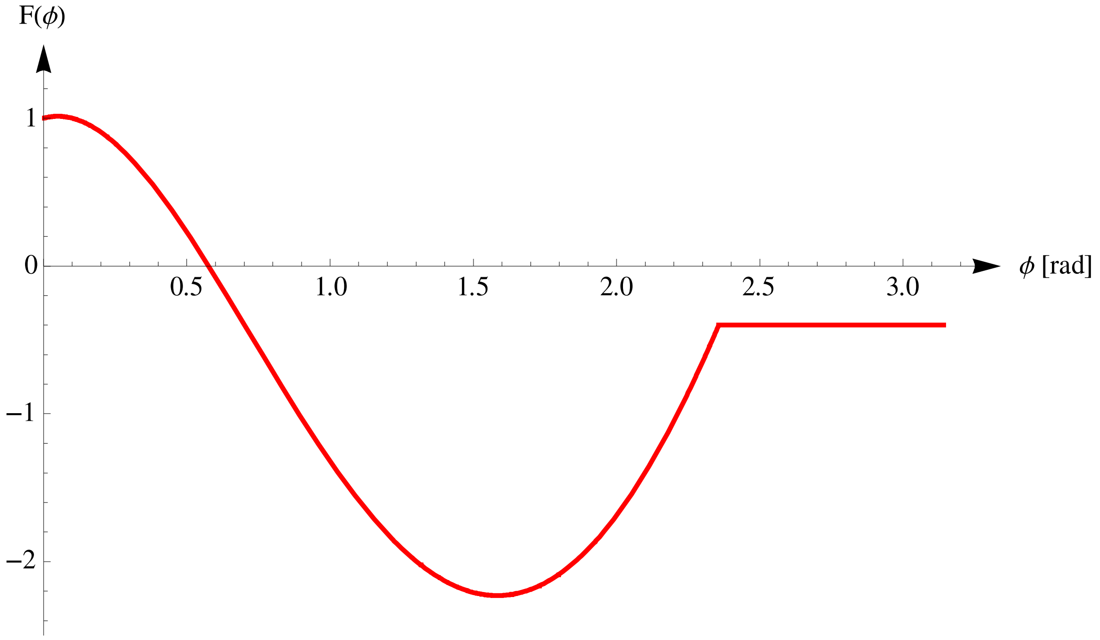


**Figure 24: Approximation of the wind pressure coefficient function**

When using the least square method, the difference or error of the sum of squares of the residual (the difference between the actual and approximated function) has to be minimal.

$$\varepsilon = \int_0^{\pi} \left( F(\phi) - c_{p0}(\phi) \right)^2 d\phi = min \tag{A.148}$$

The error obtains its extreme (in our case minimum) value, when its first derivative equals zero:

$$\frac{\partial \varepsilon}{\partial c_n} = 0$$

$$\frac{\partial \varepsilon}{\partial c_n} = -2\int_0^{\pi} \left( F(\phi) - c_{p0}(\phi) \right) \cdot \frac{\partial c_{p0}(\phi)}{\partial c_n} d\phi = 0 \tag{A.149}$$

The derivative of function $c_{p0}(\phi)$ equals:

$$\frac{\partial c_{p0}(\phi)}{\partial c_n} = \cos n\phi \tag{A.150}$$

Equation (A.149) can now be written as:

$$\int_0^{\pi} F(\phi)\cdot \cos n\phi \, d\phi - \int_0^{\pi} \left( \frac{c_0}{2} + \sum_{m=1}^{\infty} c_m \cdot \cos m\phi \right) \cdot \cos n\phi \, d\phi = 0 \tag{A.151}$$

Let us now consider the integral *I*:

$$I = \int_0^{\pi} \left( \frac{c_0}{2} + \sum_{m=1}^{\infty} c_m \cdot \cos m\phi \right) \cdot \cos n\phi \, d\phi$$

$$I = c_0 \cdot \frac{\sin n\pi}{2n} + \int_0^{\pi} \sum_{m=1}^{\infty} c_m \cdot \cos m\phi \cdot \cos n\phi \, d\phi \tag{A.152}$$

The orthogonality relationship of the cosine function[12] holds:

$$\int_0^{\pi} \cos m\phi \cdot \cos n\phi \, d\phi = \left\langle \begin{array}{l} 0, \;\; if \;\; m \neq n \\ \frac{\pi}{2}, \;\; if \;\; m = n \end{array} \right\rangle \tag{A.153}$$

Considering this relationship, we can now write the integral *I* as:

$$I = c_0 \cdot \frac{\sin n\pi}{2n} + c_n \cdot \frac{\pi}{2} \tag{A.154}$$

Equation (A.151) then simplifies into:

---

[12] The functions *i(x)* and *j(x)* are defined to be orthogonal over the range *a* to *b* if $\int_a^b i(x)\cdot j(x)\,dx = 0$ , whenever $i(x) \neq j(x)$.

$$\int_0^{\pi} F(\phi)\cdot \cos n\phi \, d\phi - c_0 \cdot \frac{\sin n\pi}{2n} - c_n \cdot \frac{\pi}{2} = 0 \qquad \text{(A.155)}$$

The coefficient $c_n$ can now be expressed as:

$$c_n = \frac{2}{\pi}\left( \int_0^{\pi} F(\phi)\cdot \cos n\phi \, d\phi - c_0 \cdot \frac{\sin n\pi}{2n} \right) = 0, \qquad \text{(A.156)}$$

where $n \in [0, 1, 2, \ldots, \infty]$.

Because *n* is a natural number, the right side of the equation in the brackets always equals *0*:

$$c_0 \cdot \frac{\sin n\pi}{2n} = 0, \quad \forall n \in [0, 1, 2, \ldots, \infty]. \qquad \text{(A.157)}$$

Equation (A.156) can thus be written as:

$$\boxed{c_n = \frac{2}{\pi}\left( \int_0^{\pi} F(\phi)\cdot \cos n\phi \, d\phi \right) = 0, \quad n \in [0, 1, 2, \ldots, \infty].} \qquad \text{(A.158)}$$

Let us now calculate the first 20 terms of the $c_{p0}(\phi)$ function, defined as a Fourier cosine series (see Equation (A.146)). The values are given in the table below.

**Table 17: First 20 terms of the $C_{p0}(\phi)$ function**

| n | $c_n$ |
|---|---|
| 0 | -1.6386 |
| 1 | 0.3720 |
| 2 | 1.2855 |
| 3 | 0.2832 |
| 4 | -0.1685 |
| 5 | 0.1106 |
| 6 | -0.0142 |

| n | $c_n$ |
|---|---|
| 10 | 0.0008 |
| 11 | 0.0135 |
| 12 | -0.0220 |
| 13 | 0.0115 |
| 14 | -0.0025 |
| 15 | -0.0098 |
| 16 | 0.0102 |

| | | | |
|---|---|---|---|
| 7 | -0.0384 | 17 | -0.0086 |
| 8 | 0.0438 | 18 | -0.0004 |
| 9 | -0.0309 | 19 | 0.0045 |

The figure below represents a comparison plot of the function $c_{p0}(\phi)$ calculated for the first 20 terms and of the polynomial approximation function $F(\phi)$. We can see that the curves match well. The biggest difference occurs in the vicinity of $\phi = \frac{3\pi}{4}$, where it equals about 20%. We can achieve a smaller difference with increasing the number of terms used for the calculation of $c_{p0}(\phi)$.

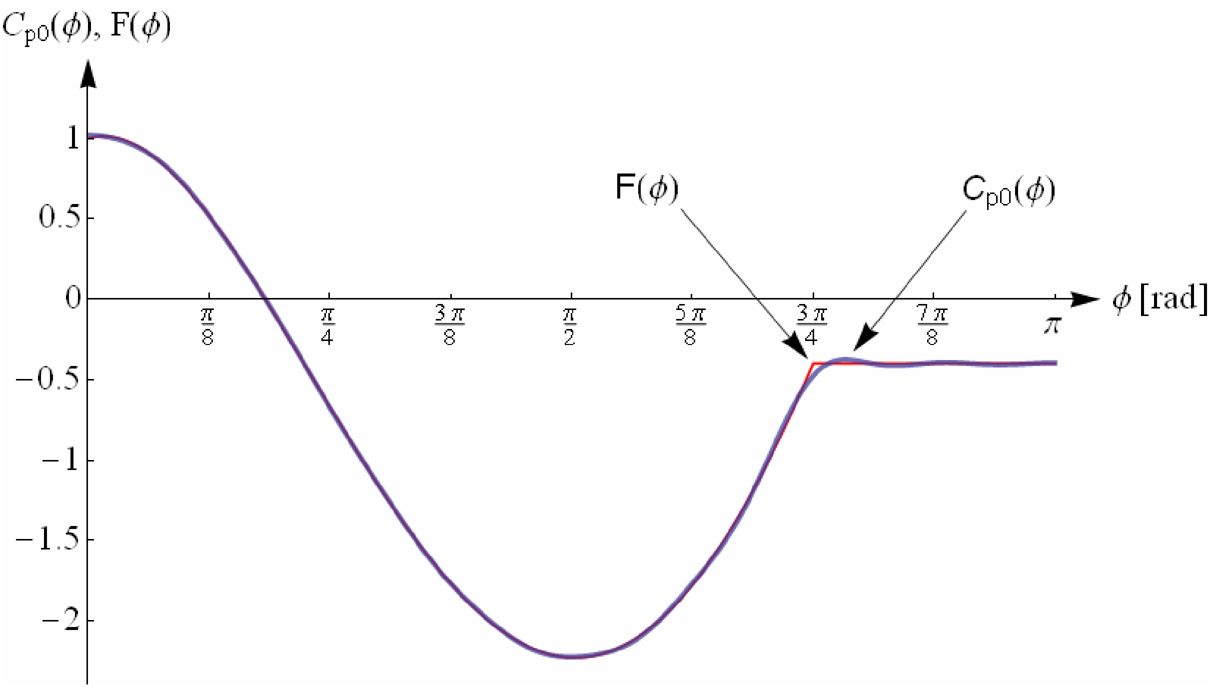


**Figure 25: Wind pressure coefficient function $C_{p0}(\phi)$ calculated by the least squares method and its polynomial approximation $F(\phi)$**

### 4.2.2 Slenderness function of typical silo structures

The calculation of external wind pressure requires the calculation of the slenderness function, which is given as a function of $\phi$ and denoted as $\psi_{\lambda\phi}(\phi)$.

For cylindrical silo structures the slenderness function is defined as:

$$\psi_{\lambda\phi}(\phi)=\left\langle \begin{array}{cl} 1 & ;\ 0° \le \phi \le 85° \\ \psi_\lambda+(1-\psi_\lambda)\cdot cos\left(\frac{3}{20}(\pi+12\phi)\right) & ;\ 85° < \phi < 135° \\ \psi_\lambda & ;\ 135° \le \phi \le 180° \end{array} \right\rangle . \qquad (A.159)$$

The parameter $\psi_\lambda$ represents the slenderness factor. The values of slenderness factor and of the slenderness function for typical silo structures are given in the table below.

**Table 18: Values of the slenderness factor and slenderness function for typical silo structures**

| **Silo cylinder type** | $\psi_\lambda$ | **$\psi_{\lambda\phi}(\phi)$, when 85°< $\phi$ < 135°** |
|---|---|---|
| Retaining silo | $0.31$ | $0.31-0.69\cdot\eta$ |
| Squat silo | $0.42$ | $0.42-0.58\cdot\eta$ |
| Intermediate slenderness silo | $0.63$ | $0.63-0.37\cdot\eta$ |
| Slender silo | $0.66$ | $0.66-0.34\cdot\eta$ |

The parameter $\eta$ stands for:

$$\eta=cos\left(\frac{3}{20}(\pi+12\phi)\right) \quad ; \quad \phi \text{ is in radians.} \qquad (A.160)$$

The slenderness factor functions are graphically represented on the figure below.

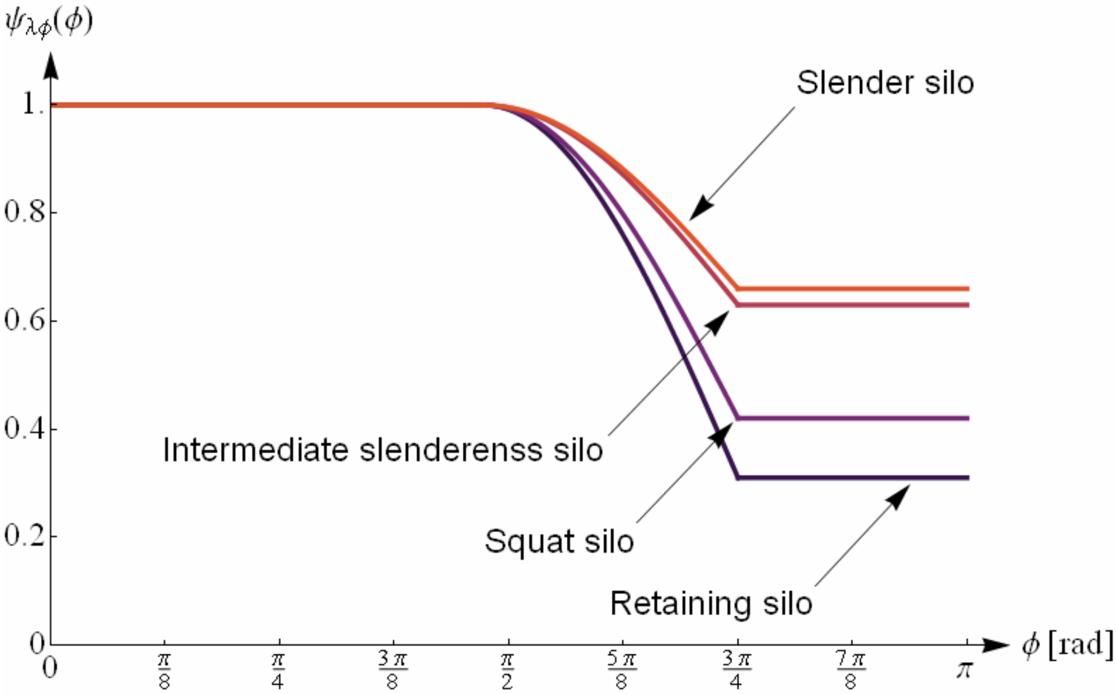


**Figure 26: Slenderness function for typical silo structures**

### 4.2.3 Formal representation of wind pressures

Let us denote the combined external pressure coefficient and slenderness function as $\overline{c_{p0}}(\phi)$:

$$\overline{c_{p0}}(\phi) = c_{p0}(\phi)\cdot\psi_{\lambda\phi}(\phi) \qquad \text{(A.161)}$$

The figure below represents the comparison between the combined external pressure distribution function $\overline{c_{p0}}(\phi)$ for different silo cylinder types and the normal, uncombined, external pressure distribution function $c_{p0}(\phi)$.

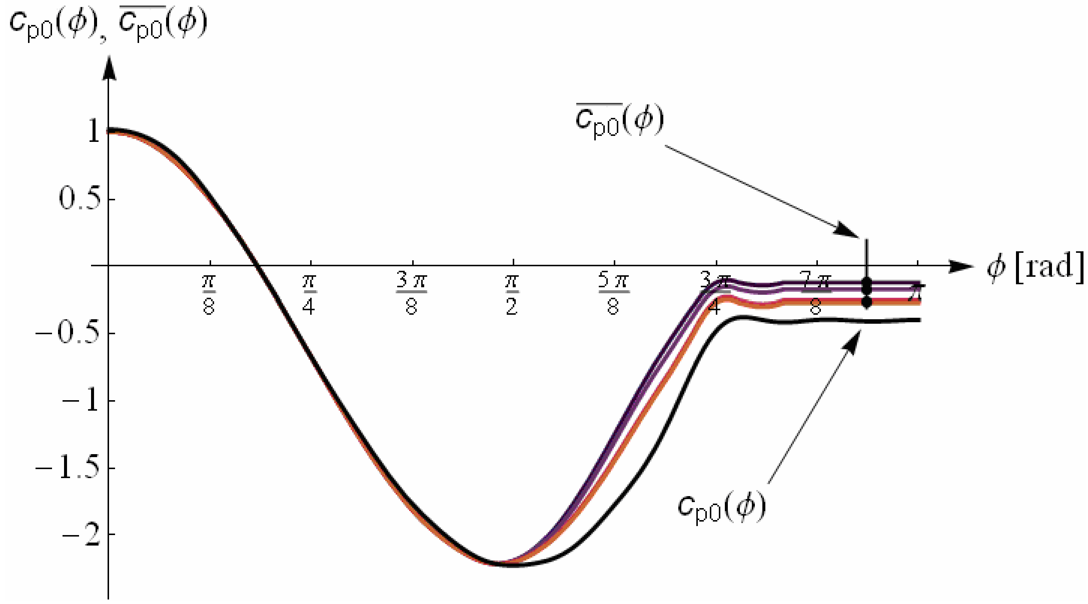


**Figure 27: Comparison of the combined and normal external wind pressure coefficient**

We can see from the figure that the slenderness function $\psi_{\lambda\phi}(\phi)$ only has an effect on the leeward side of the cylinder ($\phi > \frac{\pi}{2}$), where it reduces the wind suction. This means that we are on the safe side, if we assume that $\psi_{\lambda\phi}(\phi) = 1$ and use only the unreduced wind pressure coefficient function $c_{p0}(\phi)$.

Considering this assumption we can use the following equation for the external wind pressure:

$$\boxed{w_e(\phi) = q_p \cdot c_{p0}(\phi)} \tag{A.162}$$

The $c_{p0}(\phi)$ function can be determined in *Section 4.2.1.*

## *4.3 Relevant design situations*

The wind load on a silo structure has to be considered for two situations:

- when the silo is full,
- when the silo is empty.

In the first situation the total wind force must be taken into account in order to prevent the overturning of the silo structure. Global bending of the silo shell also needs to be considered.

In the second situation, when the silo is empty, the structure has to maintain its structural integrity also during the periods of non-operation. Here the wind pressure around the silo circumference needs to be considered, as it can cause circumferential buckling of the silo shell.

### 4.3.1 Full silo situation

Global load bearing capacity needs to be guaranteed in the full silo situation. This means that we have to consider:

- increase of support forces due to the overturning moment action, caused by the total wind force,
- global bending of the silo shell, caused by the total wind force (section forces are given in *Section 4.4.1*).

In both cases we have to consider the total wind force on the silo structure.

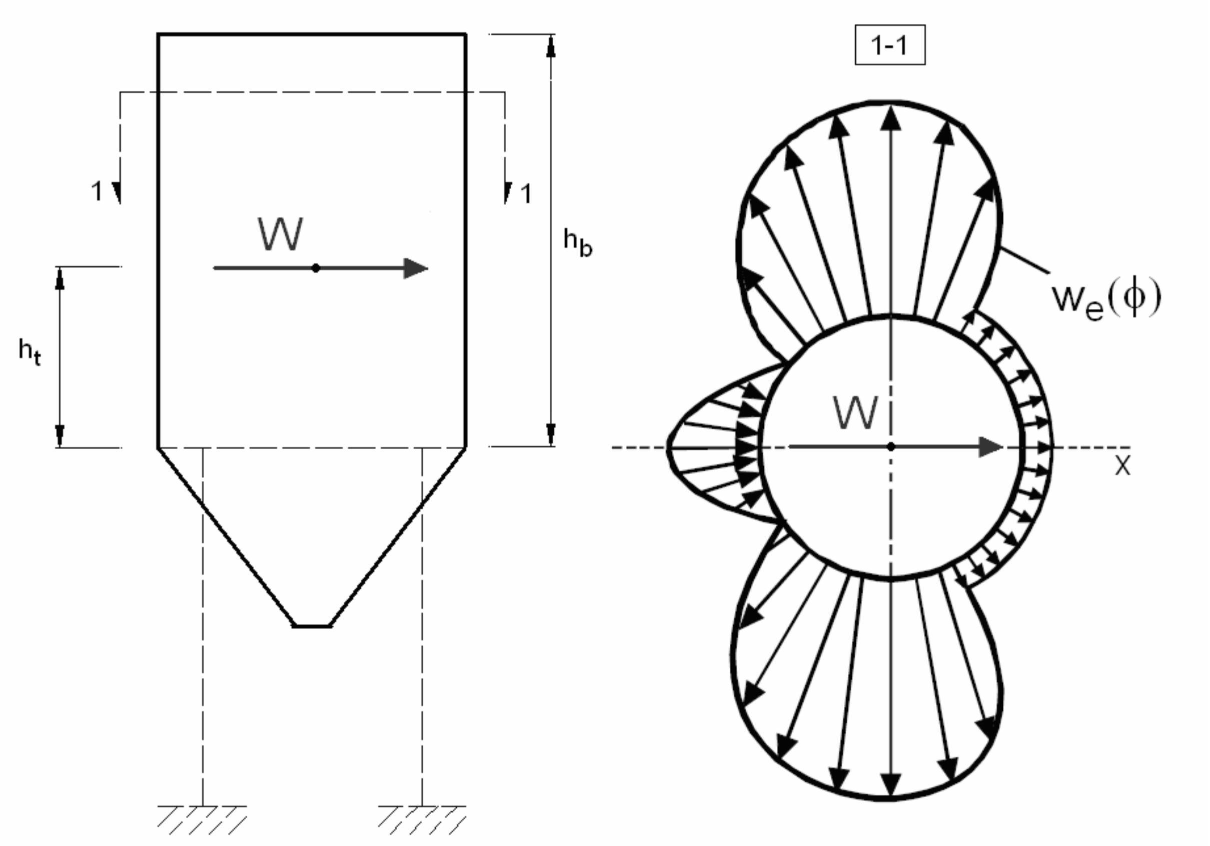


**Figure 28: Total wind force on the silo cylinder**

The total wind force *W* represents the resultant of wind pressure $w_e\,(\phi)$ acting on the silo cylinder and is calculated by the following equation:

$$W = h_c \cdot \int_0^{2\pi} w_e(\phi) \cdot cos\,\phi \cdot r\,d\phi \tag{A.163}$$

It acts at the silo cylinder center of gravity, at height $h_t$ from the transition between the silo hopper and silo cylinder.

$$h_t = \frac{h_b}{2} \tag{A.164}$$

We can see from equation (A.162) that the external wind pressure $w_e(\phi)$ is a product of parameter $q_p$ and the external wind pressure coefficient function $c_{p0}(\phi)$.

The wind force then equals:

$$W = h_c \cdot r \cdot q_p \cdot \int_0^{2\pi} c_{p0}(\phi) \cdot \cos\phi \, d\phi \tag{A.165}$$

The function $c_{p0}(\phi)$ represents the »shape« of the wind pressure distribution around the silo circumference and is defined as a Fourier cosine series (see *Section 4.2.1*).

Let us now denote with $I$ the integral $I = \int_0^{2\pi} c_{p0}(\phi) \cdot \cos\phi \, d\phi$:

$$I = \int_0^{2\pi} c_{p0}(\phi) \cdot \cos\phi \, d\phi$$

$$I = \int_0^{2\pi} \left( \frac{c_0}{2} + \sum_{m=1}^{\infty} c_m \cdot \cos m\phi \right) \cdot \cos\phi \, d\phi$$

$$I = \int_0^{2\pi} \left( \frac{c_0}{2} \cdot \cos 0\phi + \sum_{m=1}^{\infty} c_m \cdot \cos m\phi \right) \cdot \cos\phi \, d\phi \tag{A.166}$$

As in equation (A.153), the orthogonality property of the cosine function implies:

$$\int_0^{2\pi} \cos m\phi \cdot \cos n\phi \, d\phi = \left\langle \begin{array}{l} 0, \ \text{if } \ m \neq n \\ \pi, \ \text{if } \ m = n \end{array} \right\rangle \tag{A.167}$$

In our case $n = 1$.This means that integral $I$ is different from $0$ only, when $m = 1$. Integral $I$ then equals:

$$I = \int_0^{2\pi} c_1 \cdot \cos(\phi)^2 \, d\phi$$

$$I = c_1 \cdot \pi \tag{A.168}$$

The value for the total wind force then equals:

$$W = \pi \cdot h_c \cdot r \cdot c_1 \cdot q_p \tag{A.169}$$

If we consider now the external wind pressure function for typical silo structures, given in *Section 4.2.1*, we can see from
that

$$c_1 = 0.372\,. \tag{A.170}$$

The total wind force on a typical silo structure equals:

$$\boxed{W = 0.372 \cdot \pi \cdot r \cdot h_c \cdot q_p} \tag{A.171}$$

### 4.3.2 Empty silo situation

When considering the wind load on an empty silo, we need to consider the wind external wind pressure $w_e(\phi)$ for the following two design situations[13]:

- plastic limit state design (LS1)
- buckling limit state design (LS3)

#### 4.3.2.1 Wind pressure for plastic limit state design

For the plastic limit state design of the empty silo shell, we need to consider the external wind pressure $w_e(\phi)$ function as defined in equation (A.162).

The section forces for this case are given in *Section 4.4.2*.

#### 4.3.2.2 Wind pressures for buckling limit state design

For the purpose of the buckling limit state design of the empty silo shell, he external wind pressure $w_e(\phi)$ function need to be replaced by an equivalent axisymetric wind pressure distribution. This distribution is such that it accounts for the buckling design check. It takes into account silo shell imperfections and other parameters relevant for the buckling phenomena.

*Figure 29 (b)* below represents the equivalent axisymetric compressive pressure on the silo cylinder.

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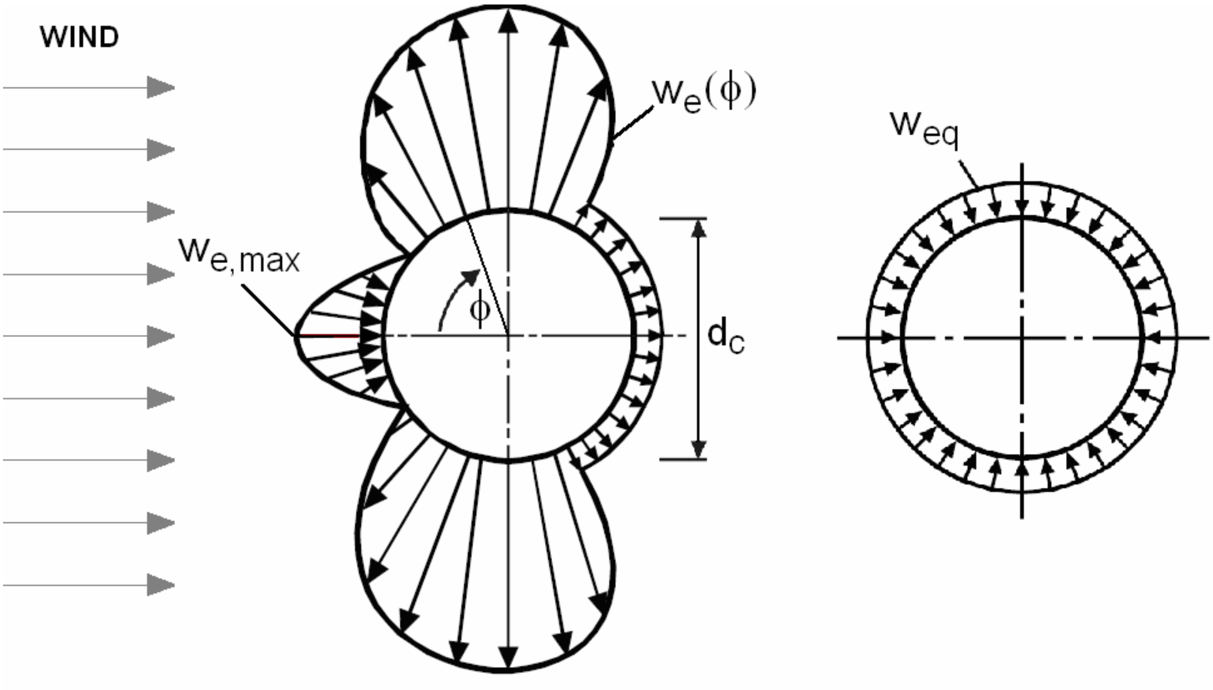
[13] See section 7.1.2 for the description of design situations.



**(a)** **(b)**

**Figure 29: Wind pressure distribution around the silo shell circumference (a) and the equivalent axisymetric pressure distribution (b)**

The external wind pressure $w_e(\phi)$ function is defined in equation (A.162). For the calculation of the equivalent axisymetric pressure $w_{eq}$ only the maximum compressive value of the wind pressure distribution $w_{e,max}$ is needed. We can see that this value occurs when $\phi = 0°$:

$$w_{e,max} = w_e(0)$$

$$w_{e,max} = q_p \cdot c_{p0}(0) \tag{A.172}$$

From *Section 4.2.1* we can see that

$$c_{p0}(0) = 1. \tag{A.173}$$

The maximum compressive value of the wind pressure distribution $w_{e,max}$ then equals:

$$w_{e,max} = q_p \tag{A.174}$$

The equivalent axisymetric pressure distribution is given by:

$$w_{eq} = k_w \cdot w_{e,max}$$

$$\boxed{w_{eq} = k_w \cdot q_p} \tag{A.175}$$

where

$$\boxed{k_w = 0.46 \cdot \left( 1 + 0.1 \sqrt{\frac{C_\Theta}{\omega} \cdot \frac{r}{t}} \right)} \tag{A.176}$$

Additionally $k_w$ should not lie outside the range:

$$\boxed{0.65 \le k_w \le 1.0} \tag{A.177}$$

Parameter $\omega$ represents the length of the shell segment and is characterized in terms of the dimensionless length :

$$\omega = \frac{h_c}{\sqrt{rt}} \tag{A.178}$$

Parameter $C_\Theta$ represents the external pressure buckling factor. It can be obtained by considering the proper silo cylinder boundary conditions, which are represented in the figure below.

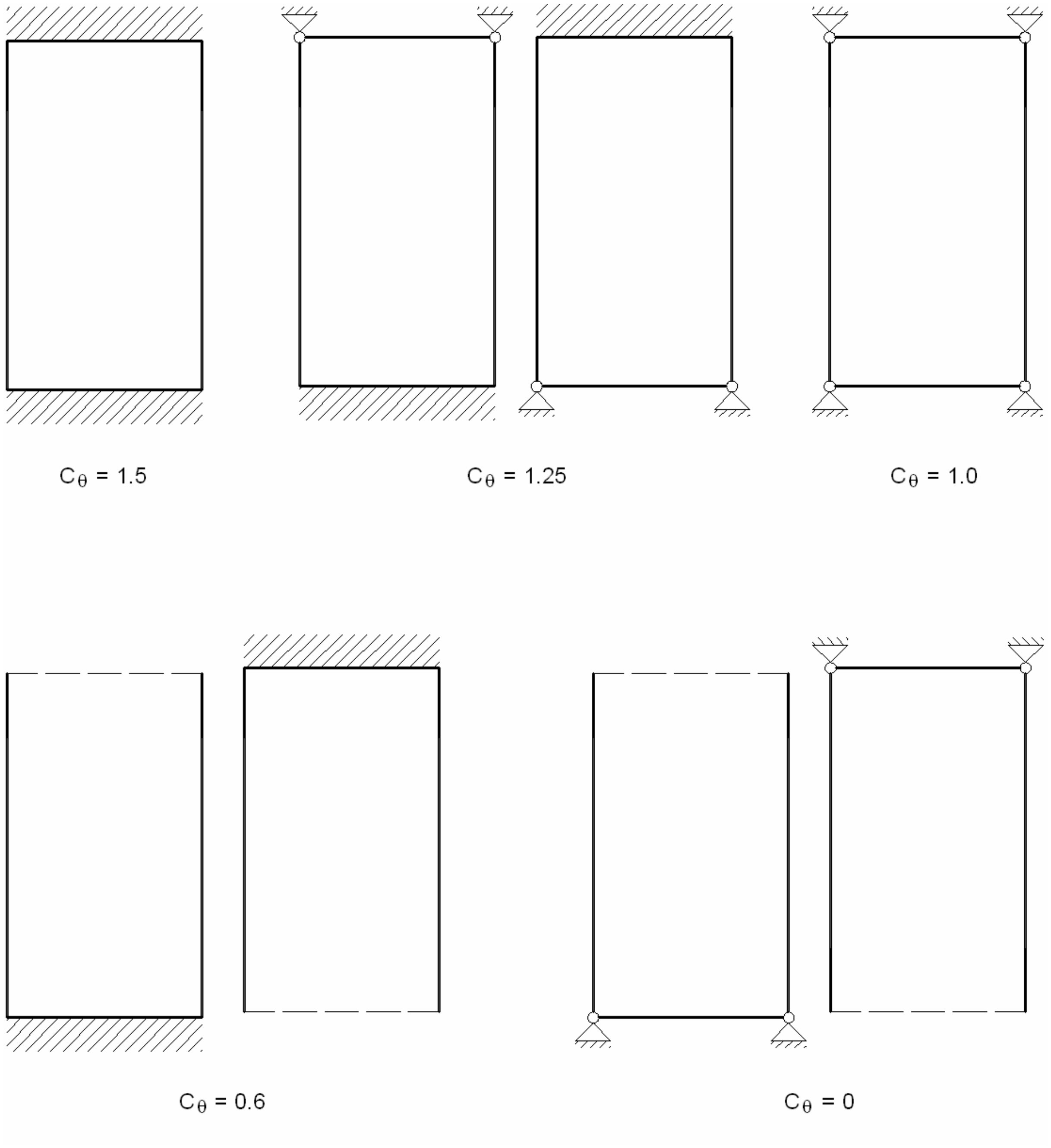


**Figure 30: External pressure buckling factor $C_\Theta$ for the various silo cylinder boundary conditions**

## *4.4 Membrane forces*

### 4.4.1 Full silo situation

Global bending of the silo shell will be considered by approximating the silo cylinder as a cantilever beam.

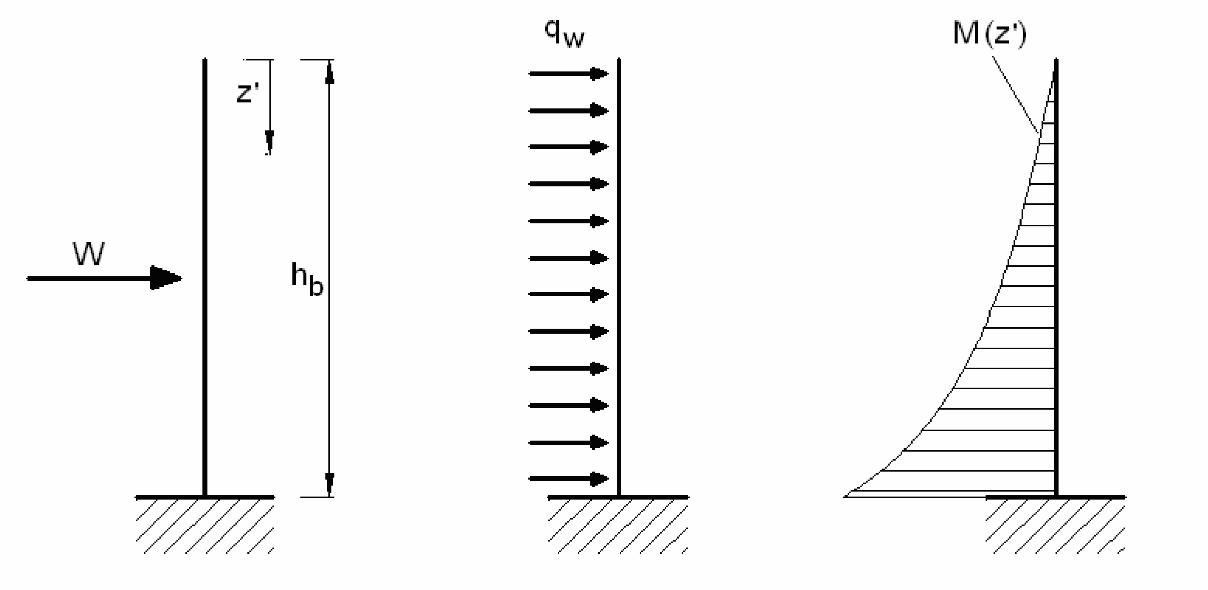


**Figure 31: Approximation of the silo cylinder as cantilever beam**

First a substitute uniform load $q_w$ must be determined as:

$$q_w = \frac{W}{d_c} \tag{A.179}$$

The coordinate *z'* runs from the top to the bottom of the silo cylinder. The bending moment of the cylinder for any given height *z* equals:

$$M(z') = q_w \cdot \frac{z'^2}{2} \tag{A.180}$$

The maximum vertical section force $n_{z,wf}$ and circumferential section force $n_{\phi,\ wf}$ due to wind on a full silo equal:

$$\boxed{n_{z,wf}(z') = \frac{M(z')}{I} \cdot r \cdot t} \tag{A.181}$$

$$\boxed{n_{\phi,wf} = 0} \tag{A.182}$$

The parameter $I$ represents the moment of inertia of the silo cylinder and is given in equation *(A.14)*.

## 4.4.2 Empty silo situation

### 4.4.2.1 Plastic limit state

For the calculation of the membrane forces for the plastic design of the empty silo shell, we will use the external wind pressure $w_e(\phi)$ function as defined in equation (A.162). The Fourier cosine series, which is used to represent the external wind pressure coefficient function $c_{p0}(\phi)$, is calculated in section 4.2.1 for only 20 terms. The formulas for the membrane forces presented here, will be given here in general form, that is for any number of terms.

The vertical section force $n_{z,we}$, the circumferential section force $n_{\phi,\ we}$ and the shear force $n_{z\phi,we}$ due to wind on an empty silo equal:

$$\boxed{n_{z,we}(z) = \frac{q_p}{2r} \cdot \left( \sum_{m=1}^{\infty} m^2 \, c_m \cos m\phi \right) \cdot z^2} \tag{A.183}$$

$$\boxed{n_{\phi,we}(z) = -\frac{q_p \, r}{2} \cdot \left( c_0 + 2 \sum_{m=1}^{\infty} c_m \cos m\phi \right)} \tag{A.184}$$

$$\boxed{n_{z\phi,we}(z) = -q_p \cdot \left( \sum_{m=1}^{\infty} m \, c_m \sin m\phi \right) \cdot z} \tag{A.185}$$

### 4.4.2.2 Buckling limit state

The equivalent axisymetric wind load $w_{eq}$ causes the following membrane forces:

$$\boxed{n_{z,we}(z)=0} \tag{A.186}$$

$$\boxed{n_{\phi,we}(z,\phi)=-w_{eq}\cdot r} \tag{A.187}$$

## 4.5 Computer program

A computer program has been written in order to aid the calculation of the wind pressure on a silo structure.

The program can be found on the CD accompanying this work. The CD is located on the inside of the back cover. To start the program open the *Wind Load on Cylindrical Silos.xls* file.

In order to run the program *Microsoft Excel 2003* (or newer) needs to be installed. Macros have to be enabled and macro security has to be set to medium or low.

Detailed instructions on how to use the program can be found in *Appendix B*, *Section 2*.

# 5 Earthquake load on silos

## 5.1 Modeling considerations for silo structures

### 5.1.1 Full Silo

The main property of the silo is that it can carry the weight of the stored material which is many times bigger than the self weight of the silo structure. In the seismic analysis, the shell of a full silo is considered as being rigid if compared to the supporting structure.

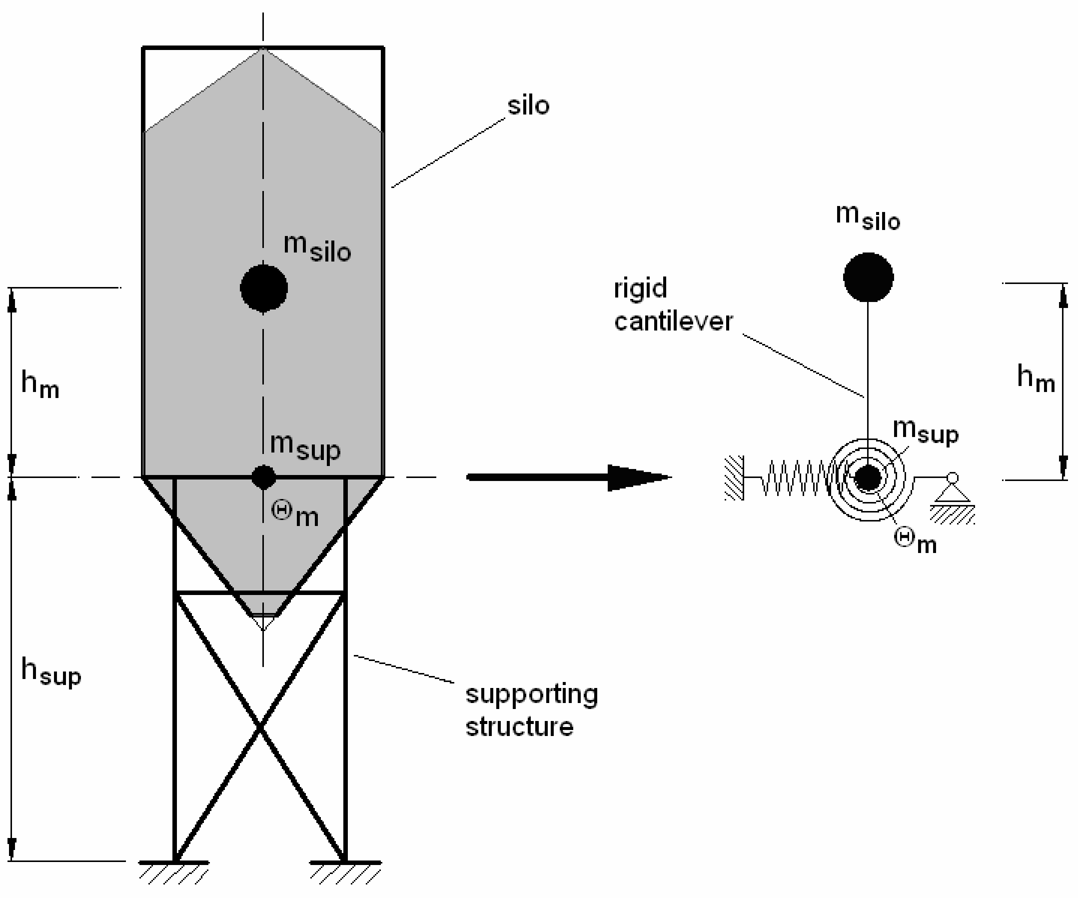


Figure 32: Rigid cantilever model for the seismic analysis

This means that there is a large mass suspended above flexible supports and that in a seismic calculation we can model the silo as a vertical rigid cantilever with a concentrated point mass at the upper end and a horizontal and rotational spring at the lower end. The springs represent the stiffness of the supporting structure. The vibration effects of the supporting structure can be considered by adding an additional transitional mass $m_{sup}$ and rotational mass $\Theta_m$ at the lower point of the rigid cantilever.

If we neglect the masses of the supporting structure, the model has only one mass supported at height $h_m$ from the transition between the silo cylinder and silo hopper and thus has only one possible vibration mode in the horizontal direction. This means that this system is a Single-Degree-of-Freedom system and therefore the "lateral force method" of analysis can be used. In the EN codes this model is also called an "inverted pendulum" model. We will call this model *Seismic model 1*.

If both masses are considered, the model has two possible vibration modes (each mass can oscillate separately) and is considered as a Two-Degree-of-Freedom system. In this case the "modal response spectrum" analysis should be used. We will call this model *Seismic model 2*.

Both methods are considered as static methods. This means that dynamic problems can be translated into static problems by using the d'Alembert's principle, which in turn implies the equilibrium condition:

$$\Sigma F + F_I = 0 \tag{A.188}$$

where:

$\Sigma F$ denotes the sum of static forces on a given system,

$F_I$ denotes the inertia force, representing the dynamic effects (2nd Newton's law):

$$F_I = - m \cdot a \tag{A.189}$$

In the following chapters both models will be analyzed more thoroughly.

### 5.1.2 Empty silo

When the silo is empty, the supporting structure is considered to be much stiffer than the silo shell. In this case section forces have to be calculated by performing a seismic analysis of the silo shell. Here contributions of higher modes significantly affect the response of the structure, so modal response spectrum analysis has to be used. According to EN 1998-1, the sum of the effective modal masses for the modes taken into account has to amount to at least 90% of the total mass of the structure.

### 5.1.3 Fundamental periods of vibration for the two models

#### 5.1.3.1 Seismic model 1

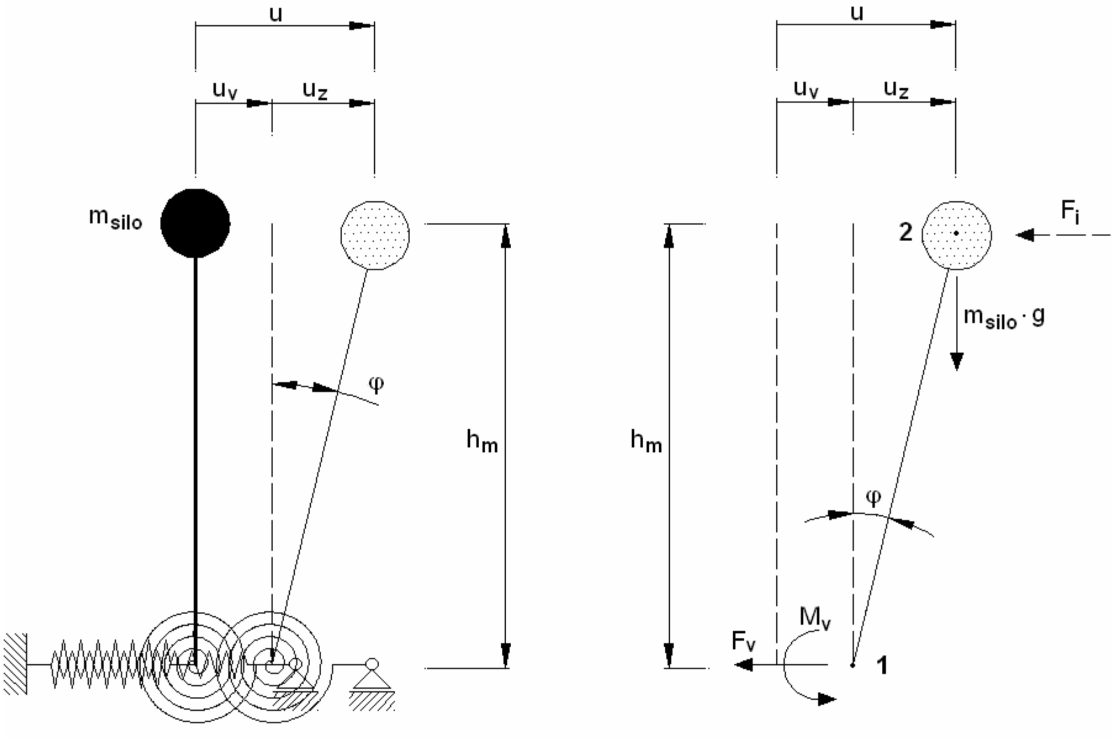


**Figure 33: Displacements and forces on Seismic model 1**

Let $u_v$ be the displacement because of the deformation of the horizontal spring and $u_z$ be the displacement because of the deformation of the rotational spring.

Like in most civil engineering problems, we can assume that the rotation $\phi$ in a silo under a seismic load is very small ($\varphi << 0.2\,rad$). The following simplification applies:

$$sin\,\varphi \cong \varphi \tag{A.190}$$

Considering the above simplification, we can now write:

$$u_z = h_m \cdot \varphi \tag{A.191}$$

The total displacement $u$ then equals:

$$u = u_v + u_z = u_v + h_m \cdot \varphi \tag{A.192}$$

Let us also denote with $k_v$ the stiffness of the horizontal spring and with $k_m$ the rotational stiffness in the rotational spring. The spring force $F_v$ and spring moment $M_v$ then equal:

$$F_v = k_v \cdot u_v \tag{A.193}$$

$$M_v = k_m \cdot \varphi \tag{A.194}$$

The force $F_i$ denotes the inertia force, representing the dynamic effects according to the d'Alembert's principle:

$$F_i = m_{silo} \cdot \ddot{u} \tag{A.195}$$

Let us now write the equilibrium condition for the sum of forces in the horizontal direction (coordinate $x$) and for the sum of moments at point $1$, considering equations (A.191) to (A.195):

$$\sum F_x = 0: \qquad F_i + F_v = 0$$

$$m_{silo} \cdot \ddot{u} + k_v \cdot u_v = 0$$

$$m_{silo} \cdot (\ddot{u}_v + \ddot{u}_z) + k_v \cdot u_v = 0$$

$$m_{silo} \cdot \ddot{u}_v + m_{silo} \cdot \ddot{u}_z + k_v \cdot u_v = 0 \tag{A.196}$$

$$\sum M^1 = 0: \quad M_v + F_i \cdot h_m - m_{silo} \cdot g \cdot u_z = 0$$

$$M_v + m_{silo} \cdot \ddot{u} \cdot h_m - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0$$

$$k_m \cdot \varphi + m_{silo} \cdot \ddot{u} \cdot h_m = m_{silo} \cdot g \cdot h_m \cdot \varphi$$

$$k_m \cdot \varphi + m_{silo} \cdot (\ddot{u}_v + \ddot{u}_z) \cdot h_m = m_{silo} \cdot g \cdot h_m \cdot \varphi$$

$$k_m \cdot \varphi + m_{silo} \cdot \ddot{u}_v \cdot h_m + m_{silo} \cdot \ddot{u}_z \cdot h_m - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0$$

$$k_m \cdot \varphi + m_{silo} \cdot \ddot{u}_v \cdot h_m + m_{silo} \cdot h_m^{\ 2} \cdot \ddot{\varphi} - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0 \qquad \text{(A.197)}$$

We can now form the system of differential equations for free vibration (vibration with no external load):

$$[M] \cdot \{\ddot{u}\} + [K] \cdot \{u\} = \{0\} \qquad \text{(A.198)}$$

where:

$[M]$ is the mass matrix, whose coefficients $m_{ij}$ represent the inertia force in point $i$ because of the unit acceleration in point $j$,

$[K]$ is the stiffness matrix, whose coefficients $k_{ij}$ represent the force in point $i$ because of the unit displacement in point $j$,

$\{\ddot{u}\}$ is the vector of accelerations,

$\{u\}$ is the vector of displacements.

Considering the equilibrium conditions given in equations (A.196) and (A.197) for the particular seismic model, we get the following system of ordinary differential equations:

$$\begin{bmatrix} m_{silo} & m_{silo} \cdot h_m \\ m_{silo} \cdot h_m & m_{silo} \cdot h_m^{\ 2} \end{bmatrix} \cdot \begin{Bmatrix} \ddot{u}_v \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} k_v & 0 \\ 0 & k_m - m_{silo} \cdot g \cdot h_m \end{bmatrix} \cdot \begin{Bmatrix} u_v \\ \varphi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \text{(A.199)}$$

Let's use the following function as a solution for *{u}*:

$$\{u\} = \{\phi\} \cdot sin(\omega t - \Theta) \qquad \text{(A.200)}$$

where:

$\{\phi\}$ is the vibration mode of the system,

$\omega$ is the angular frequency,

$\Theta$ is the phase of vibration.

The derivative $\{\ddot{u}\}$ then equals:

$$\{\ddot{u}\} = -\omega^2 \cdot \{\phi\} \cdot sin(\omega t - \Theta) \tag{A.201}$$

If we consider equations (A.200) and (A.201) with equation (A.198) we get:

$$-\omega^2 \cdot sin(\omega t - \Theta)[M]\{\phi\} + sin(\omega t - \Theta)[K]\{\phi\} = \{0\} \tag{A.202}$$

Because generally $sin(\omega t - \Theta) \neq 0$, the following must be true:

$$([K] - \omega^2 \cdot [M]) \cdot \{\phi\} = \{0\} \tag{A.203}$$

We see that the initial system of equations for vibration, translates into a problem of eigenvalues.

The eigenvalues of the free vibration system represent the natural angular frequency $\omega$. They can be obtained by solving the determinant $D$ below:

$$D = det([K] - \omega^2 \cdot [M]) = 0 \tag{A.204}$$

Considering the values for $[M]$ and $[K]$ for the given system, the determinant $D$ equals:

$$D = det\left(\begin{bmatrix} k_v & 0 \\ 0 & k_m - m_{silo} \cdot g \cdot h_m \end{bmatrix} - \omega^2 \cdot \begin{bmatrix} m_{silo} & m_{silo} \cdot h_m \\ m_{silo} \cdot h_m & m_{silo} \cdot h_m^2 \end{bmatrix}\right) = 0$$

$$D = \begin{vmatrix} k_v - \omega^2 \cdot m_{silo} & -\omega^2 \cdot m_{silo} \cdot h_m \\ -\omega^2 \cdot m_{silo} \cdot h_m & k_m - m_{silo} \cdot g \cdot h_m - \omega^2 \cdot m_{silo} \cdot h_m^2 \end{vmatrix} = 0$$

$$D = (k_v - \omega^2 \cdot m_{silo}) \cdot (k_m - m_{silo} \cdot g \cdot h_m - \omega^2 \cdot m_{silo} \cdot h_m^2) - (-\omega^2 \cdot m_{silo} \cdot h_m)^2 = 0 \tag{A.205}$$

After solving equation (A.205) we get:

$$\boxed{\omega = \sqrt{\frac{k_v}{m_{silo}} \cdot \frac{k_m - m_{silo} \cdot g \cdot h_m}{k_m + h_m\left(h_m \cdot k_v - m_{silo} \cdot g\right)}}} \tag{A.206}$$

Extreme cases:

$$k_m \to \infty : \qquad \omega = \sqrt{\frac{k_v}{m_{silo}}} \tag{A.207}$$

$$k_v \to \infty : \qquad \omega = \sqrt{\frac{k_m - m_{silo} \cdot g \cdot h_m}{m_{silo} \cdot h_m^{\ 2}}} \tag{A.208}$$

The fundamental period of vibration is defined as:

$$T = \frac{2\pi}{\omega} \tag{A.209}$$

The fundamental period of vibration of *Seismic model 1* then equals:

$$\boxed{T = \frac{2\pi}{\sqrt{\frac{k_v}{m_{silo}} \cdot \frac{k_m - m_{silo} \cdot g \cdot h_m}{k_m + h_m\left(h_m \cdot k_v - m_{silo} \cdot g\right)}}}} \tag{A.210}$$

### 5.1.3.2 Seismic model 2

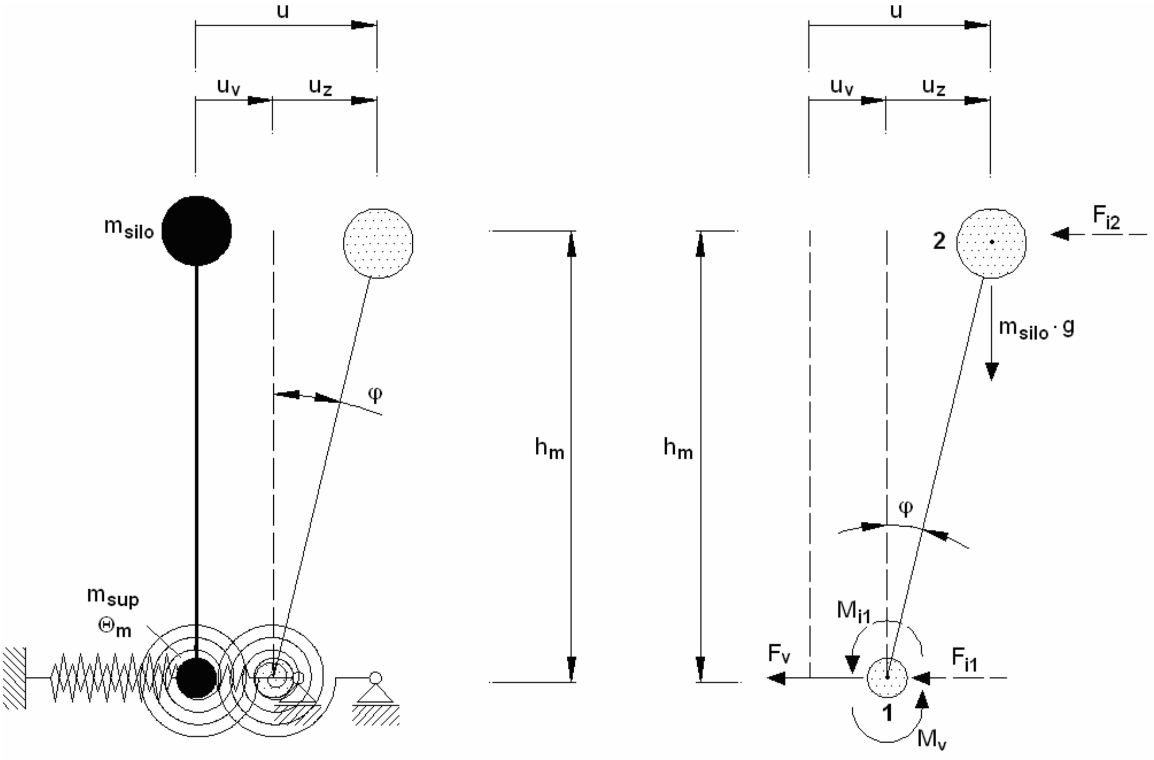


**Figure 34: Displacements and forces on Seismic model 2**

In this model we add a transitional mass $m_{sup}$ and rotational mass $\Theta_m$ at the lower point of the rigid cantilever, to represent the vibration effects of the supporting structure. They should be calculated in regard to the chosen type of supporting structure.

As an estimate, the values for the transitional mass $m_{sup}$ and rotational mass $\Theta_m$ can be calculated, by modeling the supporting structure as a supplementary uniform beam element. The diagonal mass matrix for such an element equals:

$$[m_d] = \frac{m_{sup,total}}{420} \begin{bmatrix} 210 & & & \\ & {h_{sup}}^2 & & \\ & & 210 & \\ & & & {h_{sup}}^2 \end{bmatrix} \qquad \text{(A.211)}$$

where $m_{sup,total}$ equals the total mass of the supporting structure.

This means that the translational mass $m_{sup}$ and rotational mass $\Theta_m$ of the supporting structure at each point of the supplementary beam element equal:

$$m_{sup} = 0.5\, m_{sup,total} \tag{A.212}$$

$$\Theta_m = \frac{m_{sup,total} \cdot h_{sup}^{\;2}}{420} \tag{A.213}$$

In this model we get two inertial forces: $F_{i1}$, which denotes the inertia force at point *1* and $F_{i2}$, which denotes the inertia force at point *2* (see ***Error! Reference source not found.***):

$$F_{i1} = m_{sup} \cdot \ddot{u}_v \tag{A.214}$$

$$F_{i2} = m_{silo} \cdot \ddot{u} \tag{A.215}$$

The rotational mass $\Theta_m$ at point *1* causes an inertial moment $M_{i1}$:

$$M_{i1} = \Theta_m \cdot \ddot{\varphi} \tag{A.216}$$

Equations (A.190) to (A.195) apply to this model as well. If we now write the equilibrium condition for the sum of forces in the horizontal direction (coordinate *x*) and for the sum of moments at point *1*, we get:

$$\begin{aligned}
\sum F_x = 0: \quad & F_{i1} + F_{i2} + F_v = 0 \\
& m_{sup} \cdot \ddot{u}_v + m_{silo} \cdot \ddot{u} + k_v \cdot u_v = 0 \\
& m_{sup} \cdot \ddot{u}_v + m_{silo} \cdot \left(\ddot{u}_v + \ddot{u}_z\right) + k_v \cdot u_v = 0 \\
& \left(m_{sup} + m_{silo}\right) \cdot \ddot{u}_v + m_{silo} \cdot \ddot{u}_z + k_v \cdot u_v = 0 \\
& \left(m_{sup} + m_{silo}\right) \cdot \ddot{u}_v + m_{silo} \cdot h_m \cdot \ddot{\varphi} + k_v \cdot u_v = 0
\end{aligned} \tag{A.217}$$

$$\sum M^1 = 0: \quad M_v + M_{i1} + F_{i2} \cdot h_m - m_{silo} \cdot g \cdot u_z = 0$$

$$k_m \cdot \varphi + \Theta_m \cdot \ddot{\varphi} + m_{silo} \cdot \ddot{u} \cdot h_m - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0$$

$$k_m \cdot \varphi + \Theta_m \cdot \ddot{\varphi} + m_{silo} \cdot h_m \cdot \left(\ddot{u}_v + h_m \cdot \ddot{\varphi}\right) - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0$$

$$m_{silo} \cdot h_m \cdot \ddot{u}_v + \left(\Theta_m + m_{silo} \cdot h_m^{\,2}\right) \cdot \ddot{\varphi} + k_m \cdot \varphi - m_{silo} \cdot g \cdot h_m \cdot \varphi = 0 \tag{A.218}$$

If we now form the system of ordinary differential equations for vibration, we get:

$$\begin{bmatrix} m_{silo} + m_{sup} & m_{silo} \cdot h_m \\ m_{silo} \cdot h_m & m_{silo} \cdot h_m^{\,2} + \Theta_m \end{bmatrix} \cdot \begin{Bmatrix} \ddot{u}_v \\ \ddot{\varphi} \end{Bmatrix} + \begin{bmatrix} k_v & 0 \\ 0 & k_m - m_{silo} \cdot g \cdot h_m \end{bmatrix} \cdot \begin{Bmatrix} u_v \\ \varphi \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{A.219}$$

The natural angular frequency $\omega$ is calculated by the solving the determinant $D$ below:

$$D = det\left(\begin{bmatrix} k_v & 0 \\ 0 & k_m - m_{silo} \cdot g \cdot h_m \end{bmatrix} - \omega^2 \cdot \begin{bmatrix} m_{silo} + m_{sup} & m_{silo} \cdot h_m \\ m_{silo} \cdot h_m & m_{silo} \cdot h_m^{\,2} + \Theta_m \end{bmatrix}\right) = 0$$

$$D = \begin{vmatrix} k_v - \omega^2 \cdot \left(m_{silo} + m_{sup}\right) & -\omega^2 \cdot m_{silo} \cdot h_m \\ -\omega^2 \cdot m_{silo} \cdot h_m & k_m - m_{silo} \cdot g \cdot h_m - \omega^2 \cdot \left(m_{silo} \cdot h_m^{\,2} + \Theta_m\right) \end{vmatrix} = 0 \tag{A.220}$$

After solving equation the determinant above, we get:

$$\boxed{\omega_1 = \sqrt{\frac{A - \sqrt{B - C}}{D}}} \tag{A.221}$$

$$\boxed{\omega_2 = \sqrt{\frac{A + \sqrt{B - C}}{D}}} \tag{A.222}$$

where:

$$A = k_m \left(m_{sup} + m_{silo}\right) + k_v \left(h_m^{\,2}\, m_{silo} + \Theta_m\right) - m_{silo} \cdot g \cdot h_m \left(m_{sup} + m_{silo}\right) \tag{A.223}$$

$$B = \left(m_{silo} \cdot g \cdot h_m \left(m_{sup} + m_{silo}\right) - k_m \left(m_{sup} + m_{silo}\right) - k_v \left(h_m^{\,2}\, m_{silo} + \Theta_m\right)\right)^2 \tag{A.224}$$

$$C = 4 k_v \left(k_m - m_{silo} \cdot g \cdot h_m\right) \cdot \left[h_m^{\,2}\, m_{sup}\, m_{silo} + \Theta_m \left(m_{sup} + m_{silo}\right)\right] \tag{A.225}$$

$$D = 2\left(h_m^{\,2}\, m_{sup}\, m_{silo} + \Theta_m \left(m_{sup} + m_{silo}\right)\right) \tag{A.226}$$

The fundamental periods of vibration of *Seismic model 2* then equal:

$$\boxed{T_1 = \frac{2\pi}{\omega_1}} \tag{A.227}$$

$$\boxed{T_2 = \frac{2\pi}{\omega_2}} \tag{A.228}$$

## *5.2 Load assessment procedure for a full silo*

### 5.2.1 Procedure description

The procedure below can be used to determine the earthquake force and additional wall pressures on a full silo. Only the "lateral force method" of analysis for *Seismic model 1* will be presented.

The "modal response spectrum" analysis for *Seismic model 2* is presented on the example silo in Section 3, where a comparison of the two models is also given.

The procedure is based on *EN 1998-1:2004* and *EN 1998-4:2006*.

**Table 19: Overview of procedure steps**

| Step | Task | Task location |
|---|---|---|
| 1 | Importance class | EN 1998-4, section 2.1.3(4) |
| 2 | Total mass of the silo structure | EN 1998-1, section 3.2.4(2)P |
| 3 | Stiffness of the supporting structure | - |
| 4 | Fundamental period of vibration | - |
| 5 | Ground type | EN 1998-1, table 3.1 and 3.2 |
| 6 | Design ground acceleration | EN 1998-1, section 3.2.1(3) |
| 7 | Behavior factor | EN 1998-1, section 6.3 |
| 8 | Design spectrum | EN 1998-1, section 3.2.2.2 |
| 9 | Base shear force | EN 1998-1, section 4.3.3.2.2 |
| 10 | Additional normal wall pressure | EN 1998-4, section 3.3(5) |

## 5.2.2 Description of procedure steps

### 5.2.2.1 Step 1: Importance class

In the event of an earthquake, the importance class of the silo structure specifies what consequences the failure of the silo would have on the environment, social and economic factors. There are three known importance classes. One class has to be chosen for each considered silo structure.

**Table 20: Importance classes**

| Importance class | Failure consequence |
|---|---|
| Class I | small |
| Class II | medium |
| Class III | high |

When the failure consequences are **small**, the risk to life is low and the environmental, economic and social consequences are small or negligible.

When the failure consequences are **medium**, a considerable risk to life exists and environmental, economic or social consequences are considerable.

When the failure consequences are **high**, there is a severe risk to life and the environmental, economic and social consequences are large.

After an importance class has been chosen, the proper importance factor $\gamma_I$ can then be obtained from the table below. The importance factor depends on the importance class and on the nature of the stored material.

**Table 21: Importance factor $\gamma_I$**

| Nature of the stored material | Importance Class | | |
|---|---|---|---|
| | I | II | III |
| Non-toxic, non inflammable material | 0.8 | 1 | 1.2 |
| Non-volatile toxic material | 1 | 1.2 | 1.4 |
| Explosive and other high flammability liquids | 1.2 | 1.4 | 1.6 |

It is used to multiply the total mass of the system *m* as well as in obtaining the design ground acceleration $a_g$.

### 5.2.2.2 Step 2: Total mass of the silo

The presence of masses associated to the sum *G* of permanent gravity loads (self-weight of the silo structure) and the filling load *m* (mass of the silo contents) is computed as follows taking into account the importance factor $\gamma_I$:

$$m_{silo} = \gamma_I \left( G + 0.8\, m \right) \tag{A.229}$$

### 5.2.2.3 Step 3: Stiffness of the supporting structure

The proper horizontal ($k_v$) and rotational ($k_m$) stiffness of the supporting structure has to be calculated, depending on the type of supporting structure.

### 5.2.2.4 Step 4: Fundamental period of vibration

The fundamental period of vibration should be calculated using equation (A.210):
The symbol *g* in that equation represents the acceleration of gravity:

$$g = 9.81\ m / s^2 \tag{A.230}$$

### 5.2.2.5 Step 5: Ground type

Based on the ground type, the appropriate design spectrum parameters $S$, $T_B(s)$, $T_C(s)$, $T_D(s)$ should be obtained from the table below:

**Table 22: Parameters based on the type of ground**

| Ground type | Description of stratigraphic profile | S | $T_B$(s) | $T_C$(s) | $T_D$(s) |
|---|---|---|---|---|---|
| A | Rock or other rock-like geological formation, including at most 5 m of weaker material at the surface. | 1 | 0.15 | 0.4 | 2 |
| B | Very dense sand, gravel, or very stiff clay, at least several tens of meters in thickness, characterized by a gradual increase of mechanical properties with depth. | 1.2 | 0.15 | 0.5 | 2 |
| C | Deep deposits of dense or medium dense sand, gravel or stiff clay with thickness from several tens to many hundreds of meters. | 1.15 | 0.2 | 0.6 | 2 |
| D | Deposits of loose-to-medium cohesion less soil (with or without some soft cohesive layers), or of predominantly soft-to-firm cohesive soil. | 1.35 | 0.2 | 0.8 | 2 |
| E | A soil profile consisting of a surface alluvium layer with $v_s$ values of type C or D and thickness varying between about 5 m and 20 m. (*see EN 1998-1, 3.1.1(2) for details*) | 1.4 | 0.15 | 0.5 | 2 |
| S1 | Deposits consisting – or containing a layer at least 10 m thick – of soft clays/silts with high plasticity index (PI > 40) and high water content. | special studies should provide the corresponding values | | | |
| S2 | Deposits of liquefiable soils, of sensitive clays, or any other soil profile not included in types A – E or S1. | | | | |

### 5.2.2.6 Step 6: Design ground acceleration

The design ground acceleration is defined as the design ground acceleration on *type A* ground and is given with the following equation:

$$a_g = \gamma_I \cdot a_{gR} \tag{A.231}$$

The reference peak ground acceleration on *type A* ground, $a_{gR}$, for use in a Country or its parts, may be derived from zonation maps found in its *National Annex* of the *EN 1998* standard.

### 5.2.2.7 Step 6: Behavior factor

The supporting structure of a silo shall be designed according to one of the following ductility classes. The proper behavior factor should be considered:

**Table 23: Behavior factor**

| **Ductility class** | **Behavior factor (q)** |
|---|---|
| Low (DCL) | 1.0 |
| Medium (DCM) | 2.0 or 1.25<br>*see Note below* |
| High (DCH) | |

**Note:**

– for silos supported on a single pedestal or skirt, or on irregular bracings, the upper limit of the $q$ factors is: $q = 1.0$.

– for silos supported on moment resisting frames or on regular bracings, the upper limit of the $q$ factors is: $q = 1.25$.

### 5.2.2.8 Step 8: Design spectrum for elastic analysis

For the calculated fundamental period of vibration ($T_1$) the appropriate response value ($S_d(T_1)$) should be obtained from the design spectrum for elastic analysis two limit states:

- ultimate limit state (ULS),
- damage limitation state (DLS).

The response values were calculated by using the proper viscous damping ratio.

**Table 24:Design spectrum for the ULS**

| Period of vibration | Ultimate limit state (ULS) |
|---|---|
| $0 \le T \le T_B$ | $S_d(T) = a_g \cdot S \cdot \left[\frac{2}{3} + \frac{T}{T_B} \cdot \left(\frac{2.5}{q} - \frac{2}{3}\right)\right]$ |
| $T_B < T \le T_C$ | $S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}$ |
| $T_C < T \le T_D$ | $S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C}{T}\right] \ge 0.2a_g$ |
| $T_D < T$ | $S_d(T) = a_g \cdot S \cdot \frac{2.5}{q}\left[\frac{T_C \cdot T_D}{T^2}\right] \ge 0.2a_g$ |

**Table 25: Design spectrum for the DLS**

| Period of vibration | Damage limitation state (DLS) |
|---|---|
| $0 \le T \le T_B$ | $S_d(T) = a_g \cdot S \cdot \left[\frac{1}{2} + \frac{T}{T_B} \cdot \left(\frac{1.5}{q} - \frac{1}{2}\right)\right]$ |
| $T_B < T \le T_C$ | $S_d(T) = a_g \cdot S \cdot \frac{1.5}{q}$ |
| $T_C < T \le T_D$ | $S_d(T) = a_g \cdot S \cdot \frac{1.5}{q}\left[\frac{T_C}{T}\right] \ge 0.2a_g$ |
| $T_D < T$ | $S_d(T) = a_g \cdot S \cdot \frac{1.5}{q}\left[\frac{T_C \cdot T_D}{T^2}\right] \ge 0.2a_g$ |

### 5.2.2.9 Step 9: Base shear force

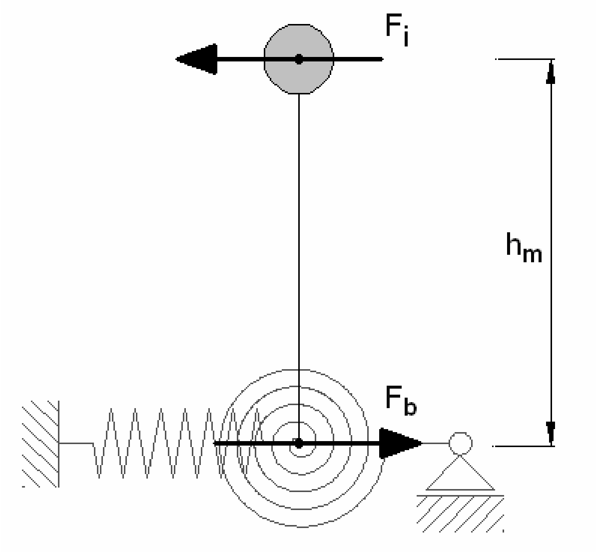


**Figure 35: Base shear force**

The base shear force acts at the supports. It represents the total earthquake force on the silo structure. Because we are using the rigid cantilever model in the seismic analysis (*Figure 32*), this force acts at the support of the cantilever. It is given by the following equation:

$$F_b = S_d(T_1) \cdot m \tag{A.232}$$

The base shear force should be calculated for both limit states (ULS and DLS), where $S_d(T_1)$ is obtained from Table 24 for ULS and Table 25 for DLS.

The force $F_i$ represents the inertia force, which equals the base shear force:

$$F_i = F_B \tag{A.233}$$

It acts at the silo centre of gravity $h_m$ given in equation (A.13).

#### 5.2.2.10 Step 10: Additional normal pressure on the silo wall

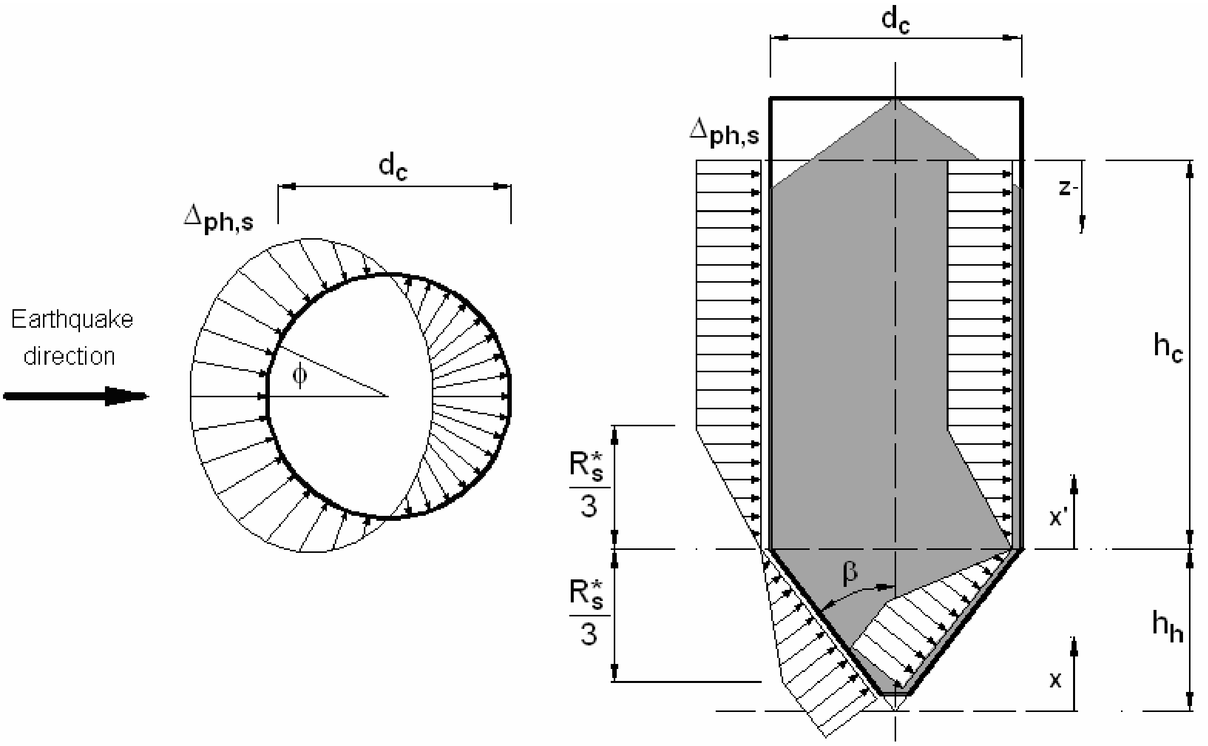


**Figure 36: Additional normal pressure distribution along the silo wall**

The procedure given in Chart 4 is to be used to determine the additional normal pressure on the silo wall because of the seismic load ($\Delta_{ph,s}$).

At any location on the silo wall this pressure is limited by the condition that the sum of the static pressures of the particulate material on the wall and the additional pressure given by the following procedure may not be taken less than zero.

This means that we have the following additional restriction:

**Cylinder:** $p_h + \Delta_{ph,s} \geq 0$ (A.234)

**Hopper:** $p_n + \Delta_{ph,s} \geq 0$ (A.235)

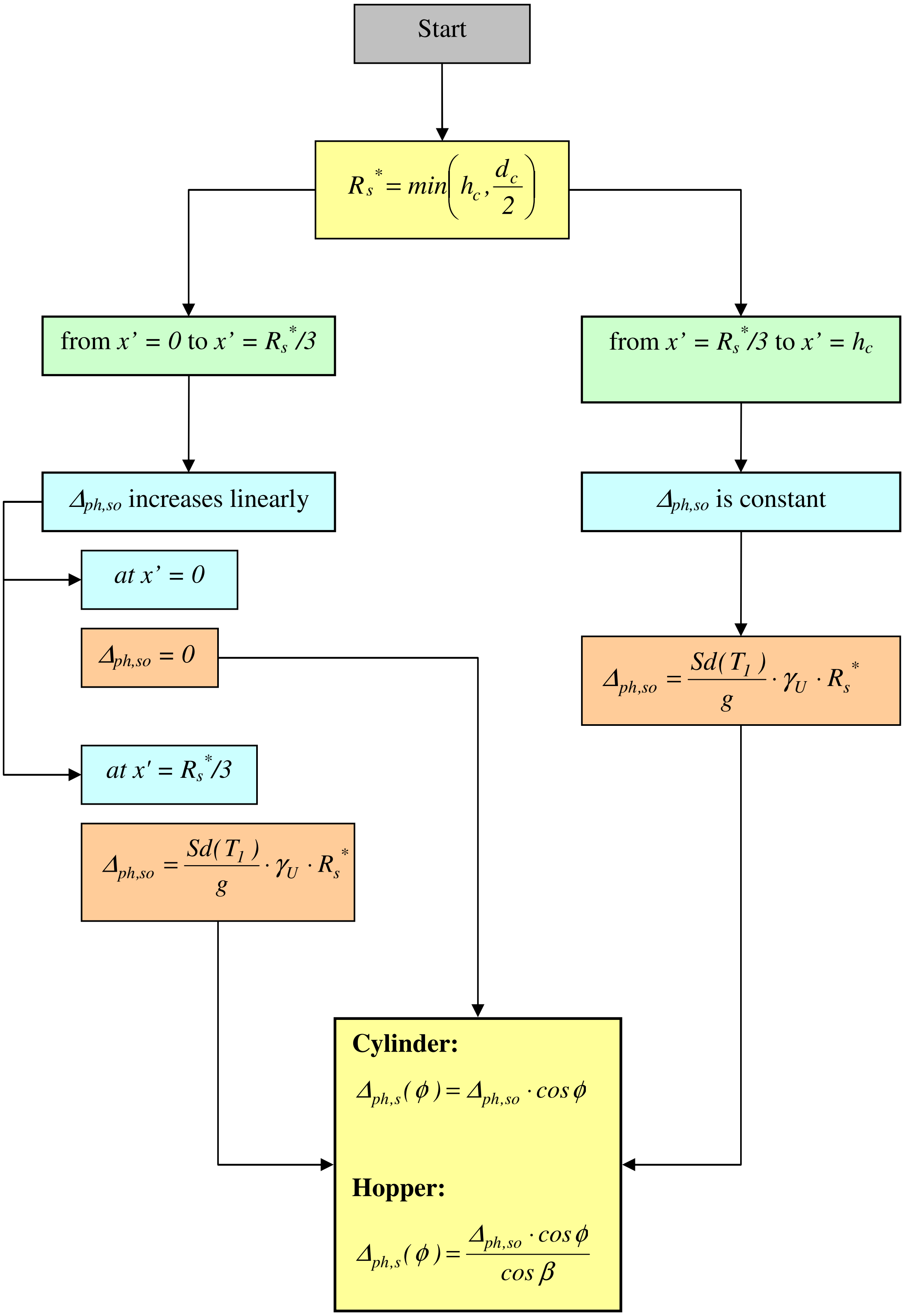
Although *EN 1998-4* implies only the distribution of pressure $\Delta_{ph,s}$ given in *Figure 36*, where the pressure first increases linearly and then has a constant value, a constant value of $\Delta_{ph,s}$ can be taken throughout the entire height of the silo cylinder and hopper, if the height $R_s^* / 3$ is much smaller than $h_c$. The results will be on the safe side, because bigger pressures will in turn produce bigger internal forces in the silo cylinder and hopper walls. It is up to the silo designer, to make the decision weather which pressure distribution to take. As a reference, the following criterion may be used:

$R_s^* / 3 < 0.1 \cdot h_c$ → constant distribution of $\Delta_{ph,s}$ throughout the entire height of the silo cylinder and hopper, which equals:

$$\Delta_{ph,so} = \frac{Sd(T_1)}{g} \cdot \gamma_U \cdot R_s^* \qquad \text{(A.236)}$$

$R_s^* / 3 \geq 0.1 \cdot h_c$ → distribution of pressure $\Delta_{ph,s}$ given in *Chart 4*.

## Chart 4


Start
$R_s^* = min\left(h_c, \frac{d_c}{2}\right)$
from $x' = 0$ to $x' = R_s^*/3$
from $x' = R_s^*/3$ to $x' = h_c$
$\Delta_{ph,so}$ increases linearly
$\Delta_{ph,so}$ is constant
at $x' = 0$
$\Delta_{ph,so} = 0$
$\Delta_{ph,so} = \frac{Sd(T_1)}{g} \cdot \gamma_U \cdot R_s^*$
at $x' = R_s^*/3$
$\Delta_{ph,so} = \frac{Sd(T_1)}{g} \cdot \gamma_U \cdot R_s^*$
Cylinder:
$\Delta_{ph,s}(\phi) = \Delta_{ph,so} \cdot cos\,\phi$
Hopper:
$\Delta_{ph,s}(\phi) = \frac{\Delta_{ph,so} \cdot cos\,\phi}{cos\,\beta}$

## *5.3 Membrane forces*

### 5.3.1 Full silo

The given membrane forces are caused by the additional normal pressure $\Delta_{ph,s}$.

#### 5.3.1.1 Cylinder

The vertical force $n_{z,efs}$, circumferential force $n_{\phi,\ efs}$ and shear force $n_{z\phi,\ efs}$ in the full silo cylinder equal:

$$\boxed{n_{z.efs}(\phi) = \frac{\Delta_{ph,s}(\phi)}{d_c} \cdot z^2} \tag{A.237}$$

$$\boxed{n_{\phi.efs} = -\Delta_{ph,s}(\phi) \cdot r} \tag{A.238}$$

$$\boxed{n_{z\phi,efs}(z) = -\Delta_{ph,so} \cdot \sin\phi \cdot z} \tag{A.239}$$

#### 5.3.1.2 Hopper

The force $n_{s,efs}$, circumferential force $n_{\phi,\ efs}$ and shear force $n_{s\phi,\ efs}$ in the hopper of a full silo equal:

$$\boxed{n_{s,efs}(\phi) = \frac{\Delta_{ph,so}}{6} \cdot \left[ \frac{1}{\sin\beta} - 3\sin\beta \right] \cdot \cos\phi \cdot x} \tag{A.240}$$

$$\boxed{n_{\phi,efs}(\phi) = -\frac{\Delta_{ph,so}}{\cos\beta} \cdot \tan\beta \cdot \cos\phi \cdot x} \tag{A.241}$$

$$\boxed{n_{s\phi}(\phi) = -\frac{\Delta_{ph,so}}{3} \cdot \sin\phi \cdot x} \tag{A.242}$$

### 5.3.2 Empty silo

It was already mentioned in *Section 5.1*, that in the case of an empty silo, the modal response spectrum analysis of the entire silo structure has to be conducted. The internal forces have to be determined regarding the sum of the effective modal masses for the modes that amount to at least 90% of the total mass of the structure. The use of a proper computer program is recommended.

# 6 Other loads

## 6.1 Silo temperature loading

If there is a considerable difference between the temperature of the stored solid and the silo structure, then the consequences of thermal effects should be considered in the silo design process. The thermal conditions should be assessed according to *EN 1991-1-5*.

## 6.2 Loading due to dust explosions

Stored solids like fertilizer, pea flour, animal feed, rubber, grain, wood dust, coal lignite, synthetic materials, ground corn, maize starch, malt, rye flour, wheat flour, milk powder, soya flour, cleaning products and sugar can produce dust that can be explosive. Dust explosions are possible, when dust particles are fine enough, distributed homogeneously in the air, and can react with oxygen to produce a continuous exothermic reaction. If no venting is present, pressures from *0.8 MPa* to *1 MPa* can arise in a dust explosion.

*EN 1991-4* gives little information on how to design silos to effectively withstand a dust explosion. The design should follow the procedures defined in *EN 26184-1*.

# 7 Loading combinations and verifications for silo design

The following design combinations apply to loads described in section 2. The symbols listed in the following table will be used to represent the characteristic values of internal forces from a specified load type:

**Table 26: Symbols representing load types**

| Symbol | Load type |
|---|---|
| $G$ | self weight |
| $Q_f$ | solids filling load |
| $Q_d$ | solid discharge load |
| $Q_s$ | snow load |
| $Q_w$ | wind load |
| $Q_t$ | thermal load |
| $Q_{ef,ULS}$ | seismic load on the full silo for the ultimate limit state |
| $Q_{ef,DLS}$ | seismic load on the full silo for the damage limitation state |
| $Q_{ee,ULS}$ | seismic load on the empty silo for the ultimate limit state |
| $Q_{ee,DLS}$ | seismic load on the empty silo for the damage limitation state |

## *7.1 Ultimate limit state (ULS)*

### 7.1.1 Design combinations

**Table 27: Design load combinations for ULS**

| Combination | Description | Formula |
|---|---|---|
| CO1 | Solids discharge | $1.35 \cdot G + 1.5 \cdot Q_d + 0.9 \cdot (Q_s + Q_w)$ |
| CO2 | Snow | $1.35 \cdot G + 1.5 \cdot Q_f + 0.9 \cdot Q_s$ |
| CO3 | Wind – full silo | $1.35 \cdot G + 1.5 \cdot Q_f + 0.9 \cdot Q_w$ |
| CO4 | Wind – empty silo | $1.35 \cdot G + 0.9 \cdot Q_{w,fs}$ |
| CO5 | Thermal | $1.35 \cdot G + 1.0 \cdot Q_f + 0.6 \cdot Q_t$ |
| CO6 | Earthquake – full silo | $1.0 \cdot G + 0.8 \cdot Q_f + Q_{ef,ULS}$ |
| CO7 | Earthquake – empty silo | $1.0 \cdot G + Q_{ee,ULS}$ |

### 7.1.2 Design checks

The design checks should be carried out in accordance with *prEN-1993-1-6*. The following limit states need to be considered:

- **Plastic limit (LS1)**
  The plastic limit accounts for the condition in which the capacity of the structure to resist the actions on it is exhausted by yielding of the material. The resistance offered by the structure at the plastic limit state may be derived as the plastic collapse load obtained from a mechanism based on small displacement theory.

- **Cyclic plasticity (LS2)**
  Here repeated cycles of loading and unloading produce yielding in tension and in compression at the same point, thus causing plastic work to be repeatedly done on the structure, eventually leading to local cracking by exhaustion of the energy absorption capacity of the material.

- **Buckling (LS3)**
  The limit state of buckling accounts for loss of stability under compressive membrane or shear membrane stresses in the silo shell. Large displacements normal to the shell surface develop, which lead to inability to sustain any increase in the stress resultants.

- **Fatigue (LS4)**
  The limit state of fatigue accounts for a condition in which repeated cycles of increasing and decreasing stress lead to the development of fatigue cracks in the silo shell. This check should be preformed, if the variable actions applied on the structure will have a reoccurrence of more than 10 000 cycles in the lifetime of the silo structure.

## *7.2 Serviceability limit state (SLS)*

In silo structures the Damage limitation state (DLS) should be considered as the Serviceability Limit State (SLS).

### 7.2.1 Design combinations

**Table 28: Design load combinations for SLS**

| Combination | Description | Formula |
|---|---|---|
| CO8 | Wind – full silo | $1.0 \cdot G + 0.9 \cdot Q_f + 0.6 \cdot Q_w$ |
| CO9 | Earthquake – full silo | $1.0 \cdot G + 0.8 \cdot Q_f + Q_{ef,DLS}$ |

### 7.2.2 Check of displacements

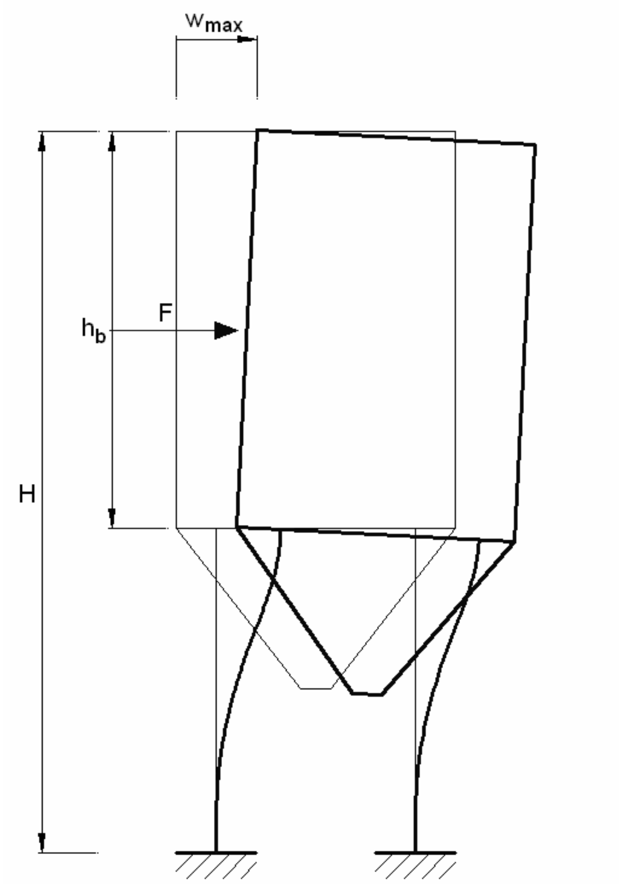


**Figure 37: Maximum horizontal displacements of a full silo**

The limiting value for global horizontal displacements equals:

$$w_{max} = 0.02 \cdot H \,, \tag{A.243}$$

where *H* is the total height of the silo structure measured from the foundation to the roof of the silo.

Horizontal displacement of the top part of the silo caused by the design value of the wind force or the earthquake force on the full silo needs to be considered. The design value of the two forces is denoted with *F* in the figure above.

### 7.2.2.1 Maximum displacement due to wind load

The design value of the wind force $W_d$ has to be calculated according to combination *CO8*, given in table *Table 28*.

Considering the values for the horizontal stiffness $k_v$ and rotational stiffness $k_m$ of the supporting structure, we can now calculate the displacement at the top of a full silo because of the wind load $w_W$:

$$w_W = W_d \left( \frac{1}{k_v} + \frac{{h_b}^2}{2\,k_m} \right) \tag{A.244}$$

The displacement of the top of a full silo due to wind load has to be smaller than the maximum displacement:

$$\boxed{w_W \leq w_{max}} \tag{A.245}$$

### 7.2.2.2 Maximum displacement due to earthquake load

The design value of the earthquake force $F_{ed}$ has to be calculated according to combination *CO9*, given in table *Table 28*.

Considering the values for the horizontal stiffness $k_v$ and rotational stiffness $k_m$ of the supporting structure and the behavior factor $q$, we can calculate the displacement at the top of a full silo because of the earthquake load $w_{Fe}$:

$$w_{Fe} = F_{ed} \cdot q \cdot \left( \frac{1}{k_v} + \frac{{h_b}^2}{2\,k_m} \right) \tag{A.246}$$

The displacement of the top of a full silo due to the earthquake load has to be smaller than the maximum displacement:

$$\boxed{w_{Fe} \leq w_{max}} \tag{A.247}$$

# Priloga B: Angleški prevod III. dela

# *Appendix B: English translation of Part III*

## Table of contents

PART III – Computer programs

# PART III – Computer programs

## 1 Stored Solid Load on Cylindrical Silos

The program determines the axisymetric pressure distributions and section forces in the silo cylinder and hopper wall due to the stored particulate solid material, by using the procedure described in *Appendix A, section 3.1.1.* Section forces are calculated by equations given in section *Appendix A, section 3.3.1.* The assumptions given in *Appendix A, section 1.3* apply for the program.

### *Running the program*

To start the program, open the *Stored Solid Load on Cylindrical Silos.xls* file located on the CD accompanying this work. The CD is located on the inside of the back cover.

*Microsoft Excel 2003* (or newer) needs to be installed on the computer running the program.

In order for the program to run properly, macro security has to be set to medium or low. You can do this by opening *Excel* and selecting menus *Tools/Macro/Security*. In the new window set the security to *Medium* or *Low* and then re-open the program.

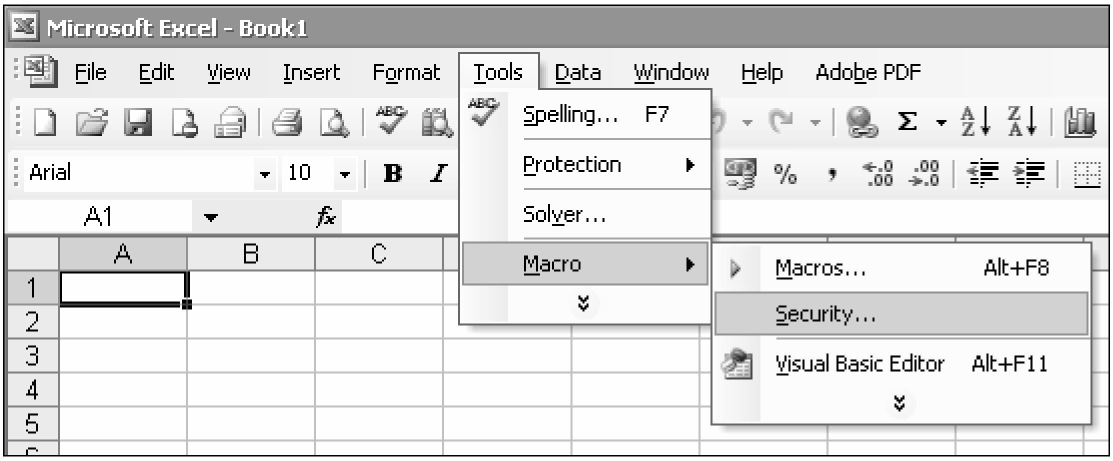


Figure 38: Accessing macro security settings in Microsoft Excel 2003

○ Very High. Only macros installed in trusted locations will be allowed to run. All other signed and unsigned macros are disabled.

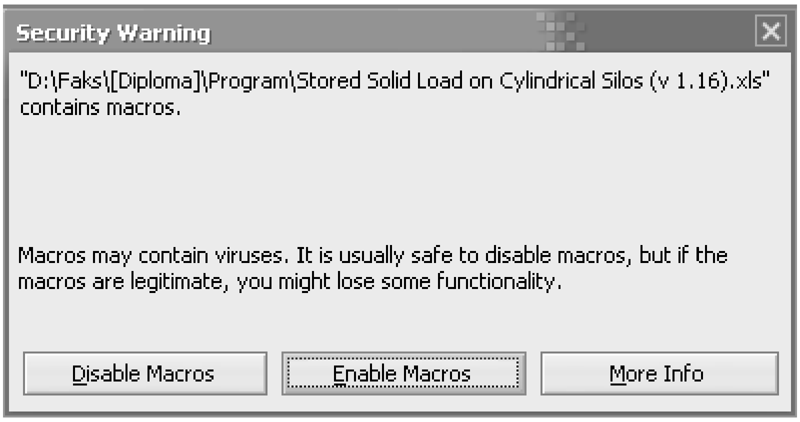
○ High. Only signed macros from trusted sources will be allowed to run. Unsigned macros are automatically disabled.

● Medium. You can choose whether or not to run potentially unsafe macros.

○ Low (not recommended). You are not protected from potentially unsafe macros. Use this setting only if you have virus scanning software installed, or you have checked the safety of all documents you open.

Figure 39: Macro security settings window

If the macro security is set to *Medium*, a security pop-up window will appear after re-opening the program. Click on *Enable macros* button.



"D:\Faks\[Diploma]\Program\Stored Solid Load on Cylindrical Silos (v 1.16).xls" contains macros.

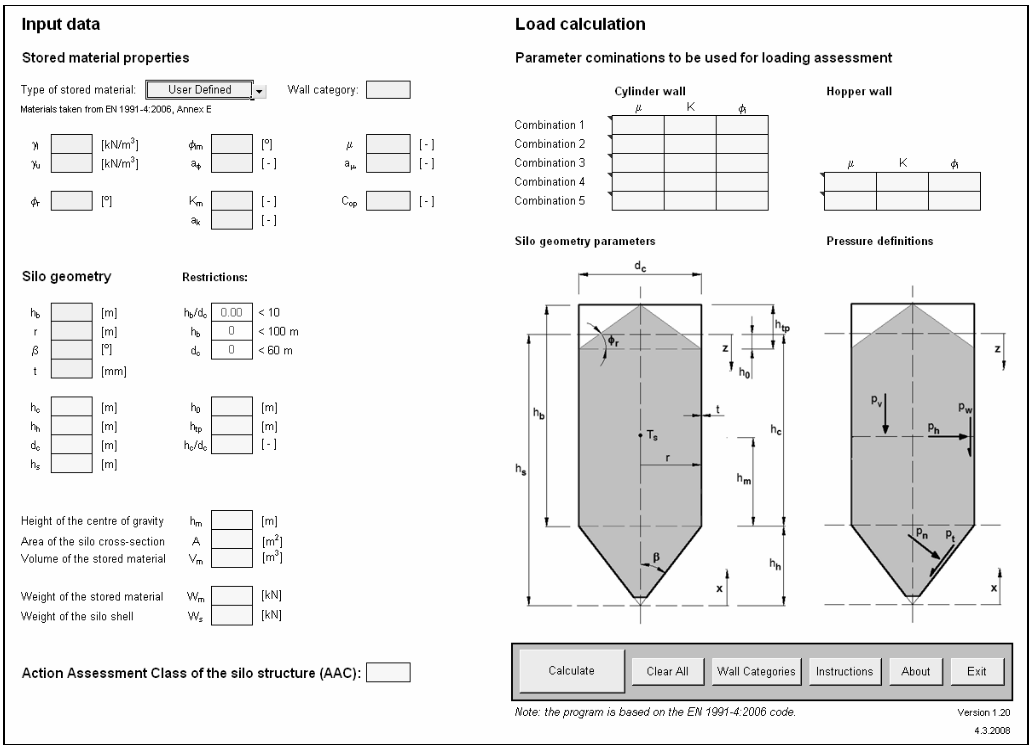
Macros may contain viruses. It is usually safe to disable macros, but if the macros are legitimate, you might lose some functionality.

**Figure 40: Enabling macros**

# *Using the program*

## Data input

When the program is opened, the input sheet appears. Use this sheet to input the data needed for the calculation.



Type of stored material: User Defined

Materials taken from EN 1991-4:2006, Annex E

Restrictions:

$h_b/d_c$ 0.00 < 10

$h_b$ 0 < 100 m

$d_c$ 0 < 60 m

Note: the program is based on the EN 1991-4:2006 code.

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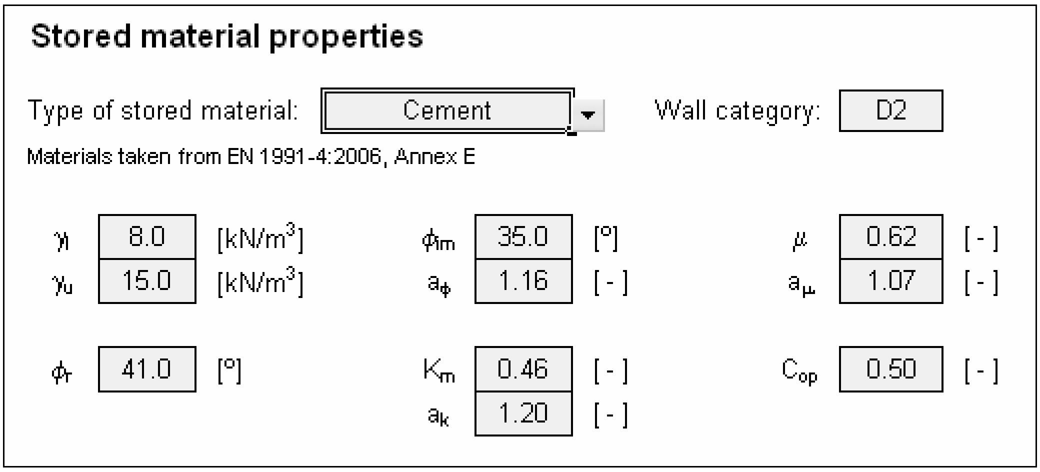
**Figure 41: Input sheet**

Appropriate values need to be entered in the *green* cells only. Values in other cells will be calculated automatically.

## Stored material properties



**Stored material properties**

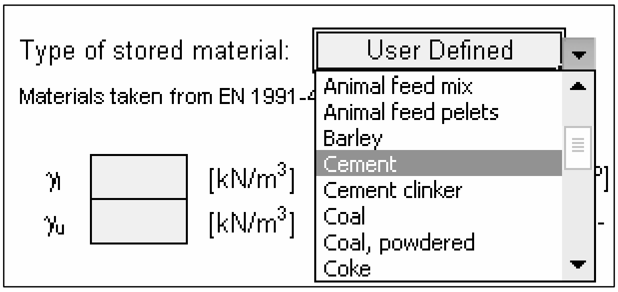
Type of stored material: Cement
Wall category: D2
Materials taken from EN 1991-4:2006, Annex E

$\gamma_l$ 8.0 [kN/m$^3$]
$\gamma_u$ 15.0 [kN/m$^3$]
$\phi_r$ 41.0 [°]
$\phi_{im}$ 35.0 [°]
$a_\phi$ 1.16 [ - ]
$K_m$ 0.46 [ - ]
$a_K$ 1.20 [ - ]
$\mu$ 0.62 [ - ]
$a_\mu$ 1.07 [ - ]
$C_{op}$ 0.50 [ - ]

**Figure 42: Stored material properties input section of the program**

First the values for the stored material properties must be given. The program includes a list of predefined materials, taken from *EN 1991-4:2006, Annex E, Table E.1*.

The predefined materials are given in a drop-down list. When a material is selected, the values for the required material parameters will be entered automatically.



Type of stored material: User Defined
Materials taken from EN 1991-4

$\gamma_l$ [kN/m$^3$]
$\gamma_u$ [kN/m$^3$]

**Figure 43: Drop-down list of predefined materials**

For each material the proper wall category has to be selected. Use the drop-down list and select a wall category. The names of the wall categories correspond to the names given in

*Table 8*. You can also view the table defining the wall categories from the program, by clicking on the *Wall categories* button in the program control box.

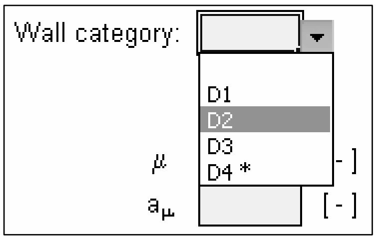
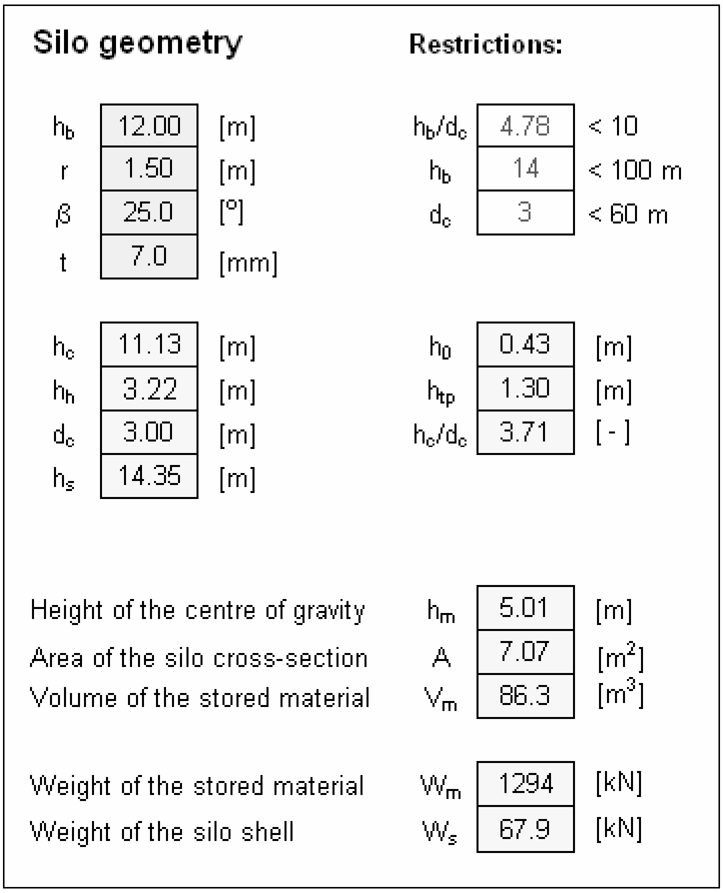


**Figure 44: Drop-down list of wall categories**

If you wish to use a custom material, choose *User defined* material from the drop-down list of predefined materials and enter the required material parameters manually. You do not need to specify a wall category in this case.

**Silo geometry**



Silo geometry

$h_b$ 12.00 [m]
r 1.50 [m]
$\beta$ 25.0 [°]
t 7.0 [mm]

Restrictions:

$h_b/d_c$ 4.78 < 10
$h_b$ 14 < 100 m
$d_c$ 3 < 60 m

$h_c$ 11.13 [m]
$h_h$ 3.22 [m]
$d_c$ 3.00 [m]
$h_s$ 14.35 [m]

$h_0$ 0.43 [m]
$h_{tp}$ 1.30 [m]
$h_c/d_c$ 3.71 [ - ]

Height of the centre of gravity $h_m$ 5.01 [m]
Area of the silo cross-section A 7.07 [$m^2$]
Volume of the stored material $V_m$ 86.3 [$m^3$]

Weight of the stored material $W_m$ 1294 [kN]
Weight of the silo shell $W_s$ 67.9 [kN]

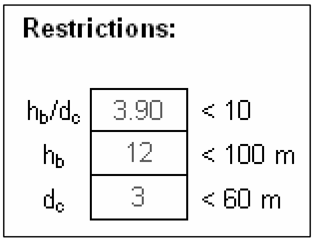
**Figure 45: Silo geometry input section of the program**

The only geometry parameters that need to be entered are:

- $h_b$, which represents the total height of the silo cylinder,
- $r$ , which represents the radius of the silo cylinder
- $\beta$, which represents the hopper apex half-angle and
- $t$ , which represents the thickness of the silo shell. If the silo is composed out of sections of varying wall thickness, the mean value of the wall thickness can be entered.

All other geometry parameters will be calculated automatically, using equations defined in *section* ***3.1.1*** .

The program also tests if the silo geometry meets the requirements defined in *EN 1991-4:2006, 1.1.2(3)*. These requirements are given in the form of three restrictions.



**Restrictions:**

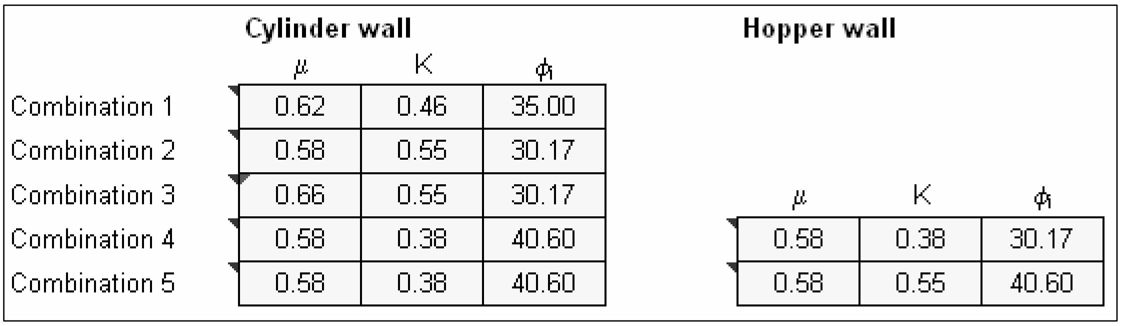
| | | |
|---|---|---|
| $h_b/d_c$ | 3.90 | < 10 |
| $h_b$ | 12 | < 100 m |
| $d_c$ | 3 | < 60 m |

**Figure 46: Silo geometry restrictions**

If a given parameter meets a certain restriction criteria, its value will be displayed in green, otherwise it will be displayed in red. In this case the silo geometry must be altered, until all three restriction criteria are met.

## Load calculation

The program first determines the proper minimum, maximum or mean values of parameters $\mu$, $K$ and $\phi_L$., regarding the parameter combinations defined in *Table 11* and *Table 12*. These values will then be used in the calculation.



| | Cylinder wall | | | Hopper wall | | |
|---|---|---|---|---|---|---|
| | $\mu$ | K | $\phi_i$ | $\mu$ | K | $\phi_i$ |
| Combination 1 | 0.62 | 0.46 | 35.00 | | | |
| Combination 2 | 0.58 | 0.55 | 30.17 | | | |
| Combination 3 | 0.66 | 0.55 | 30.17 | | | |
| Combination 4 | 0.58 | 0.38 | 40.60 | 0.58 | 0.38 | 30.17 |
| Combination 5 | 0.58 | 0.38 | 40.60 | 0.58 | 0.55 | 40.60 |

**Figure 47: Parameter combinations to be used in the calculation**

The calculation is initiated by pressing the *Calculate* button in the program control box.

## Program controls

The program is operated by using the buttons from the program control box, located on the lower-right corner of the input sheet.

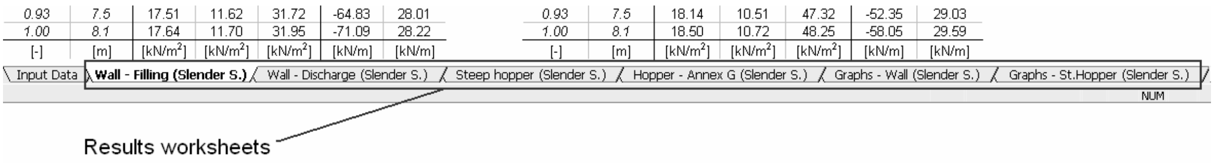
**Figure 48: Program control box**

Their functions are:

- **Calculate** – use this button to initiate the calculation,
- **Clear All** – use this button to clear all of the results and input data,
- **Wall Categories** – here you can view the definitions of the wall categories,
- **Instructions** – here you can find instructions on how to use the program,
- **About** – here you can view some information about the author of the program,
- **Exit** – use this button to exit the program.

## Viewing the results

The results are displayed on separate worksheets, which appear after the calculation has been initiated. The results appear separately for the cylinder and hopper wall.



| | | | | | | |
|---|---|---|---|---|---|---|
| 0.93 | 7.5 | 17.51 | 11.62 | 31.72 | -64.83 | 28.01 |
| 1.00 | 8.1 | 17.64 | 11.70 | 31.95 | -71.09 | 28.22 |
| [-] | [m] | [kN/m²] | [kN/m²] | [kN/m²] | [kN/m] | [kN/m] |

| | | | | | | |
|---|---|---|---|---|---|---|
| 0.93 | 7.5 | 18.14 | 10.51 | 47.32 | -52.35 | 29.03 |
| 1.00 | 8.1 | 18.50 | 10.72 | 48.25 | -58.05 | 29.59 |
| [-] | [m] | [kN/m²] | [kN/m²] | [kN/m²] | [kN/m] | [kN/m] |

Figure 49: Results worksheets

The results for the *slender silo*, *intermediate slenderness silo* and *squat silo* are given in the following result worksheets:

- **Wall – Filling:** includes pressures on the silo cylinder wall because of the filling of the stored solid material,
- **Wall – Discharge:** includes pressures on the silo cylinder wall because of the discharge of the stored solid material,
- **Steep/Shallow Hopper:** includes pressures on the silo hopper wall because of the filling and discharge of the particulate solid material,
- **Hopper – Annex G:** includes pressures on the silo hopper wall because of the filling and discharge of the particulate solid material according to *Annex G* of *EN 1991-4*,
- **Graphs – Wall:** includes graphical representations of pressures on the silo cylinder wall because of the filling and discharge of the stored solid material,
- **Graphs – Steep/Shallow Hopper:** includes graphical representations of pressures on the hopper wall because of the filling and discharge of the stored solid material. Pressures calculated under the *Annex G* are also included.

The results for *retaining silos* have only two results worksheets:

- **Wall:** includes pressures on the silo cylinder wall because of the filling of the stored solid material,
- **Graphs – Wall:** includes graphical representations of pressures on the silo cylinder wall because of the filling of the stored solid material.

All the results for pressures and section forces are given in table form. The height of the considered silo cylinder is divided into 16 parts (heights) and the height of the hopper is divided into 11 parts (heights). The results are given for each height, where as the results for the heights in-between two parts can be calculated by linearly interpolating the results from the neighboring parts.

## *License information*

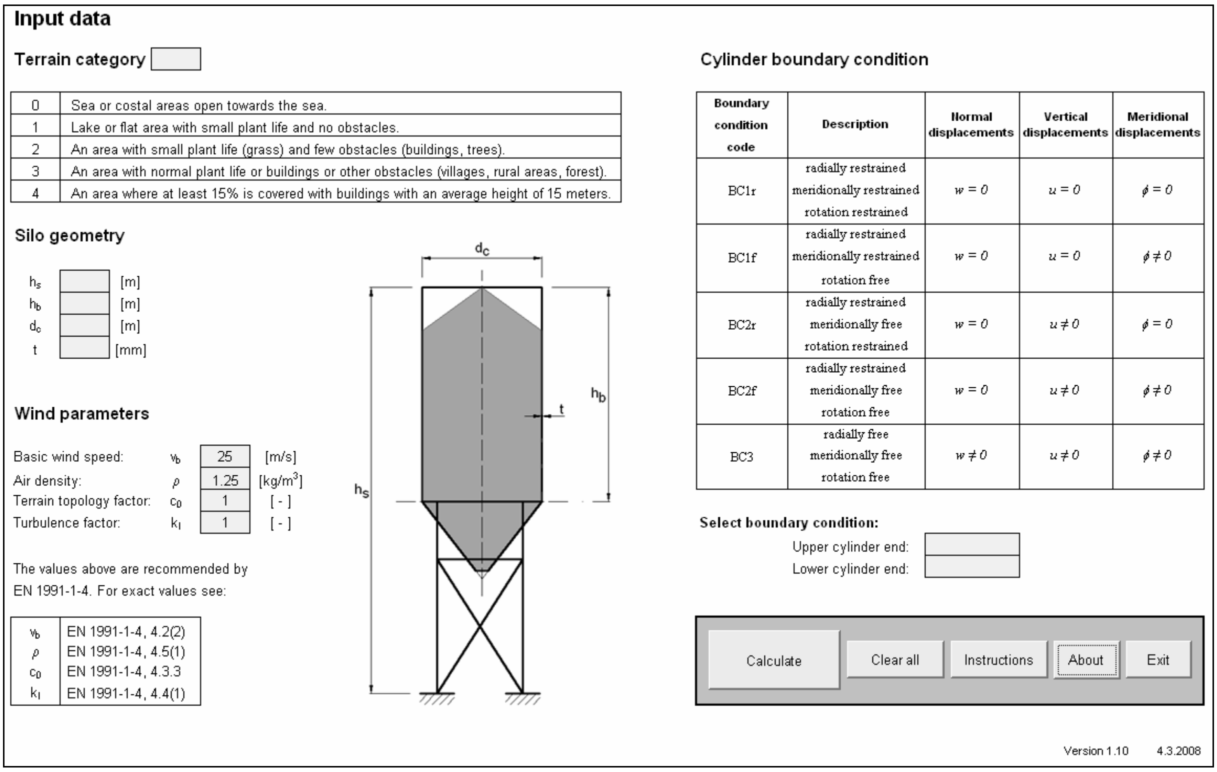
# 2 Wind Load on Cylindrical Silos

The program calculates the external wind pressure and total wind force on the silo structure using the guidelines described in *Appendix A, section 4.2*. The program calculates the actual external wind pressure distribution along with the equivalent axisymetric distribution and the total wind force. All three curves given in the figure in *Appendix A, section 4.2.1* are built in the program.

## *Running the program*

The file *Wind Load on Cylindrical Silos.xls* needs to be open. See the *Running the program* section under *Appendix B, section 1.*

## *Using the program*



**Input data**

Terrain category

| | |
|---|---|
| 0 | Sea or costal areas open towards the sea. |
| 1 | Lake or flat area with small plant life and no obstacles. |
| 2 | An area with small plant life (grass) and few obstacles (buildings, trees). |
| 3 | An area with normal plant life or buildings or other obstacles (villages, rural areas, forest). |
| 4 | An area where at least 15% is covered with buildings with an average height of 15 meters. |

Silo geometry

$h_s$ [m]
$h_b$ [m]
$d_c$ [m]
t [mm]

Wind parameters

| | | | |
|---|---|---|---|
| Basic wind speed: | $v_b$ | 25 | [m/s] |
| Air density: | $\rho$ | 1.25 | [kg/m$^3$] |
| Terrain topology factor: | $c_0$ | 1 | [ - ] |
| Turbulence factor: | $k_I$ | 1 | [ - ] |

The values above are recommended by EN 1991-1-4. For exact values see:

| | |
|---|---|
| $v_b$ | EN 1991-1-4, 4.2(2) |
| $\rho$ | EN 1991-1-4, 4.5(1) |
| $c_0$ | EN 1991-1-4, 4.3.3 |
| $k_I$ | EN 1991-1-4, 4.4(1) |

Cylinder boundary condition

| Boundary condition code | Description | Normal displacements | Vertical displacements | Meridional displacements |
|---|---|---|---|---|
| BC1r | radially restrained<br>meridionally restrained<br>rotation restrained | $w = 0$ | $u = 0$ | $\phi = 0$ |
| BC1f | radially restrained<br>meridionally restrained<br>rotation free | $w = 0$ | $u = 0$ | $\phi \neq 0$ |
| BC2r | radially restrained<br>meridionally free<br>rotation restrained | $w = 0$ | $u \neq 0$ | $\phi = 0$ |
| BC2f | radially restrained<br>meridionally free<br>rotation free | $w = 0$ | $u \neq 0$ | $\phi \neq 0$ |
| BC3 | radially free<br>meridionally free<br>rotation free | $w \neq 0$ | $u \neq 0$ | $\phi \neq 0$ |

**Select boundary condition:**

Upper cylinder end:
Lower cylinder end:

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Figure 50: Input sheet

## Data input

When the program is opened, the input sheet appears. Use this sheet to input the data needed for the calculation. Appropriate values need to be entered in the *green* cells.

### Silo geometry

First the geometry parameters need to be entered. They include:

- $h_s$, which represents the total height of the silo structure,
- $h_b$, which represents the height of the silo cylinder,
- $d_c$ , which represents the diameter of the silo cylinder,
- $t$ , which represents the thickness of the silo shell. If the silo is composed out of sections of varying wall thickness, the mean value of the wall thickness can be entered.

### Wind parameters

The wind parameters that need to be entered are:

- $v_b$, which represents the basic wind speed,
- $\rho$ , which represents the density of air,
- $c_0$ , which represents the terrain topology factor,
- $k_I$ , which represents the turbulence factor.

The program gives recommended values, which are taken from *EN 1991-1-4.* If you want to use the exact values, refer to the proper sections of *EN 1991-1-4* for their calculation.

### Cylinder boundary conditions

From the two drop-down lists select the proper boundary conditions for the upper and lower cylinder end.

## Program controls

The program is operated by using the buttons from the program control box, located on the lower-right corner of the input sheet.

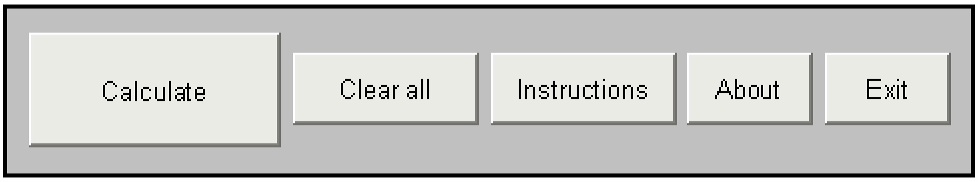


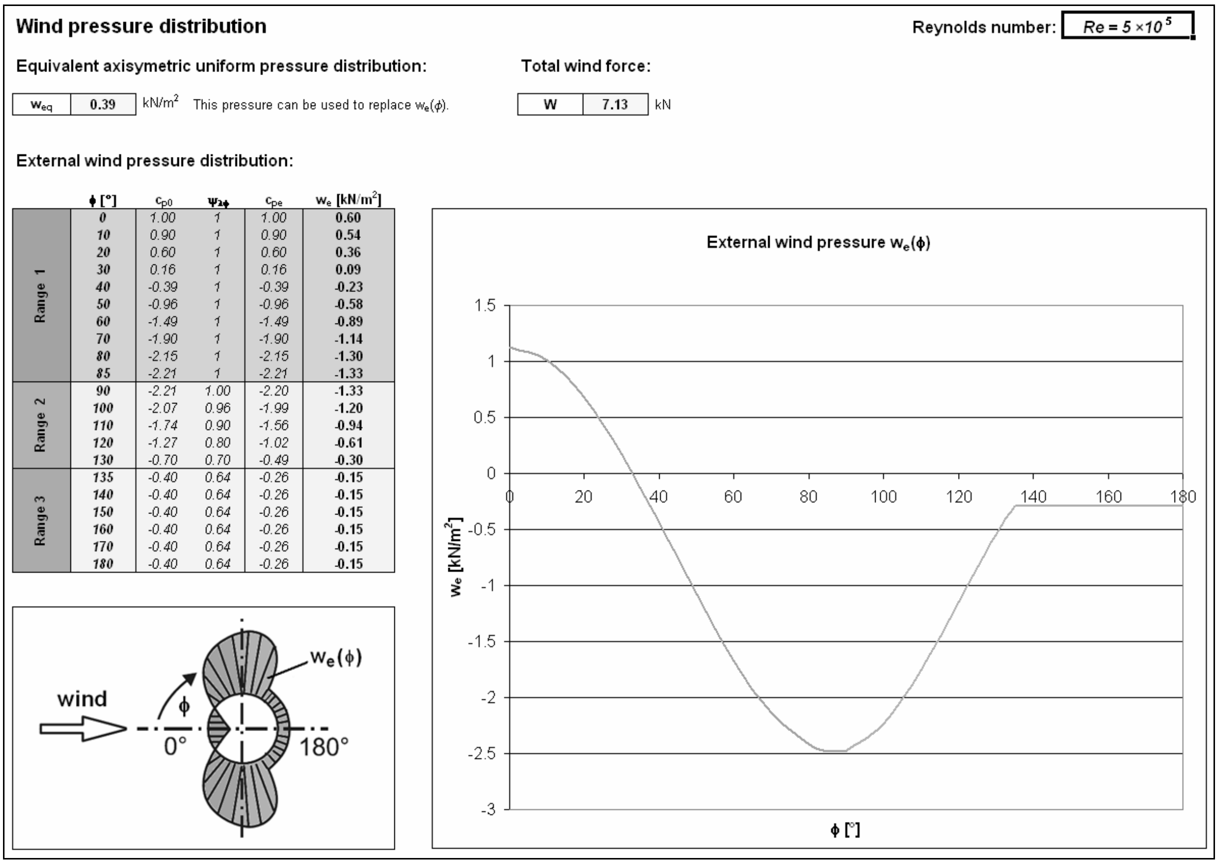
**Figure 51: Program control box**

Their functions are:

- **Calculate** – use this button to initiate the calculation,
- **Clear All** – use this button to clear all of the results and input data,
- **Instructions** – here you can find instructions on how to use the program,
- **About** – here you can view some information about the author of the program,
- **Exit** – use this button to exit the program.

## Viewing the results

The results are displayed on a separate worksheet, which appears after the calculation has been initiated.



**Wind pressure distribution**

Reynolds number: $Re = 5 \times 10^5$

Equivalent axisymetric uniform pressure distribution:

$w_{eq}$ 0.39 kN/m$^2$ This pressure can be used to replace $w_e(\phi)$.

Total wind force:

W 7.13 kN

External wind pressure distribution:

|  | $\phi$ [°] | $c_{p0}$ | $\psi_{\lambda\phi}$ | $c_{pe}$ | $w_e$ [kN/m$^2$] |
|---|---|---|---|---|---|
| Range 1 | 0 | 1.00 | 1 | 1.00 | 0.60 |
|  | 10 | 0.90 | 1 | 0.90 | 0.54 |
|  | 20 | 0.60 | 1 | 0.60 | 0.36 |
|  | 30 | 0.16 | 1 | 0.16 | 0.09 |
|  | 40 | -0.39 | 1 | -0.39 | -0.23 |
|  | 50 | -0.96 | 1 | -0.96 | -0.58 |
|  | 60 | -1.49 | 1 | -1.49 | -0.89 |
|  | 70 | -1.90 | 1 | -1.90 | -1.14 |
|  | 80 | -2.15 | 1 | -2.15 | -1.30 |
|  | 85 | -2.21 | 1 | -2.21 | -1.33 |
| Range 2 | 90 | -2.21 | 1.00 | -2.20 | -1.33 |
|  | 100 | -2.07 | 0.96 | -1.99 | -1.20 |
|  | 110 | -1.74 | 0.90 | -1.56 | -0.94 |
|  | 120 | -1.27 | 0.80 | -1.02 | -0.61 |
|  | 130 | -0.70 | 0.70 | -0.49 | -0.30 |
| Range 3 | 135 | -0.40 | 0.64 | -0.26 | -0.15 |
|  | 140 | -0.40 | 0.64 | -0.26 | -0.15 |
|  | 150 | -0.40 | 0.64 | -0.26 | -0.15 |
|  | 160 | -0.40 | 0.64 | -0.26 | -0.15 |
|  | 170 | -0.40 | 0.64 | -0.26 | -0.15 |
|  | 180 | -0.40 | 0.64 | -0.26 | -0.15 |

**Figure 52: Results sheet**

The results for the external wind pressure $w_e(\alpha)$ are given in table form. Because the wind distribution is symmetrical to the wind direction, the external wind pressure is calculated from $\alpha = 0°$ to $\alpha = 180°$.

## *License information*

# Priloga C: Preglednica z lastnostmi shranjenih materialov

# *Appendix C: Table of stored material properties*

Spodnja preglednica podaja fizikalne lastnosti tipičnih materialov, ki se shranjujejo v silosih.

*The table below specifies the physical properties for the most common materials stored in silo structures.*

| Material name | $\gamma_l$ | $\gamma_u$ | $\phi_r$ | $\phi_{im}$ | $a_\phi$ | $K_m$ | $a_K$ | $\mu_m$ (D1) | $\mu_m$ (D2) | $\mu_m$ (D3) | $a_\mu$ | $C_{op}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| ***Default material*** | 6.00 | 22.00 | 40.00 | 35.00 | 1.30 | 0.50 | 1.50 | 0.32 | 0.39 | 0.50 | 1.40 | 1.00 |
| **Aggregate** | 17.00 | 18.00 | 36.00 | 31.00 | 1.16 | 0.52 | 1.15 | 0.39 | 0.49 | 0.59 | 1.12 | 0.40 |
| **Alumina** | 10.00 | 12.00 | 36.00 | 30.00 | 1.22 | 0.54 | 1.20 | 0.41 | 0.46 | 0.51 | 1.07 | 0.50 |
| **Animal feed mix** | 5.00 | 6.00 | 39.00 | 36.00 | 1.08 | 0.45 | 1.10 | 0.22 | 0.30 | 0.43 | 1.28 | 1.00 |
| **Animal feed pellets** | 6.50 | 8.00 | 37.00 | 35.00 | 1.06 | 0.47 | 1.07 | 0.23 | 0.28 | 0.37 | 1.20 | 0.70 |
| **Barley** | 7.00 | 8.00 | 31.00 | 28.00 | 1.14 | 0.59 | 1.11 | 0.24 | 0.33 | 0.48 | 1.16 | 0.50 |
| **Cement** | 13.00 | 16.00 | 36.00 | 30.00 | 1.22 | 0.54 | 1.20 | 0.41 | 0.46 | 0.51 | 1.07 | 0.50 |
| **Cement clinker** | 15.00 | 18.00 | 47.00 | 40.00 | 1.20 | 0.38 | 1.31 | 0.46 | 0.56 | 0.62 | 1.07 | 0.70 |
| **Coal** | 7.00 | 10.00 | 36.00 | 31.00 | 1.16 | 0.52 | 1.15 | 0.44 | 0.49 | 0.59 | 1.12 | 0.60 |
| **Coal, powdered** | 6.00 | 8.00 | 34.00 | 27.00 | 1.26 | 0.58 | 1.20 | 0.41 | 0.51 | 0.56 | 1.07 | 0.50 |
| **Coke** | 6.50 | 8.00 | 36.00 | 31.00 | 1.16 | 0.52 | 1.15 | 0.49 | 0.54 | 0.59 | 1.12 | 0.60 |
| **Fly ash** | 8.00 | 15.00 | 41.00 | 35.00 | 1.16 | 0.46 | 1.20 | 0.51 | 0.62 | 0.72 | 1.07 | 0.50 |
| **Flour** | 6.50 | 7.00 | 45.00 | 42.00 | 1.06 | 0.36 | 1.11 | 0.24 | 0.33 | 0.48 | 1.16 | 0.60 |
| **Iron ore pellets** | 19.00 | 22.00 | 36.00 | 31.00 | 1.16 | 0.52 | 1.15 | 0.49 | 0.54 | 0.59 | 1.12 | 0.50 |
| **Lime, hydrated** | 6.00 | 8.00 | 34.00 | 27.00 | 1.26 | 0.58 | 1.20 | 0.36 | 0.41 | 0.51 | 1.07 | 0.60 |
| **Limestone powder** | 11.00 | 13.00 | 36.00 | 30.00 | 1.22 | 0.54 | 1.20 | 0.41 | 0.51 | 0.56 | 1.07 | 0.50 |
| **Maize** | 7.00 | 8.00 | 35.00 | 31.00 | 1.14 | 0.53 | 1.14 | 0.22 | 0.36 | 0.53 | 1.24 | 0.90 |
| **Phosphate** | 16.00 | 22.00 | 34.00 | 29.00 | 1.18 | 0.56 | 1.15 | 0.39 | 0.49 | 0.54 | 1.12 | 0.50 |
| **Potatoes** | 6.00 | 8.00 | 34.00 | 30.00 | 1.12 | 0.54 | 1.11 | 0.33 | 0.38 | 0.48 | 1.16 | 0.50 |
| **Sand** | 14.00 | 16.00 | 39.00 | 36.00 | 1.09 | 0.45 | 1.11 | 0.38 | 0.48 | 0.57 | 1.16 | 0.40 |
| **Slag clinkers** | 10.50 | 12.00 | 39.00 | 36.00 | 1.09 | 0.45 | 1.11 | 0.48 | 0.57 | 0.67 | 1.16 | 0.60 |
| **Soya beans** | 7.00 | 8.00 | 29.00 | 25.00 | 1.16 | 0.63 | 1.11 | 0.24 | 0.38 | 0.48 | 1.16 | 0.50 |
| **Sugar** | 8.00 | 9.50 | 38.00 | 32.00 | 1.19 | 0.50 | 1.20 | 0.46 | 0.51 | 0.56 | 1.07 | 0.40 |
| **Sugar beet pellets** | 6.50 | 7.00 | 36.00 | 31.00 | 1.16 | 0.52 | 1.15 | 0.35 | 0.44 | 0.54 | 1.12 | 0.50 |
| **Wheat** | 7.50 | 9.00 | 34.00 | 30.00 | 1.12 | 0.54 | 1.11 | 0.24 | 0.38 | 0.57 | 1.16 | 0.50 |

**(Povzeto po standardu EN 1991-4:2006, Preglednica E.1, stran 99)**

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**END OF TERMS AND CONDITIONS**

# How to Apply These Terms to Your New Programs

*If you develop a new program, and you want it to be of the greatest possible use to the public, the best way to achieve this is to make it free software which everyone can redistribute and change under these terms.*

*To do so, attach the following notices to the program. It is safest to attach them to the start of each source file to most effectively state the exclusion of warranty; and each file should have at least the "copyright" line and a pointer to where the full notice is found.*

```
<one line to give the program's name and a brief idea of what it does.>
Copyright (C) <year> <name of author>

This program is free software: you can redistribute it and/or modify
it under the terms of the GNU General Public License as published by the
Free Software Foundation, either version 3 of the License, or (at your
option) any later version.

This program is distributed in the hope that it will be useful,
but WITHOUT ANY WARRANTY; without even the implied warranty of
MERCHANTABILITY or FITNESS FOR A PARTICULAR PURPOSE.  See the
GNU General Public License for more details.

You should have received a copy of the GNU General Public License
along with this program.  If not, see <http://www.gnu.org/licenses/>.
```

*Also add information on how to contact you by electronic and paper mail.*

*If the program does terminal interaction, make it output a short notice like this when it starts in an interactive mode:*

```
<program>  Copyright (C) <year>  <name of author>
This program comes with ABSOLUTELY NO WARRANTY; for details type `show w'.
This is free software, and you are welcome to redistribute it
under certain conditions; type `show c' for details.
```

*The hypothetical commands `show w' and `show c' should show the appropriate parts of the General Public License. Of course, your program's commands might be different; for a GUI interface, you would use an "about box".*

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