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Mechanical analysis of glulam beams exposed to changing humidity

Abstract: The paper deals with the mechanical analysis of glulam beams during changing relative humidity of the surrounding air. The computational part of the paper includes two separate numerical procedures. First, the diffusion equation is solved in order to determine the temporal and spatial distribution of water content in the cross-section of the beam. The results of the first computational stage are used as the input data for the numerical analysis of mechanical response of the beam. The displacements and stress distribution at some characteristic cross-sections are presented. In the article some experimentally determined values of vertical displacements in the middle of span are shown and compared to the results of numerical analysis.

Introduction

A particular characteristic of wooden load bearing elements is that their deformations strongly depend upon the changes of relative humidity and temperature of the surrounding air. In constant climatic conditions the total deformation after a certain time consists mainly of the deformation parts due to mechanical load and the normal viscous creep where the intensiveness of the normal creep depends on the constant level of temperature and relative humidity of the environment. However, if the relative humidity of the surrounding air changes during the time, two more phenomena can be observed in wood as well: shrinking and swelling and, as a coupled effect of mechanical load and changing water content, the so called mechano–sorptive effect. The phenomena of mechano–sorptive effect was first observed and reported by Hearmon

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and Paton (1964) . Several other authors dealt with mechano-sorptive effects (e.g.: Bažant 1985, Grossman 1976, Hoffmeyer and Davidson 1989, Hunt and Shelton 1988, Leicester 1971, Mohager and Toratti 1993, Mukudai and Yata 1987, Olsson et al. 2007, Ranta Maunus 1995, Toratti 1992). In their papers experimental results as well as numerical models are presented. The effect on moisture content on longitudinal creep was studied by Jönsson (2005) and Kojima and Yamamoto (2005). The analysis of timber elements considering the effect of viscous creep, shrinkage amd mechano-sorptive effect was also reported by several authors (e.g.: Kang et al. 2004, Moutee et al. 2007).

When analysing the behaviour of wooden beams in changing climatic conditions we have to deal basically with a coupled physical problem which includes the nonstationary heat and water transfer over the element and the mechanical response of the beam. However, assuming that the deformation of the structure does not significantly affect the heat and water transfer in wood, the numerical procedure simplifies considerably. In this manner the coupled computational procedure splits into two separate phases. In the first phase, the spatial and temporal distribution of water content and temperature over the element has to be determined according to relative humidity and temperature of the surrounding air. Luikov (1966) has given the governing equations for simultaneous heat and moisture transfer in porous materials. However, in the present work the numerical results are compared with results of the experiment during which the temperature has been kept practically constant. Due to that the pure diffusion problem has been considered. Assuming the homogeneity of the humidity field around the beam, the water content is constant along the beam axis thus only the distribution of the water content over the cross–section needs to be evaluated. The obtained results are used as input data in the second phase of analysis in which the mechanical response of the beam to the mechanical load and changing humidity is determined.

Experimental methods

Long–term tests of straight glulam beams (size $5 \times 10 \times 180\text{cm}^3$) in different constant and changing climatic conditions were completed in January 2000.

The specimens were made from European spruce (*Picea abies*) and glued with resorcin–phenol–formaldehyde adhesive. Each beam consists of 7 laminations (the thickness of the inner laminations was 16 mm and the thickness of the outer laminations was 10 mm). Some control specimens were kept at constant humidity, others were exposed to changing humidity but were not loaded. The dimensions of the specimen and loading are shown in Fig. **??**. The modulus of elasticity in static bending was measured on six specimens at short term bending test. The average value was 13850 MPa.

Figure 1

Specimens were exposed to four point bending load as shown in Fig. **??**. The acting force was 2×1.525 kN, so that the maximum stress at the mid-span of the beam was 10 MPa in tension and in compression. The relative humidity was set to 95% for the period of one week (or two or four weeks), then to 65% for another week (or two or four weeks). These cycles were repeated four times. The deflection of the beam at the mid-span was measured during the experiment. In addition, some special specimens kept unloaded in the same climatic conditions were periodically cut and the water contents in the outer layer and in the core were measured by ordinary gravimetric method. Some details concerning the experiment have already been published (Srpčič et al. 2000). The experiment was performed at the Slovenian National Building and Civil Engineering Institute in Ljubljana.

Water content distribution

The water transfers through the porous media predominantly by diffusion. Therefore, the distribution of the water content was estimated by solving the partial diferential equation of transient moisture diffusion through the cross-section $\mathscr{A}$ of the beam

$$\mathscr{A}: \qquad \frac{\partial}{\partial x_i}\left(D_{ij}\frac{\partial w}{\partial x_j}\right) - \frac{\partial w}{\partial t} = 0\,. \tag{1}$$

There w and D_{ij} are water content (kg/m^3) and diffusion coefficient (m^2/s), respectively. In order to solve equation (**??**) the initial and boundary conditions have to be given. Initial conditions are defined by

$$\mathscr{A}: \qquad w(x, y, z, 0) = w_0(x, y, z). \tag{2}$$

Boundary conditions on the countoure $\mathscr{C}$ of the cross-section are in general given by the following equations

$$\mathscr{C}: \qquad D_{ij}\frac{\partial w}{\partial x_j} n_i - q_w = 0 \qquad \text{or} \qquad w = w_p \tag{3}$$

where q_w, n_i, and w_p are water flow through the boundary surface (kg/m^{2}s), components of the normal to the boundary surface, and prescribed water content at the boundary, respectively. If we assume isotropy, equation (**??**) simplify to the following form

$$\mathscr{A}: \qquad \frac{\partial}{\partial y}\left(D\frac{\partial w}{\partial y}\right) + \frac{\partial}{\partial z}\left(D\frac{\partial w}{\partial z}\right) = \frac{\partial w}{\partial t} \tag{4}$$

with corresponding boundary conditions $\mathscr{C}$:

$$D\frac{\partial w}{\partial y} e_{ny} + D\frac{\partial w}{\partial z} e_{nz} = q_w \qquad \text{or} \qquad w = w_p. \tag{5}$$

The water flow boundary condition (**??**) or (**??**) can be simplified by the assumption that the water flow is a linear function of the difference between water content at the boundary w_S and equivalent water content in the surrounding air w_A, which depends on relative humidity of surrounding air and the type of wood, i. e.,

$$\mathscr{A}: \qquad q_w = S\,(w_A - w_S), \tag{6}$$

where S is surface emissivity.

The equivalent ambient water content depends on the relative humidity of the surrounding air and type of wood used in the experiment. Equation (**??**) or (**??**) with corresponding initial and boundary conditions is generally non-linear and can be rarely solved analytically. Therefore, numerical methods have to be employed. In our case the computer program HUMID (Hribar, 2000) based on finite element method was used to solve 2-D diffusivity problem.

When the diffusion problem of a beam is modelled by equation (4) with the boundary condition (5) the estimation of material parameters D and S and equivalent surrounding air water content w_A is of essential importance. If the beam is protected against the climatic impact by some special protective coating the effect of the protection may be included in surface emissivity S or some additional finite elements need to be added. A specific problem which needs to be addressed in modelling diffusion in a glulam beam is the effect of glue on water content distribution. There are two limiting cases: (*i*) glue has the same permeability as wood, and (*ii*) glue has zero permeability, which means that water content distributions in all laminates are mutually independent. The reality is somewhere between these limiting cases.

Mechanical analysis

The basic assumption in mechanical analysis of beam elements is that imaginary longitudinal filaments of an element are exposed to uniaxial stress state. This assumption significantly facilitates the task because the results of uniaxial tests can be directly used for the formulation of constitutive relations. This means that we are dealing with physical values of stresses and strains which refer, in the sense of the Lagrange description, to the initial non–deformed state of the element.

In order to consider the geometrical and material non-linear behaviour of an element, the relation between strain ε, moisture w, longitudinal normal stress σ and time t shall be expressed in an incremental form.

$$d\sigma = d\sigma(\sigma_0, \varepsilon_0, w_0, d\varepsilon, dw, dt). \tag{7}$$

In this work the additive principle is adopted in which the total geometrical strain increment $d\varepsilon$ is expressed as a sum of shrinkage/swelling $d\varepsilon_s$, normal creep $d\varepsilon_c$, mechano–sorptive $d\varepsilon_{ms}$ and mechanical strain increment $d\varepsilon_m$

$$d\varepsilon = d\varepsilon_s + d\varepsilon_c + d\varepsilon_{ms} + d\varepsilon_e\,. \tag{8}$$

Due to incremental approach and assuming that all the values involved are sufficiently small, in our numerical evaluation the infinitesimal stress and strain increments as well as the increments of water content and time are replaced by the finite ones

$$\mathit{\Delta}\varepsilon = \mathit{\Delta}\varepsilon_s + \mathit{\Delta}\varepsilon_c + \mathit{\Delta}\varepsilon_{ms} + \mathit{\Delta}\varepsilon_m\,. \tag{9}$$

Shrinkage and swelling deformation is assumed to be a linear function of water content. Thus the increment of shrinkage/swelling deformation is

$$\varDelta\varepsilon_s = \alpha_s \varDelta w \,, \tag{10}$$

where α_s is a constant shrinkage coefficient parallel to the grain at the actual constant temperature.

Normal creep depends on the time only, i.e. the changes of moisture content have no effect on normal creep. Various creep models for wood can be found in the literature. In our research two of them are considered in detail and included into the present paper.

Model A (Fig. **??**a) represents the so called standard solid which is described by viscoelastic rule

$$\varepsilon_c = \sigma_0 \left(a_1 e^{-a_2 t} + a_3 \right) \,, \tag{11}$$

where $a_1 = -\dfrac{1}{E_2}$, $a_2 = \dfrac{E_2}{\mu_3}$, and $a_3 = \dfrac{E_1 + E_2}{E_1 E_2}$.

However, in practice there is no need to determine the rheological parameters E_1 , E_2 and μ_3 but the parameters a_1 , a_2 , a_3 shall be directly calibrated according to experimental results. σ_0 is the stress at the beginning t_0 of the time step $\varDelta t = t_1 - t_0$. By this means the normal creep strain increment is

$$\varDelta\varepsilon_c = \sigma_0 a_1 \left(e^{-a_2 t_1} - e^{-a_2 t_0} \right) \,. \tag{12}$$

Figure 2

Model B, shown in Fig. **??**b, is described mathematically by the following equation

$$\varDelta\varepsilon_c = \sigma_0 \Phi_0^r \sum_{i=1}^{6} \Phi_i \left(1 - e^{-\varDelta t/\tau_i} \right) ,$$

where Φ_i and τ_i are the final compliances and retardation times of Kelvin elements, approximated on the basis of experiments. These elements are assembled by the assumption that Boltzmann principle is valid. Φ_0^r is the reference compliance relative to elastic compliance, and σ_0 is the stress at the beginning of the time step.

The increment of mechano-sorptive deformation is expressed by

$$\varDelta\varepsilon_{ms} = \sigma_0 \, \Phi^{\infty} \left(1 - e^{-c|\varDelta w|} \right) , \tag{13}$$

where c is generally different for sorption and desorption ($c^+ \neq c^-$), and Φ^∞ is the reference compliance. The increment of mechanical strain $\varDelta\varepsilon_m$ which does not explicitly depend on time and water content is obtained by subtraction of shrinkage/swelling-, viscous-, and mechano–sorptive part from the increment of total strain. The mechanical part of the deformation consists of elastic part only, i.e. plasticity effect is negligible

$$\varDelta\varepsilon_m = \varDelta\varepsilon_e = \varDelta\varepsilon - \varDelta\varepsilon_s - \varDelta\varepsilon_c - \varDelta\varepsilon_{ms} \tag{14}$$

On the other hand, based on Hooke's law, the increment of elastic strain can be expressed as follows

$$\varDelta\varepsilon_e = \frac{1}{E_1}\left(\varDelta\sigma - \varDelta E\,\varepsilon_{e0}\right). \tag{15}$$

In equation (16) as well as in following equations subscripts "$_0$" and "$_1$" denote the quantities at the begining and at the end of the time step respectivelly. In equation (16) the elastic strain increment $\varDelta\varepsilon_e$ depends on the stress increment $\varDelta\sigma$ and also on the modification of elastic modulus $\varDelta E$. Namely, the elastic modulus $E(w)$ belonging to water content w refers to the reference modulus E_{ref} by linear rule

$$E(w) = E_{ref}(1 - c_E\,w). \tag{16}$$

Comparing equations (**??**) and (**??**) the stress increment $\varDelta\sigma$ can be expressed in a simple form

$$\varDelta\sigma = E_1\,\varDelta\varepsilon_e + \varDelta E\,\varepsilon_{e0} \tag{17}$$

and the stress σ_1 at the end of the time step is

$$\sigma_1 = \sigma_0 + \varDelta\sigma. \tag{18}$$

Equations (**??**–**??**) represent a specific constitutive model which was incorporated into computer program NonWood. The program is based on the finite element method , which enables geometrically and materially non-linear analysis of planar beams and frames. Kinematic equations used in the formulation of this element allow consideration of large displacements and rotations and moderate deformations. The basic equations were developed by the mixed variational principle in which, besides the transversal displacements and rotations, the axial force is taken as an independent parameter. This element improves the convergence of numerical procedures involved in mechanical analysis.

Numerical example

The results obtained by numerical evaluations were compared to those measured during experiments, which were performed at the Slovenian National Building and Civil Engineering Institute. The geometry and the mechanical load are shown in Fig. **??**. The temperature was kept constant at 21°C. The relative humidity of surrounding air changed in two, four, or eight weeks cycles (the results of eight weeks cycles are shown in Fig. **??**). The material parameters governing the water content distribution were determined after literature survey. The values used in numerical estimation are shown in Table **??**.

Table 1

The diffusion problem was solved by the computer program HUMID, where the rectangular cross-section was modelled by 800 finite elements using uniform size grid.

The results of water content distributions are shown in Fig. **??** and Fig. **??**. The measured results are represented by squares for the edge and by triangles for the core. In Fig. **??** the average water contents in the core and at the edges are shown. These values are compared to water content measured by gravimetrical method. Only the results for the eight weeks cycles are shown here.

As it can be seen from Fig. **??** where computed values are presented by solid lines, the numerical results are relatively close to the measured values of water content. A few points, where the discrepancy is more apparent, can be explained by experimental error. Generally the difference is below 1% of water content.

Figure 3

The literature survey (see e.g. Brewis et al. (1987), Schultz and Kelly (1980) and Šega et al. (2005)) revealed that there are no conclusive evidence about the influence of glue on permeability of glulam elements. The results obtained by different authors differ considerably. Therefore, at the next stage the influence of the permeability of the glue is examined by introducing two extreme models of the glue contact. In the first case it is assumed that the glue has the same permeability as wood. In the second it is assumed that the glue is impermeable. The thin lines in Fig. **??** correspond to the impermeable glue model. The differences between the two cases of glue permeability are more evident for the core area whereas for the edge area they are negligible. The differences shown in Fig. **??** for the two cases are less pronounced also because only the average over the area is taken in consideration.

The distributions of the moisture for both cases reveal that at some points of the cross section the differences are quite large. Four characteristic time steps are chosen: 63rd day at the beginning of the sorption phase, 77th day at the middle of the sorption phase, 91st day at the beginning of the desorption phase, and 105th day at the middle of the desorption phase. The moisture distributions for both glue permeability models are shown in Fig. **??**. The differences between the two cases are clearly visible in the outer laminates at the top and at the bottom of the beam. In the inner laminates the differences are almost negligible. In the second laminates from top and bottom the differences are up to 1.5% of water content whereas the differences in the inner three laminates are less than 0.5% of water content.

Figure 4

Water content distribution was used as input data for mechanical analysis, which was performed by computer program NonWood. The material parameters are shown in Table **??**.

Table 2

In Table **??** the values of the parameters of the two creep models used in mechanical analysis are shown.

Figure 5

Table 3

Fig. **??** shows the calculated stress distributions in the middle cross-section of the beam. Due to shrinkage, mechano-sorptive effect and non-linear water content distribution significant redistribution of longitudinal normal stress occurs in the cross-section. This effect is illustrated by the difference between the actual stress σ and linear stress which would develop if only mechanical load was applied (Fig. ??). The differences between the stresses are as high as 25% at the corners of the cross-section. There are no significant differences in stress distribution for both cases of glue permeability. This may be attributed to the computational error caused by the fact that the values of moisture were averaged for six characteristic parts of the cross-section, and these average values were used in the mechanical analysis.

Figure 6

When observing the computational results for stress distributions at characteristic time points it is interesting that considerable differences between the actual and linear steresses are noted at the 77th and 91st day which correspond to the second half of the sorption phase while at the 63th and 105th day the differences are less perceivable.

Displacements at the mid-span of the beam obtained by the computer program NonWood using two normal creep models and two glue permeability models were compared to displacements obtained by the experiment (Fig. ??). All models exhibit very good agreement with experimental data for up to 90 days. Afterwards the experimental displacements decreased considerably which could not be accounted for by the mathematical model employed. The shape of the experimental displacement curve indicates that the effects of water content changes are more pronounced during the desorption than during the sorption phase. Displacements are almost equal for both cases of glue permeability.

Figure 7

Figure 8

In Fig. ?? and ?? the development of longitudinal stress and different constituants of total strain over time are shown for two characteristic points at the top and at the bottom of the mid cross-section of the beam. The comparison between total, elastic, normal creep, shrinkage, and mechano-sorptive strains is very interesting. The partial effects of all these phenomenon can be assessed. Since our experiment involved relatively large specimens the mechano-sorptive effect was not as important as in the case of very small specimens reported in the literature.

Figure 9

Conclusions

The aim of the present work was to establish an adequate mathematical model for describing mechanical behaviour of glulam beams exposed to changing climatic conditions. The proposed material model combines constitutive laws which relate elastic, shrinkage, normal creep and mechano-sorptive strains to water content changes in wood. The comparison between numerical and experimental results indicates that it is possible to obtain a successful mathematical model for moisture diffusion as well as for mechanical behaviour. The analysis of contributions of different strains to the total strain implies that all effects induced by water content changes are more pronounced in beams with relatively small cross-sections. The main problem in mechanical analysis remains the experimental evaluation and verification of parameters involved in numerical procedures.

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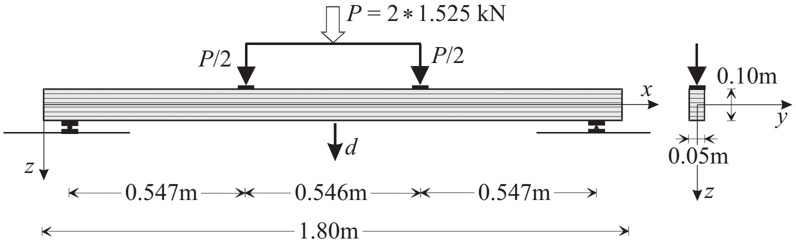


Figure 1: Dimensions of the specimen.

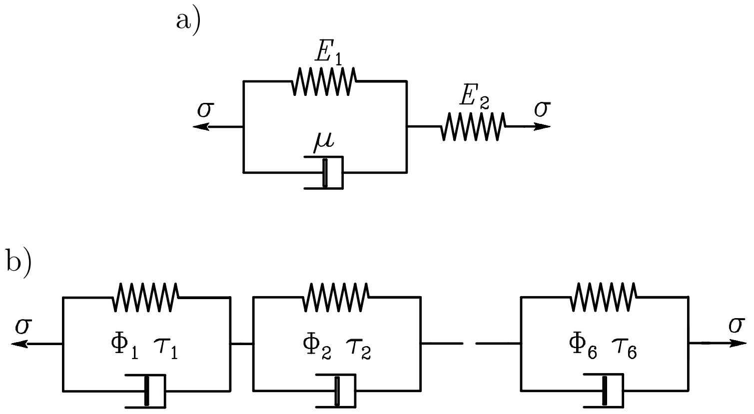


Figure 2: a) Creep model A; b) Creep model B

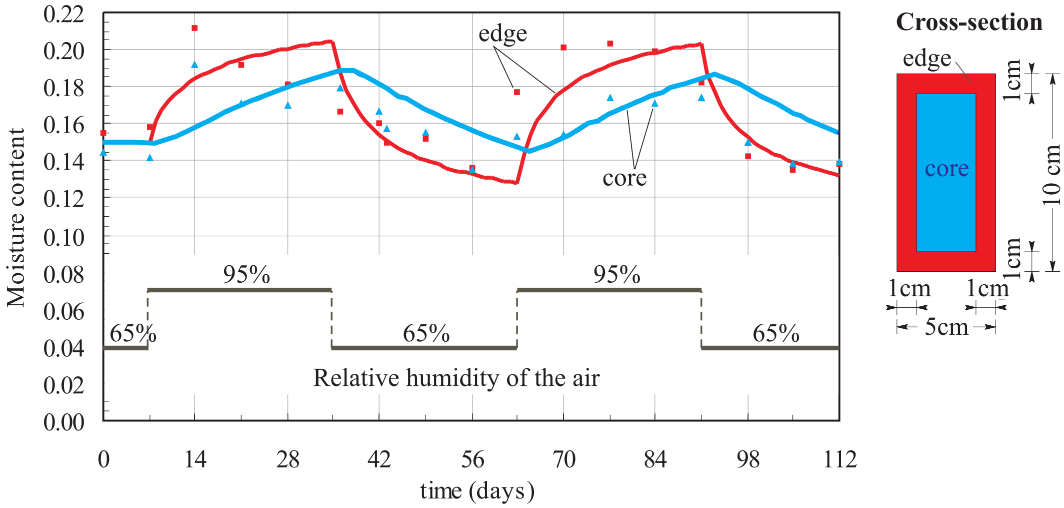
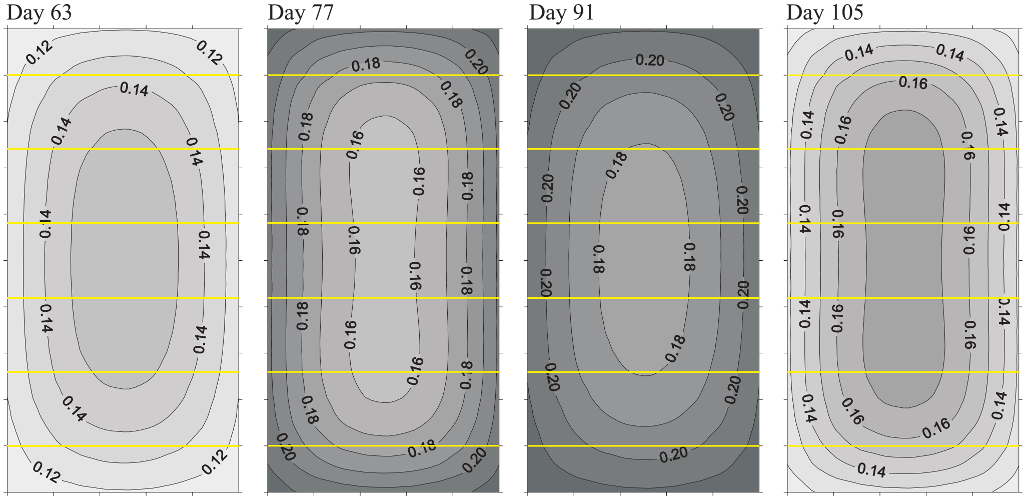


Figure 3: Water content in two zones of the cross-section of the beam.

Glue with the same permeability as wood



Glue with zero permeability

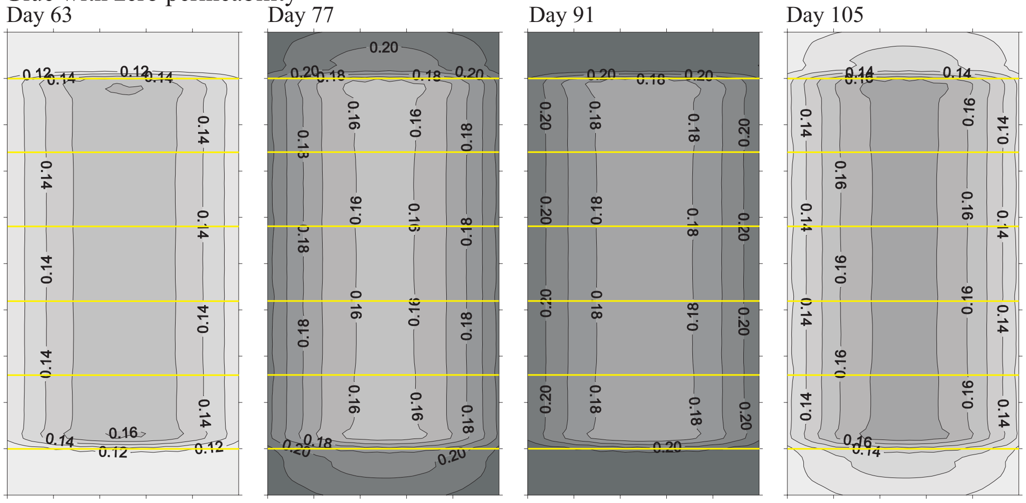
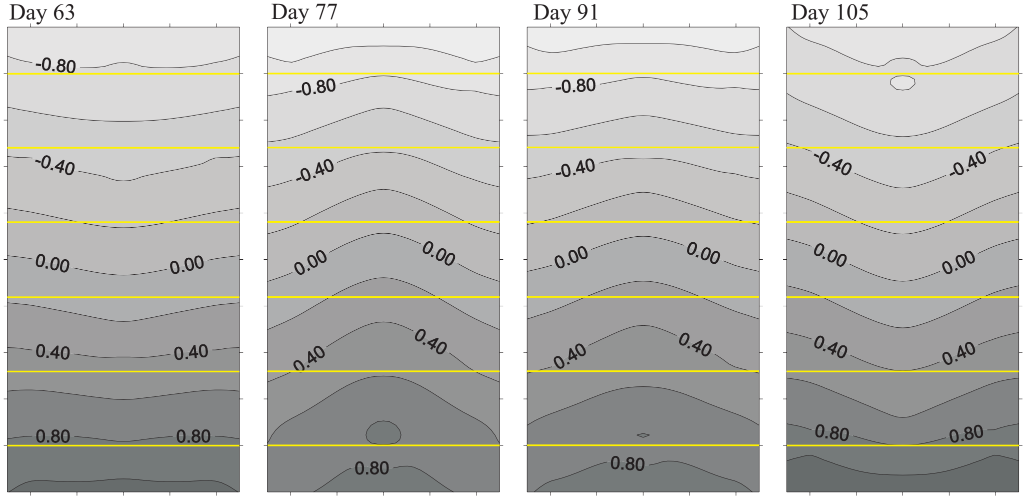
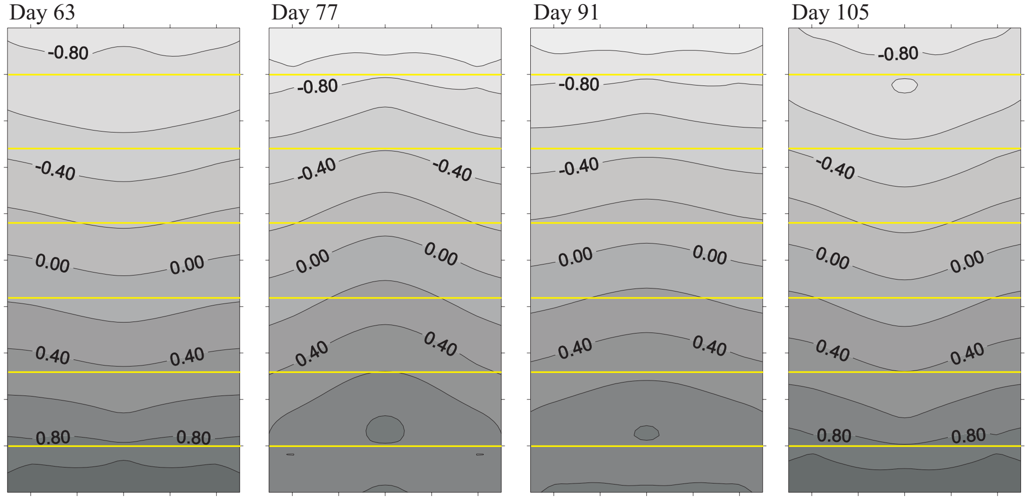


Figure 4: Moisture distribution.


Glue with the same permeability as wood

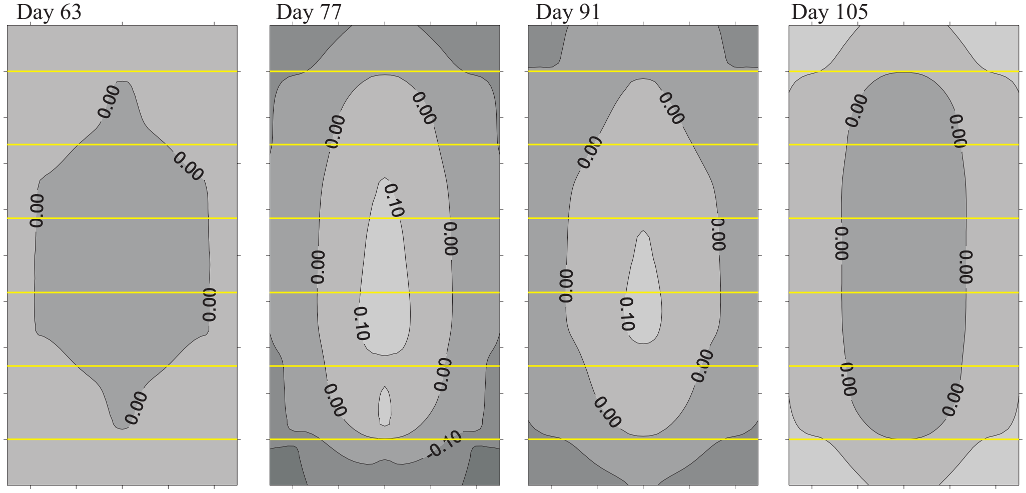
Day 63 Day 77 Day 91 Day 105


Glue with zero permeability

Day 63 Day 77 Day 91 Day 105

Figure 5: Stress distribution (creep model A).

Glue with the same permeability as wood



Glue with zero permeability

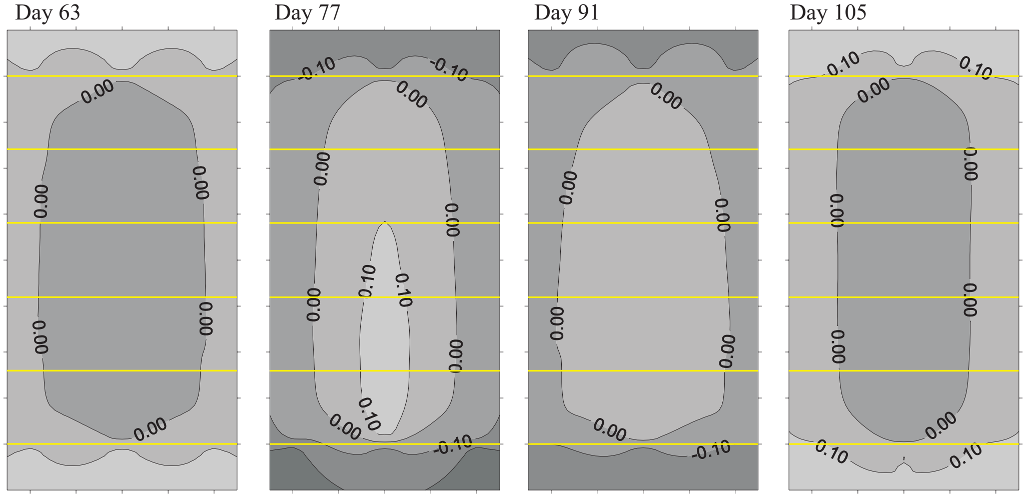


Figure 6: Stress differences (creep model A).

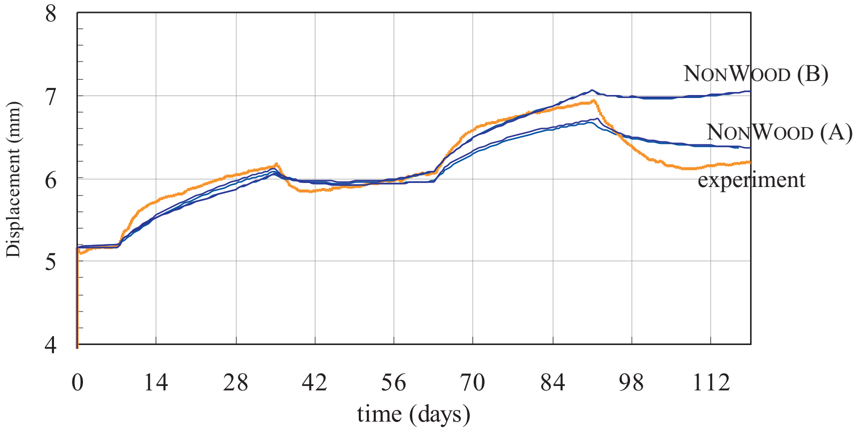


Figure 7: Displacements at the mid-span.

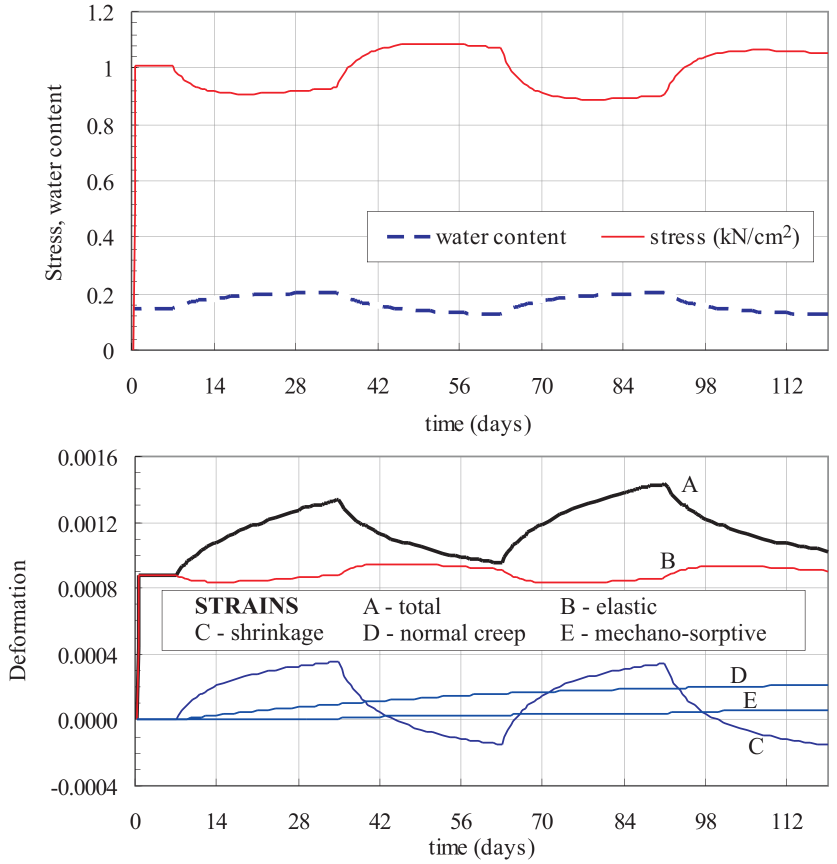


Figure 8: Stress and deformations at the bottom of the beam.

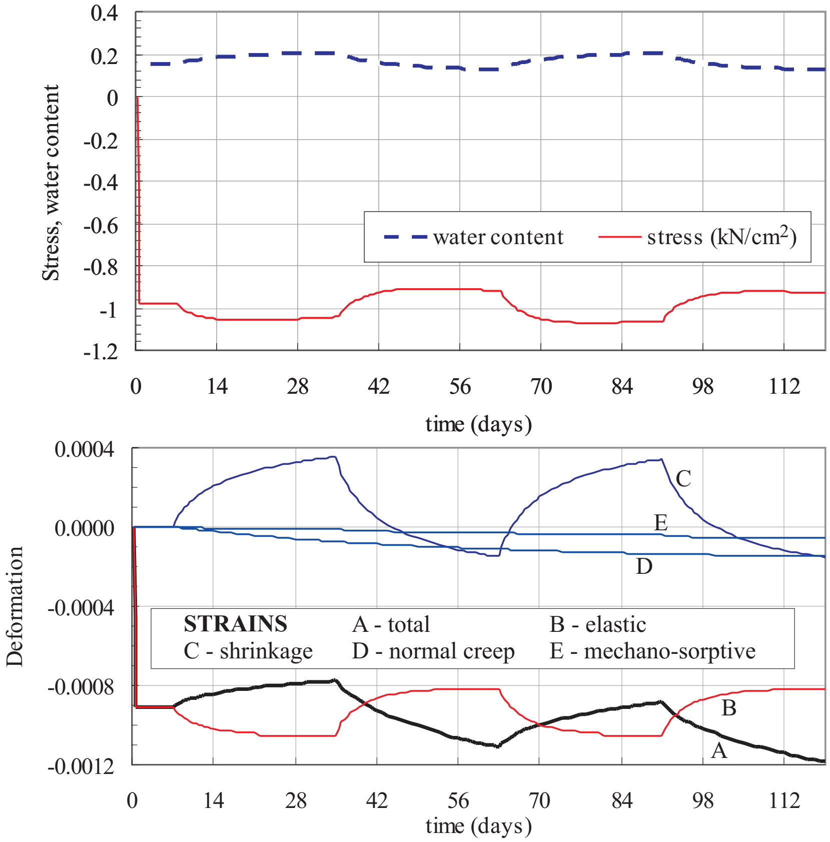


Figure 9: Stress and deformations at the top of the beam.

Table 1: Data for water content evaluation

diffusivity	$D = 10^{-10}\,\mathrm{m}^2/\mathrm{s}$
surface emissivity	$S = 2 \cdot 10^{-8}\,\mathrm{m/s}$
equivalent water content (95% relative humidity)	$w_A = 0.22$
equivalent water content (65% relative humidity)	$w_A = 0.11$

Table 2: Material parameters

elastic modulus (tension)	$E_{ref} = 13850\,\mathrm{MPa}$
elastic modulus (compression)	$E_{ref} = 13000\,\mathrm{MPa}$
elastic modulus parameter	$c_E = 1.15$
shrinkage parameter	$\alpha_s = 6.25 \cdot 10^{-5}/w[\%]$
mechano-sorptive parameter	$\Phi^{\infty} = 0.0001$
mechano-sorptive parameter (sorption)	$c^{+} = 1.6$
mechano-sorptive parameter (desorption)	$c^{-} = 2.4$

Table 3: Creep parameters for two normal creep models

Model A			
	$a_1 \cdot 10^4$	$a_2 \cdot 10^2$	$a_3 \cdot 10^4$
tension	-0.2719	1.975	0.9927
compression	-0.1820	2.017	1.0080

Model B							
i	1	2	3	4	5	6	Φ_0^r
Φ_i	0.0686	-0.0056	0.0716	0.0404	0.2073	0.5503	0.000018
τ_i	0.01	0.1	1.0	10.0	100	5000	